Polynomial Homework Answer Key

5-1 Free Response

1. $$\lim_{x\to 3}\frac{x^2+x-12}{x^2-9}=\frac{0}{0}$$

$$=\lim_{x\to 3}\frac{(x-3)(x+4)}{(x-3)(x+3)}=\lim_{x\to 3}\frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}(x+3)}=\lim_{x\to 3}\frac{x+4}{x+3}=\frac{7}{6}$$

3. $$\lim_{x\to 2}\frac{x^2+4x-12}{3x-1}=\frac{0}{5}=0$$

5. $$\lim_{x\to -2}\frac{4x+8}{x^2+6x+8}=\frac{0}{0}$$

$$=\lim_{x\to -2}\frac{4(x+2)}{(x+2)(x+4)}=\lim_{x\to -2}\frac{4\cancel{(x+2)}}{\cancel{(x+2)}(x+4)}=\lim_{x\to -2}\frac{4}{x+4}=2$$

7. $$\lim_{x\to 1}\frac{x^3-1}{x^2+3x-4}=\frac{0}{0}$$

$$=\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)(x+4)}=\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+4)}=\lim_{x\to 1}\frac{x^2+x+1}{x+4}=\frac{3}{5}$$

9. $$\lim_{x\to 3/2}\frac{2x^3+x^2-8x+3}{6x^3-7x^2-7x+6}=\frac{0}{0}$$

Use synthetic division or your calculator to factor the numerator and denominator.

$$\begin{array}{r|rrrr} 3/2 & 2 & 1 & -8 & 3 \\ & & 3 & 6 & -3 \\ \hline & 2 & 4 & -2 & \cancel{0} \end{array}$$

$$\Rightarrow \left(x-3/2\right)\left(2x^2+4x-2\right)$$

$$=2(2x-3)\left(x^2+2x-1\right)$$

$$
\begin{array}{c|cccc}
\tfrac{3}{2} & 6 & -7 & -7 & 6 \\
 & & 9 & 3 & -6 \\
\hline
 & 6 & 2 & -4 & \cancel{0}
\end{array}
$$

$$\Rightarrow \left(x - \tfrac{3}{2}\right)\left(6x^2 + 2x - 4\right)$$

$$= 2(2x-3)\left(3x^2 + x - 2\right)$$

$$= \lim_{x \to 3/2} \frac{2(2x-3)\left(x^2+2x-1\right)}{2(2x-3)\left(3x^2+x-2\right)}$$

$$= \lim_{x \to 3/2} \frac{\cancel{2(2x-3)}\left(x^2+2x-1\right)}{\cancel{2(2x-3)}\left(3x^2+x-2\right)}$$

$$= \lim_{x \to 3/2} \frac{x^2+2x-1}{3x^2+x-2}$$

$$= \frac{17}{25}$$

11. $$\lim_{x\to\sqrt{2}} \frac{x^4+4x^2-12}{x^3+x^2-2x-2} = \frac{0}{0}$$

$$= \lim_{x\to\sqrt{2}} \frac{\left(x^2-2\right)\left(x^2+6\right)}{x^2\left(x+1\right)-2\left(x+1\right)}$$

$$= \lim_{x\to\sqrt{2}} \frac{\left(x^2-2\right)\left(x^2+6\right)}{\left(x+1\right)\left(x^2-2\right)}$$

$$= \lim_{x\to\sqrt{2}} \frac{\cancel{\left(x^2-2\right)}\left(x^2+6\right)}{\left(x+1\right)\cancel{\left(x^2-2\right)}}$$

$$= \lim_{x\to\sqrt{2}} \frac{x^2+6}{x+1}$$

$$= \frac{8}{\sqrt{2}+1}$$

13. $$\lim_{x\to 0} \frac{\sqrt{5-x}-\sqrt{5}}{x} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\sqrt{5-x}-\sqrt{5}}{x} \bullet \frac{\sqrt{5-x}+\sqrt{5}}{\sqrt{5-x}+\sqrt{5}}$$

$$= \lim_{x\to 0} \frac{5-x-5}{x\left(\sqrt{5-x}+\sqrt{5}\right)}$$

$$= \lim_{x\to 0} \frac{\cancel{5}-x\cancel{-5}}{x\left(\sqrt{5-x}+\sqrt{5}\right)}$$

$$= \lim_{x\to 0} \frac{-\cancel{x}}{\cancel{x}\left(\sqrt{5-x}+\sqrt{5}\right)}$$

$$= \lim_{x\to 0} \frac{-1}{\sqrt{5-x}+\sqrt{5}}$$

$$= -\frac{1}{2\sqrt{5}}$$

15. $\lim_{x\to 1}\frac{1-x^2}{x^3-3x+2}=\frac{0}{0}$

Use synthetic division or your calculator to factor the denominator.

