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Introduction: General Overview of Calculus

Calculus as a subject is fundamentally different from the topics of math that came before it. Where Arithmetic, Algebra, Geometry, and even Trigonometry deal with what is, Calculus deals with what might be. At its core, Calculus is founded on something that you were told in previous math classes could not be done: dividing by zero and working with infinity. These could not be done in the context of earlier math, but that does not mean they cannot be done at all. We just need to expand our horizons to include the possibility that they can be done and then figure out how.

Since the advent and widespread use of the graphing calculator, there has been a split within the mathematical community over the nature and accessibility of the Calculus. So-called "traditional calculus" emphasizes the algebraic aspects of the topic and considers Calculus to be the study of functions, as Euler did in the 17th century. As such it is an extension of analytic geometry into the real of analysis that requires rigorous use of theorems and proofs as well as long intricate algebraic manipulations. This view has led many to consider college calculus classes to be a gatehouse, or weeding-out, class that restricts access to science, pre-med, and engineering programs.

The Reform Calculus Movement of the 1990s emphasizes open access to the Calculus by highlighting understanding of the basic concepts and implications and by de-emphasizing the rigor. The graphing calculator can make the algebra superfluous. This approach tends to view Calculus as the study of change.

As with any dichotomous situation, the answer is somewhere in between. Both positions are correct and certainly are not mutually exclusive. In good Ignatian tradition, we will take a flexible approach to the subject and prepare the student for whichever situation they might face in future courses.

There are four operations involved in Calculus:

- Limits
- Derivatives
- Indefinite integrals (or Anti-Derivatives)
- Definite Integrals

Limits are the foundation of the other three and are what was referred to above in terms of "what might be." A limit is an operation that allows one to consider what previously had been numerical impossibilities, like dividing by zero. It is an extrapolation of known values to a logical conclusion.

The derivative is a function that yields the slope of the tangent line. If the line is an expression of distance and time, the slope becomes a rate of change. In other words, the context expands the implications.

The indefinite integral is, as noted above, an anti-derivative, meaning it is the inverse operation of the derivative. This is one of the ideas adopted by AP from the Reform Calculus. Older calculus books do not use the phrase "anti-derivative" and emphasize the link between the two kinds of integrals. Traditional calculus tends to break the Calculus into differential calculus and integral calculus, a distinction still found in many college programs. The indefinite integral is the bridge between the derivatives and definite integrals.

The definite integral is an operation that yields the accumulation of area under a curve. This is an area where the graphing calculator has had a high impact.

How does this relate to the AP Calculus syllabus? The AB and BC designations for the AP calculus tests hearken back to the old college Calculus 1A, 1B and 1C courses. The 1A material is comprised of the limits and derivatives. The 1B class covered basics indefinite and definite integrals with their applications. The 1C course covered advanced integration techniques, parametric and polar mode, and sequences and series. The BC Calculus test (and class) actually cover all three areas—A, B, and C—which is why passing the test carried one year of credit. (The 1A, 1B and 1C used to be quarter classes, not semester classes.)

Course Goals

There are two main goals for the course:

1. Learn the subject of the Calculus.
2. Pass the AP test.

These goals are not identical. It is possible to know the subject and not do well on the test due to the particular emphases and vocabulary of the test. It is possible to do well on the test with a modicum of knowledge and an arsenal of test-taking skills.

There is a third, more important goal:

3. Begin to think about your own learning process.

We teachers must regrettably acknowledge that, while you are all capable of success in higher mathematics, this may be the last math class you take. By preparing you for advanced calculus, we are keeping your college options open, but we know your interests might lie elsewhere in the future. Therefore, we want you to understand that the material is somewhat secondary to the process. We would like for you to become life-long learners of whatever subjects interest you, including math, and know you have the tools to explore any field that peaks your interest.

Enduring Understandings

What does this third goal mean in terms of the subject of Calculus? What do we really want you to carry forward? What understanding do we hope you achieve and maintain?

1. At a basic level, you will become conversant in the language and notation of math. You will be able to translate mathematical concepts into "normal" language and vice versa.

2. You will be able to reason logically through a problem.

3. You will begin to see the underlying structure of the mathematical system so as to recognize the assumptions, flaws, and logical conclusions. You may begin to see alternatives to the system. Ideally, you will recognize this systemic approach in other fields and apply the same analysis to them.

4. You will hopefully go beyond the utilitarian mode of learning ("When are we going to use this?" or "Is this going to be on the test?") and begin to learn new things for the sake of learning them. Ultimately, you might begin to appreciate the power and the intrinsic beauty people find in the material.

5. You will begin to develop a meta-cognitive approach to learning. That is, you will begin to think about your thinking process and how that affects your learning.

6. You will develop a confidence and tenacity when approaching lengthy and intricate math problems. By breaking down a problem into its component parts, analyzing and resolving each part, and then reassembling the whole, you will develop a sense that no problem is beyond your grasp.

7. You will develop the ability to manage large amounts of information by learning to prioritize time, tasks, and goals.

The AP program has set forward nine Goals in their course description for Calculus that are more material-oriented:

- Students should be able to work with functions represented in a variety of ways: Graphical, numerical, verbal, and analytic (algebraic). They should understand the connections between these representations.
- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as a net accumulation of change and should be able to use integrals to solve a variety of problems.
- Students should understand the relationship between the derivative and the definite integral as expressed in the Fundamental Theorem of Calculus.
- Students should be able to communicate mathematics and explain solutions to problems both verbally and in written sentences.
- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
- Students should be able to use technology to help solve problems, experiment, interpret results, and support conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measure.
- Students should develop an appreciation of Calculus as a coherent body of knowledge and as a human accomplishment.

The first eight are tested in very specific ways on the AP exam. Obviously, this last one is impossible to test. In order for us to achieve OUR first two goals, we must address the AP goals in the manner in which they will be tested. Certain sections of this book are written in a manner that should help us to achieve this.