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \not{0} \end{array}$$

$\Rightarrow (x-1)(x^2+x-2)$

$$=\lim_{x\to 1}\frac{(1-x)(1+x)}{(x-1)(x^2+x-2)}=\lim_{x\to 1}\frac{-\cancel{(x-1)}(1+x)}{\cancel{(x-1)}(x^2+x-2)}=\lim_{x\to 1}\frac{-(1+x)}{x^2+x-2}=\frac{-2}{0}=\text{DNE}$$

17. $\lim_{x\to -1}\frac{(1-x^2)^2}{x^3-2x^2-3x}=\frac{0}{0}$

$$=\lim_{x\to -1}\frac{(1-x^2)(1-x^2)}{x(x^2-2x-3)}=\lim_{x\to -1}\frac{(1-x)(1+x)(1-x)(1+x)}{x(x-3)(x+1)}=\lim_{x\to -1}\frac{(1-x)\cancel{(1+x)}(1-x)(1+x)}{x(x-3)\cancel{(x+1)}}$$

$$=\lim_{x\to -1}\frac{(1-x)(1-x)(1+x)}{x(x-3)}=\frac{0}{4}=0$$

19. $\lim_{x\to -1}\frac{x^4-1}{x^4+x^3+x^2+2x+1}=\frac{0}{0}$

Use synthetic division or your calculator to factor the denominator.

$$\begin{array}{c|ccccc} -1 & 1 & 1 & 1 & 2 & 1 \\ & & -1 & 0 & -1 & -1 \\ \hline & 1 & 0 & 1 & 1 & \not{0} \end{array}$$

$\Rightarrow (x-1)(x^3+x+1)$

$$= \lim_{x \to -1} \frac{(x^2-1)(x^2+1)}{(x+1)(x^3+x+1)} = \lim_{x \to -1} \frac{(x-1)(x+1)(x^2+1)}{(x+1)(x^3+x+1)} = \lim_{x \to -1} \frac{(x-1)\cancel{(x+1)}(x^2+1)}{\cancel{(x+1)}(x^3+x+1)}$$

$$= \lim_{x \to -1} \frac{(x-1)(x^2+1)}{x^3+x+1} = \frac{-4}{-1} = 4$$

5-1 Multiple Choice

1. B

$$\lim_{x \to a} \frac{x^2-a^2}{x^4-a^4} = \frac{0}{0}$$

$$= \lim_{x \to a} \frac{x^2-a^2}{(x^2-a^2)(x^2+a^2)}$$

$$= \lim_{x \to a} \frac{\cancel{x^2-a^2}}{\cancel{(x^2-a^2)}(x^2+a^2)}$$

$$= \lim_{x \to a} \frac{1}{x^2+a^2}$$

$$= \frac{1}{2a^2}$$

3. A

$$\lim_{x \to 0} \frac{\frac{1}{x-1}+1}{x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\frac{1}{x-1}+1 \bullet \frac{x-1}{x-1}}{x} = \lim_{x \to 0} \frac{\frac{1+x-1}{x-1}}{x} = \lim_{x \to 0} \frac{\frac{x}{x-1}}{x} = \lim_{x \to 0} \frac{x}{x-1} \div x$$

$$= \lim_{x \to 0} \frac{x}{x-1} \bullet \frac{1}{x} = \lim_{x \to 0} \frac{\cancel{x}}{x-1} \bullet \frac{1}{\cancel{x}} = \lim_{x \to 0} \frac{1}{x-1} = -1$$

5. C

$$\lim_{x \to 8} \frac{x^2-64}{x-8} = \frac{0}{0}$$

$$= \lim_{x\to 8} \frac{(x-8)(x+8)}{x-8} = \lim_{x\to 8} \frac{\cancel{(x-8)}(x+8)}{\cancel{x-8}} = \lim_{x\to 8} x+8 = 16$$

5-2 Free Response

1. $$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 + 5(x+h) + 13 - (x^2 + 5x + 13)}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 5x + 5h + 13 - x^2 - 5x - 13}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{5x} + 5h + \cancel{13} \cancel{- x^2} \cancel{- 5x} \cancel{- 13}}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 + 5h}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h+5)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h+5)}{\cancel{h}}$$

$$= \lim_{h\to 0} (2x+h+5)$$

$$= 2x+5$$

$$\therefore f'(x) = 2x+5$$

3. $f'(x)=\lim_{h\to 0}\frac{(x+h)^3+4(x+h)-(x^3+4x)}{h}$

Use Pascal's Triangle to multiply $(x+h)^3$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3+4x+4h-x^3-4x}{h}$

$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3+\cancel{4x}+4h\cancel{-x^3}\cancel{-4x}}{h}$

$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3+4h}{h}$

$=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2+4)}{h}$

$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2+4)}{\cancel{h}}$

$=\lim_{h\to 0}(3x^2+3xh+h^2+4)$

$=3x^2+4$

$\therefore f'(x)=3x^2+4$

5. $f'(x) = \lim_{h \to 0} \frac{(x+h)^6 - x^6}{h}$

Use Pascal's Triangle to multiply $(x+h)^6$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$= \lim_{h \to 0} \frac{x^6 + 6x^5h + 15x^4h^2 + 20x^3h^3 + 15x^2h^4 + 6xh^5 + h^6 - x^6}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^6} + 6x^5h + 15x^4h^2 + 20x^3h^3 + 15x^2h^4 + 6xh^5 + h^6 \cancel{- x^6}}{h}$$

$$= \lim_{h \to 0} \frac{6x^5h + 15x^4h^2 + 20x^3h^3 + 15x^2h^4 + 6xh^5 + h^6}{h}$$

$$= \lim_{h \to 0} \frac{h\left(6x^5 + 15x^4h + 20x^3h^2 + 15x^2h^3 + 6xh^4 + h^5\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(6x^5 + 15x^4h + 20x^3h^2 + 15x^2h^3 + 6xh^4 + h^5\right)}{\cancel{h}}$$

$$= \lim_{h \to 0} \left(6x^5 + 15x^4h + 20x^3h^2 + 15x^2h^3 + 6xh^4 + h^5\right)$$

$$= 6x^5$$

$$\therefore f'(x) = 6x^5$$

7. $$f'(x)=\lim_{h\to 0}\frac{\sqrt{(x+h)^2-1}-\sqrt{x^2-1}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{(x+h)^2-1}-\sqrt{x^2-1}}{h}\bullet\frac{\sqrt{(x+h)^2-1}+\sqrt{x^2-1}}{\sqrt{(x+h)^2-1}+\sqrt{x^2-1}}$$

$$=\lim_{h\to 0}\frac{\left(\sqrt{(x+h)^2-1}\right)^2-\left(\sqrt{x^2-1}\right)^2}{h\left(\sqrt{(x+h)^2-1}+\sqrt{x^2-1}\right)}$$

$$=\lim_{h\to 0}\frac{(x+h)^2-1-(x^2-1)}{h\left(\sqrt{(x+h)^2-1}+\sqrt{x^2-1}\right)}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-1-x^2+1}{h\left(\sqrt{(x+h)^2-1}+\sqrt{x^2-1}\right)}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2\cancel{-1}\cancel{-x^2}+\cancel{1}}{h\left(\sqrt{(x+h)^2-1}+\sqrt{x^2-1}\right)}$$

$$=\lim_{h\to 0}\frac{2xh+h^2}{h\left(\sqrt{(x+h)^2-1}+\sqrt{x^2-1}\right)}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}\left(\sqrt{(x+h)^2-1}+\sqrt{x^2-1}\right)}$$

$$=\lim_{h\to 0}\frac{2x+h}{\sqrt{(x+h)^2-1}+\sqrt{x^2-1}}$$

$$=\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-1}}$$

$$\therefore f'(x)=\frac{x}{\sqrt{x^2-1}}$$

9. $f'(x)=\lim\limits_{h\to 0}\dfrac{(x+h)^3-x^3}{h}$

$=\lim\limits_{h\to 0}\dfrac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$

$=\lim\limits_{h\to 0}\dfrac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-x^3}}{h}$

$=\lim\limits_{h\to 0}\dfrac{3x^2h+3xh^2+h^3}{h}$

$=\lim\limits_{h\to 0}\dfrac{\cancel{h}\left(3x^2+3xh+h^2\right)}{\cancel{h}}$

$=\lim\limits_{h\to 0}\left(3x^2+3xh+h^2\right)$

$=3x^2$

$\therefore f'(x)=3x^2$

$\Rightarrow f'(3)=3(3)^2=27$

OR

$f'(3)=\lim\limits_{x\to 3}\dfrac{f(x)-f(3)}{x-3}$

$=\lim\limits_{x\to 3}\dfrac{x^3-27}{x-3}=\dfrac{0}{0}$

$=\lim\limits_{x\to 3}\dfrac{(x-3)(x^3+3x+9)}{x-3}$

$=\lim\limits_{x\to 3}\dfrac{\cancel{(x-3)}(x^3+3x+9)}{\cancel{x-3}}$

$=\lim\limits_{x\to 3}\left(x^3+3x+9\right)$

$=27$

$\therefore f'(3)=27$

11. $f'(x)=\lim_{h\to 0}\frac{(x+h)^3-1-(x^3-1)}{h}$

$=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-1-x^3+1}{h}$

$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-1}\cancel{-x^3}\cancel{+1}}{h}$

$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}$

$=\lim_{h\to 0}\frac{\cancel{h}\left(3x^2+3xh+h^2\right)}{\cancel{h}}$

$=\lim_{h\to 0}\left(3x^2+3xh+h^2\right)$

$=3x^2$

$\therefore f'(x)=3x^2$

$\Rightarrow f'(1)=3(1)^2=3$

OR

$f'(1)=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$

$=\lim_{x\to 1}\frac{x^3-1-0}{x-1}$

$=\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{x-1}$

$=\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}$

$=\lim_{x\to 1}\left(x^2+x+1\right)$

$=3$

$\therefore f'(1)=3$

13. $f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2} \frac{(x^2 + 4x + 3) - 15}{x - 2}$

$= \lim_{x \to 2} \frac{(x-2)(x+6)}{x - 2}$

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+6)}{\cancel{x-2}}$

$= \lim_{x \to 2} (x+6)$

$= 8$

$\therefore f'(2) = 8$

OR

$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h \to 0} \frac{(2+h)^2 + 4(2+h) + 3 - 15}{h}$

$= \lim_{h \to 0} \frac{4 + 4h + h^2 + 8 + 4h + 3 - 15}{h}$

$= \lim_{h \to 0} \frac{\cancel{15} + 8h + h^2 \cancel{-15}}{h}$

$= \lim_{h \to 0} \frac{8h + h^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(8+h)}{\cancel{h}}$

$= \lim_{h \to 0} (8+h)$

$= 8$

$\therefore f'(2) = 8$

15. $$f'(2)=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$$

$$=\lim_{x\to 2}\frac{x^3-8-0}{x-2}$$

$$=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x-2}$$

$$=\lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$$

$$=\lim_{x\to 2}(x^2+2x+4)$$

$$=12$$

$$f'(2)=12$$

OR

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{(2+h)^3-8-0}{h}$$

$$=\lim_{h\to 0}\frac{(2+h)(4+4h+h^2)-8}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{8}+12h+6h^2+h^3\cancel{-8}}{h}$$

$$=\lim_{h\to 0}\frac{12h+6h^2+h^3}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(12+6h+h^2)}{\cancel{h}}$$

$$=\lim_{h\to 0}(12+6h+h^2)$$

$$=12$$

$$\therefore f'(2)=12$$

5-2 Multiple Choice

1. A

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Let's try option (a):

$$f(2)=8(2)^3$$

$$f(2+h)=8(2+h)^3$$

$$\Rightarrow f'(2)=\lim_{h\to 0}\frac{8(2+h)^3-8(2)^3}{h}$$

See this fail for options (b)–(e)

Option (b):

$$f(8)=2(8)^3$$

$$f(8+h)=2(8+h)^3$$

$$\Rightarrow f'(8)=\lim_{h\to 0}\frac{2(8+h)^3-2(8)^3}{h}$$

$$f(-2)=8(-2)^3$$

$$f(-2+h)=8(-2+h)^3$$

Option (c):

$$\Rightarrow f'(-2)=\lim_{h\to 0}\frac{8(-2+h)^3-8(-2)^3}{h}$$

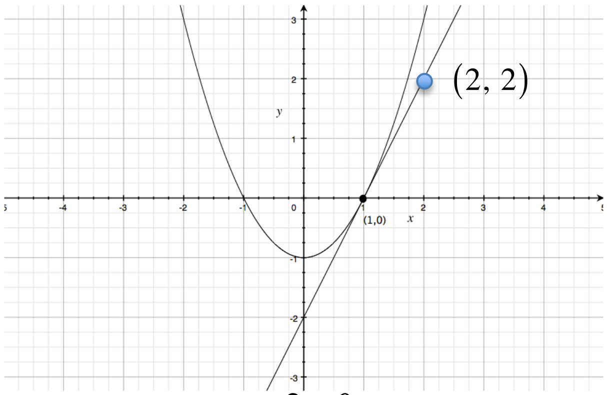
Option (d):

$$f(-8)=2(-8)^3$$

$$f(-8+h)=2(-8+h)^3$$

$$\Rightarrow f'(-8)=\lim_{h\to 0}\frac{2(-8+h)^3-2(-8)^3}{h}$$

3. C



$$f'(1) \approx m = \frac{2-0}{2-1} = 2$$

5. E

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\Rightarrow f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2} \frac{f(x^2 - 2) - 2}{x - 2} = \frac{0}{0}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

$$= \lim_{x \to 2} (x+2) = 4$$

5-3 Free Response

1. $D_x\left[4x^2 - 3x + 1\right] = 2(4x) - 3(1) + 0 \ = 8x - 3$

3. $\frac{d}{dx}\left[2x^3 + 16x^2 - 3x + 16\right] = 3(2x^2) + 2(16x) - 3(1) + 0 = 6x^2 + 32x - 3$

5. $f(x)=x^2+4x-\pi \Rightarrow f'(x)=2(x)+4(1)-0=2x+4$

7. $y=\frac{1}{x^4}+\frac{4}{x^3}-\frac{5}{x}=x^{-4}+4x^{-3}-5x^{-1}$

$$\frac{dy}{dx}=-4x^{-5}+4(-3)x^{-4}-5(-1)x^{-2}=-4x^{-5}-12x^{-4}+5x^{-2}=-\frac{4}{x^5}-\frac{12}{x^4}+\frac{5}{x^2}$$

9. $D_x\left[3x^2-4x^3+\frac{2}{x^2}-\frac{1}{\sqrt[3]{x}}+6^4\right]$

$$=D_x\left[3x^2-4x^3+2x^{-2}-\frac{1}{x^{1/3}}+6^4\right]=D_x\left[3x^2-4x^3+2x^{-2}-x^{-1/3}+6^4\right]$$

$$=3(2)x-4(3)x^2+2(-2)x^{-3}-\left(-\frac{1}{3}\right)x^{-4/3}+0$$

$$=6x-12x^2-4x^{-3}+\frac{1}{3}x^{-4/3}$$

$$=6x-12x^2-\frac{4}{x^3}+\frac{1}{3x^{4/3}}$$

$$=6x-12x^2-\frac{4}{x^3}+\frac{1}{3\sqrt[3]{x^4}}$$

11. $f(x)=\left[\sqrt[3]{x^5}+\frac{9}{x^3}-2\ \sqrt[5]{x^7}-\pi x\right]$

$$=x^{5/3}+9x^{-3}-2x^{7/5}-\pi x$$

$$f'(x)=\frac{5}{3}x^{2/3}+9(-3)x^{-4}-2\left(\frac{7}{5}\right)x^{2/5}-\pi$$

$$=\frac{5}{3}x^{2/3}-27x^{-4}-\frac{14}{5}x^{2/5}-\pi$$

$$=\frac{5}{3}\sqrt[3]{x^2}-\frac{27}{x^4}-\frac{14}{5}\sqrt[5]{x^2}x-\pi$$

5-3 Multiple Choice

1. C

$$f(x) = x^{3/2}$$

$$f'(x) = \frac{3}{2}x^{1/2}$$

$$f'(4) = \frac{3}{2}\left(4^{1/2}\right) = \frac{3}{2}(2) = 3$$

3. A

$$f(x) = \frac{1}{2x} + \frac{1}{x^2}$$

$$= \frac{1}{2} \bullet \frac{1}{x} + \frac{1}{x^2}$$

$$= \frac{1}{2}x^{-1} + x^{-2}$$

$$f'(x) = -\frac{1}{2}x^{-2} - 2x^{-3}$$

$$= -\frac{1}{2x^2} - \frac{2}{x^3}$$

5. B

$$y = 3x^3 + 6x^2 - 4x$$

$$\frac{dy}{dx} = 9x^2 + 12x - 4$$

$$\frac{d^2y}{dx^2} = 18x + 12$$

$$\frac{d^3y}{dx^3} = 18$$

5-4 Free Response

1. $f(x)=x^5-5x+1$

$f(-2)=(-2)^5-5(-2)+1=-21$

$\therefore$ point: $(-2,\ -21)$

$f'(x)=5x^4-5$

$f'(-2)=5(-2)^4-5=75$

$\therefore$ slope = 75

Tangent line: $y+21=75(x+2)$

3. $f(x)=3x^4+4x^3-12x^2+5$

$f(0)=3(0)^4+4(0)^3-12(0)^2+5=5$

$\therefore$ point: $(0,\ 5)$

$f'(x)=12x^3+12x^2-24x$

$f'(0)=12(0)^3+12(0)^2-24(0)=0$

$\therefore$ slope = 0

Tangent line: $y-5=0(x+0)\Rightarrow y=5$

5. $g(x)=x^3-x^2+4x-4$

$g(-3)=(-3)^3-(-3)^2+4(-3)-4=-52$ $\therefore$ point: $(-3,\ -52)$

$g'(x)=3x^2-2x+4$

$g'(-3)=3(-3)^2-2(-3)+4=37$

$\therefore$ slope = 37

Tangent line: $y+52=37(x+3)$

Normal line: $y+52=-\dfrac{1}{37}(x+3)$

7. $y = 2x^3 - x^2 + 3x - 4$

$y\big|_{x=-1} = 2(-1)^3 - (-1)^2 + 3(-1) - 4 = -10$ ∴ point: $(-1,\ -10)$

$y' = 6x^2 - 2x + 3$

$y'\big|_{x=-1} = 6(-1)^2 - 2(-1) + 3 = 11$

∴ slope = 11

Tangent line: $y + 10 = 11(x+1)$

Normal line: $y + 10 = -\frac{1}{11}(x+1)$

9. Tangent line (from FR #1): $y + 21 = 75(x+2)$

$f(x) \approx -21 + 75(x+2)$

$f(-1.9) \approx -21 + 75(-1.9+2) = -13.5$

$f(-1.9) \approx -13.5$

11. Tangent line (from FR #3): $y = 5$

$f(x) \approx 5$

$f(-0.1) \approx 5$

13. (3, –4)

$\frac{dy}{dx} = 2x - 3 = +3$

$2x = 6$

$x = 3$

$y(3) = 4$

15. $y+64=12(x-4)$ and $y+64=12(x-4)$

$$\frac{dy}{dx}=6x^2-18x-12=12$$

$$6x^2-18x-24=0$$

$$x^2-3x-4=0$$

$$(x+1)(x-4)=0$$

$$x=-1,4$$

$$y(-1)=1\rightarrow y-1=12(x+1)$$

$$y(4)=-64\rightarrow y+64=12(x-4)$$

5-4 Multiple Choice

1. A

$$y=x^3+3x^2+2$$

$$y\big|_{x=-1}=(-1)^3+3(-1)^2+2=4$$

$\therefore$ point: $(-1,\ 4)$

$$y'=3x^2+6x$$

$$y'\big|_{x=-1}=3(-1)^2+6(-1)=-3$$

$\therefore$ slope = –3

Tangent line: $y-4=-3(x+1)\Rightarrow y=-3x+1$

3. C

Tangent line: $y-2=5(x-3)$

$$0-2=5(x-3)$$

$$13=5x$$

$$x=2.6$$

5. A

$$y = x + \frac{1}{x} = x + x^{-1}$$

$$y' = 1 - x^{-2} = 1 - \frac{1}{x^2}$$

$$y'\Big|_{x=5} = 1 - \frac{1}{5^2} = \frac{24}{25}$$

Tangent line: $y - \frac{26}{5} = \frac{24}{25}(x-5)$

$$y - \frac{26}{5} = \frac{24}{25}x - \frac{24}{5}$$

$$25\left(y - \frac{26}{5} = \frac{24}{25}x - \frac{24}{5}\right)$$

$$25y - 130 = 24x - 120$$

$$24x - 25y = -10$$

5-5 Free Response

1a. $v(t) = 6t^2 - 42t + 60 = 6(t^2 - 7t + 10) = 6(t-2)(t-5) = 0 \Rightarrow t = 2,\ 5$ sec

b. $v(3) = 6(3)^2 - 42(3) + 60 = -12 < 0 \therefore$ moving left

c. $x(3) = 2(3)^3 - 21(3)^2 + 60(3) + 4 = 49$

The particle is 49 units right of the origin.

d. $a(t) = 12t - 42 \qquad a(3) = 12(3) - 42 = -6$

3a. $v(t) = 36t^3 - 12t^2 - 480t + 576$

$= 12(3t^3 - t^2 - 40t + 48) = 12(3t-4)(t-3)(t+4) = 0 \Rightarrow t = -4,\ \frac{4}{3},\ 3$ sec

b. $v(3) = 36(3)^3 - 12(3)^2 - 480(3) + 576 = 0$

The particle is stopped.

c. $x(3) = 9(3)^4 - 4(3)^3 - 240(3)^2 + 576(3) - 48 = 141$

The particle is 141 units right of the origin.

d. $a(t) = 108t^2 - 24t - 480$

$a(3) = 108(3)^2 - 24(3) - 480 = 420$

5a. $v(t) = 15t^2 - 5t^4 = 5t^2(3 - t^2) = 0 \Rightarrow t = 0,\ t = \pm\sqrt{3}$ sec

b. $v(3) = 15(3)^2 - 5(3)^4 = -270 < 0 \therefore$ moving left

c. $s(3) = 5(3)^3 - (3)^5 = -108$

d. $a(t) = 30t - 20t^3$

$a(3) = 30(3) - 20(3)^3 = -450$

7. $x(t) = t^2 - 5t + 4 \to v(t) = 2t - 5 = 0 \to t = 2.5$

$x(2.5) = -2.25$

9. $x(t) = 6t^5 - 15t^4 - 8t^3 + 24t^2 + 12 \to x'(t) = 30t^4 - 60t^3 - 24t^2 + 48t = 0$

$x'(t) = 6t(5t^3 - 10t^2 - 4t + 8) = 6t[5t^2(t-2) - 4(t-2)] = 0$

$6t(t-2)[5t^2 - 4] = 0$

$t = 0,\ 2,\ 0.894,\ -0.894$

$x(0) = 12;\ x(2) = -4;\ \ x(0.894) = -19.090;\ x(-0.894) = 23.890$

11. $x(t) = t^3 - 6t^2 + 12t + 5 \to v(t) = 3t^2 - 12t + 12 \to a(t) = 6t - 12 = 0$

$t = 2$

$x(2) = 13;\ v(2) = 0$

10. $x(t) = 2t^3 - 21t^2 + 60t + 4$

$v(t) = 6t^2 - 42t + 60$

$a(t) = 12t - 42 = 0 \Rightarrow t = 3.5$

$x(3.5) = 2(3.5)^3 - 21(3.5)^2 + 60(3.5) + 4 = 42.5$

$v(3.5) = 6(3.5)^2 - 42(3.5) + 60 = -13.5$

11. $x(t) = t^3 - 6t^2 - 63t + 4$

$v(t) = 3t^2 - 12t - 63 = 3(t-7)(t+3) = 0 \Rightarrow t = 7,\ -3$

$x(7) = (7)^3 - 6(7)^2 - 63(7) + 4 = -388$

$x(-3) = (-3)^3 - 6(-3)^2 - 63(-3) + 4 = 112$

12. $x(t)=9t^4-4t^3-240t^2+576t-48$

$v(t)=36t^3-12t^2-480t+576$

$a(t)=108t^2-24t-480=0$

$=12(9t^2-2t-40)=0$

$=12(9t-20)(t+2)=0$

$\Rightarrow t=20/9,\ -2$

$x(20/9)=9(20/9)^4-4(20/9)^3-240(20/9)^2+576(20/9)-48=222.398$

$v(20/9)=36(20/9)^3-12(20/9)^2-480(20/9)+576=-154.864$

$x(-2)=9(-2)^4-4(-2)^3-240(-2)^2+576(-2)-48$

$=-1984$

$v(-2)=36(-2)^3-12(-2)^2-480(-2)+576$

$=1200$

13. $h(t)=-16t^2+64t+25$

$v(t)=-32t+64=0\Rightarrow t=2$

$h(2)=-16(2)^2+64(2)+25=89$ feet

15. Mars: $s=1.86t^2\rightarrow v=3.72t=27.8\Rightarrow t=7.473$ sec

Jupiter: $s=11.44t^2\rightarrow v=22.88t=27.8\Rightarrow t=1.215$ sec

<u>5-5 Multiple Choice</u>

1. B $v=3t^2+2t\Rightarrow a=6t+2=0\Rightarrow t=-1/3$

3. C $v(t)=2t-6=0\Rightarrow t=3$

5. A $v(t)=18t^2-14t-9\Rightarrow a(t)=36t-14\rightarrow a(9)=36(9)-14=310$

5-6 Free Response

1a. $x'(t) = 2t - 5,\ y'(t) = -8t \Rightarrow \langle 2t - 5,\ -8t \rangle$

b. $x''(t) = 2,\ y''(t) = -8 \Rightarrow \langle 2,\ -8 \rangle$

c. speed $= \sqrt{(2t-5)^2 + (-8t)^2}$

$\text{speed}\big|_{t=2} = \sqrt{(-1)^2 + (-16)^2} = \sqrt{257} \approx 16.031$

3a. $x'(t) = 6t^2 - 42t + 60,\ y'(t) = 2t \Rightarrow \langle 6t^2 - 42t + 60,\ 2t \rangle$

b. $x''(t) = 12t - 42,\ y''(t) = 2 \Rightarrow \langle 12t - 42,\ 2 \rangle$

c. speed $= \sqrt{(6t^2 - 42t + 60)^2 + (2t)^2}$

$\text{speed}\big|_{t=2} = \sqrt{(0)^2 + (4)^2} = \sqrt{16} \approx 4$

5. $x'(t) = 4t^3 - 8t = 0$

$4t(t^2 - 2) = 0$

$t = 0,\ \pm\sqrt{2}$

v_x	$-$	0	$+$	0	$-$	0	$+$
t		$-\sqrt{2}$		0		$\sqrt{2}$	

$y'(t) = 6 - 3t^2 = 0$

$3(2 - t^2) = 0$

$t = \pm\sqrt{2}$

v_y	$-$	0	$+$	0	$-$
t		-2		$\sqrt{2}$	

Particle traveling left and up for $t \in (0,\ \sqrt{2})$.

7a. $x'(t) = 8t,\ y'(t) = -2t^{-2} - 2t \Rightarrow v(2) = \langle 16,\ -4.5 \rangle$

b. $x''(t) = 8,\ y''(t) = 4t^{-3} - 2 \Rightarrow a(2) = \langle 8,\ -1.5 \rangle$

9. Same x position:

$x_A(t) = x_B(t)$

$t - 2 = \frac{3}{2}t - 4 \Rightarrow t = 4$

Same y position:

$y_A(t) = y_B(t)$

$(t-2)^2 = \frac{3}{2}t - 2$

$\Rightarrow t = 1.5,\ 4$

Particles in the same position at the same time when $t = 4$.

<u>5-6 Multiple Choice</u>

1. B____ $a(t) = \langle 2t,\ 5\rangle \Rightarrow a(3) = \langle 6,\ 5\rangle$

3. E

$a_x = 2t - 1 \qquad a_y = 2$

$a_x\big|_{t=1} = 1 \qquad a_y\big|_{t=1} = 2$

$\Rightarrow (1,\ 2)$

5. C

$p'(t) = \left\langle t^2 + t - 2,\ 2t^2 + 3t - 2 \right\rangle$

$t^2 + t - 2 = 0$	$2t^2 + 3t - 2 = 0$
$(t+2)(t-1) = 0$	$(2t-1)(t+2) = 0$
$t = \boxed{-2},\ 1$	$t = \frac{1}{2},\ \boxed{-2}$

Polynomial Practice Test Answer Key

Multiple Choice

1. C

$y' = 3x^2$

$y'|_{x=2} = 3 \bullet 2^2 = 12$

slope = 12

$\Rightarrow y - 8 = 12(x-2)$

$\Rightarrow y = 12x - 16$

2. D

$f(x) = 2x^{1/2} \Rightarrow f(c) = 2c^{1/2}$

$f'(x) = x^{-1/2} \Rightarrow f'(c) = c^{-1/2}$

$f(c) = f'(c) \Rightarrow 2c^{1/2} = c^{-1/2}$

$\Rightarrow 1 = 2c$

$\Rightarrow c = 1/2$

3. C

$$\lim_{x\to 1} \frac{f(x+1) - f(2)}{x^2 - 1} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{\left[(x+1)^2 - 1\right] - \left[2^2 - 1\right]}{x^2 - 1} = \lim_{x\to 1} \frac{x^2 + 2x + \cancel{1} - \cancel{1} - 3}{x^2 - 1} = \lim_{x\to 1} \frac{x^2 + 2x - 3}{x^2 - 1} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{(x+3)(x-1)}{(x+1)(x-1)} = \lim_{x\to 1} \frac{(x+3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \lim_{x\to 1} \frac{(x+3)}{(x+1)} = 2$$

4. A

$y|_{x=0} = 7 \bullet 0^4 + 2 \bullet 0^3 + 0^2 + 2 \bullet 0 + 5 = 5$, so the point: $(0,\ 5)$

$y' = 28x^3 + 6x^2 + 2x + 2$ $y'|_{x=0} = 28 \bullet 0^3 + 6 \bullet 0^2 + 2 \bullet 0 + 2 = 2$, so the slope = 2

$$m_{normal} = -\frac{1}{2} \Rightarrow y - 5 = -\frac{1}{2}(x - 0) \Rightarrow \frac{1}{2}x + y = 5 \Rightarrow x + 2y = 10$$

5. C $\quad v = 2t - 2 \rightarrow v\big|_{t=1} = 2 \bullet 1 - 2 = 0$

6. E

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(e) = \lim_{h\to 0} \frac{f(e+h) - f(e)}{h} = \lim_{h\to 0} \frac{e^{e+h} - e^{e}}{h}$$

Free Response

1. "Above the x-axis" means y is positive:

$$y = 8x^3 - x^4 = x^3(8 - x)$$

$v(t)$	−	0	+	0	−
t		0		8	

"Decreasing" means the derivative is negative:

$$\frac{dy}{dx} = 24x^2 - 4x^3 = 4x^2(6 - x)$$

$v(t)$	+	0	+	0	−
t		0		6	

Both occur on $x \in (6, 8)$

3a) $$\lim_{x\to 1} \frac{x^2 + 2x - 3}{x^2 - 1} = \lim_{x\to 1} \frac{(x-1)(x+3)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x+3}{x+1} = \frac{4}{2} = 2$$

3b) $$\lim_{x\to 3} \frac{x^3 - 27}{x^4 - 14x^2 + 45} = \lim_{x\to 3} \frac{(x-3)(x^2+3x+3)}{(x^2-9)(x^2-5)} = \lim_{x\to 3} \frac{(x-3)(x^2+3x+3)}{(x-3)(x+3)(x^2-5)} =$$

$$\lim_{x\to 3} \frac{(x^2+3x+3)}{(x+3)(x^2-5)} = \frac{27}{24} = \frac{9}{8}$$

3c) $\lim_{x\to -1} \frac{3x^2+2x-1}{x^3+x^2+4x+4} = \lim_{x\to -1} \frac{(3x-1)(x+1)}{(x^2+4)(x+1)} = \lim_{x\to -1} \frac{3x-1}{x^2+4} = -\frac{4}{5}$

5a. $\frac{d}{dx}\left[y = 6x^7 - 19x^4 + 3x^2 - 12x - 13\right]$

$$\frac{dy}{dx} = 7\cdot(6)x^{7-1} - 4\cdot 19x^{4-1} + 2\cdot(3)x^{2-1} - 1\cdot(12)x^{1-1} - 0\cdot(13)x^{0-1}$$

$$\frac{dy}{dx} = 42x^6 - 76x^3 + 6x - 12$$

5b) $D_x\left[\sqrt[4]{x^7} - \frac{6}{x^5} - \sqrt[3]{x} + \pi^2 - x\right] = D_x\left[x^{7/4} - 6x^{-5} - x^{1/3} + \pi^2 - x^1\right]$

$$\frac{dy}{dx} = \frac{7}{4}x^{7/4-1} - 6(-5)x^{-5-1} - \frac{1}{3}x^{1/3-1} + 0(\pi^2)x^{0-1} - 1x^{1-1}$$

$$= \frac{7}{4}x^{3/4} + 30x^{-6} - \frac{1}{3}x^{-2/3} - 1$$

$$= \frac{7}{4}x^{3/4} + \frac{30}{x^6} - \frac{1}{3x^{2/3}} - 1$$

5c) $\frac{d}{dx}\left[x^7 - 4\sqrt[8]{x^7} + 7^3 - \frac{1}{\sqrt[7]{x^4}} + \frac{1}{5x}\right] = \frac{d}{dx}\left[x^7 - 4x^{7/8} + 7^3x^0 - x^{-4/7} + \frac{1}{5}x^{-1}\right]$

$$= 7x^{7-1} - \left(\frac{7}{8}\right)4x^{7/8-1} + 0(7^3)x^{0-1} - \frac{-4}{7}x^{-4/7-1} + \frac{1}{5}(-1)x^{-1-1}$$

$$= 7x^6 - \frac{7}{2}x^{-1/8} + \frac{4}{7}x^{-11/7} - \frac{1}{5}x^{-2}$$

$$= 7x^6 - \frac{7}{2x^{1/8}} + \frac{4}{7x^{11/7}} - \frac{1}{5x^2}$$

5d) $y = 4x^{17} + 6x^3 - 2x + 17 + \pi + \sqrt[3]{x^8} - \frac{2}{x^3}$

$$= (17)4x^{17-1} - (3)6x^{3-1} - (1)2x^{1-1} + 0(17)x^{0-1} + 0(\pi)x^{0-1} - (-3)x^{-3-1}$$

$$= 68x^{16} - 18x^2 - 2 + 3x^{-4}$$