

	Page
Introduction: General Preview of Analytic Geometry and Calculus	v
Chapter 0: Triangle Trigonometry	
Chapter 0 Overview: Triangle Trigonometry	<i>i</i>
0-1: Right Triangle Review	<i>ii</i>
0-2: The Law of Cosines	<i>vii</i>
0-3: Area and the Law of Sines	<i>xii</i>
0-4: SSA--The Ambiguous Case	<i>xvii</i>
0-5: Mathematical Modeling with Triangles	<i>xxii</i>
Triangle Trigonometry Practice Test	<i>xxvi</i>
Triangle Trigonometry Answer Key	<i>xxix</i>
Chapter 1: Analytic Trigonometry	
Chapter 1 Overview	1
1-1 Analytic Trigonometry	2
1-2 The Unit Circle and Special Triangles	13
1-3 Calculator Use	21
1-4 Vectors	28
Analytic Trigonometry Practice Test	39
Analytic Trigonometry Answer Key	44
Chapter 2: Parent Trigonometric Functions	
Chapter 2 Overview: Traits of Parent Trigonometric Functions	49
2-1: Graphing Trigonometric Functions by Calculator	51
2-2: Solving Sinusoidal Equations	69
2-3: Mathematical Modeling with Sinusoidals	75
2-4: Reciprocal and Quotient Curves	88
Parent Trigonometric Functions Practice Test	98
Parent Trigonometric Functions Homework Answer Key	102

Chapter 3: Trigonometric Identities	
Chapter 10 Overview	123
3-1: Factoring Review	124
3-2: Quotient, Reciprocal, and Pythagorean Identities	130
3-3: Composite Argument and Even/Odd Rules	140
3-4: Double Angle Rules	148
3-5: Half Angle Rules	155
3-6: Solving Equations with Identities and Algebra	164
Trigonometric Identities Practice Test	177
Trigonometric Identities Answer Key	185
Chapter 4: PreCalculus Basics	
Chapter 4 Overview	209
4-1: Review of Linear Functions	210
4-2: Review of Quadratic Functions	216
4-3: Finding Complete Graphs of Polynomials	226
4-4: Optimization by Calculator	239
4-5: Zeros of Higher Order Polynomials by Synthetic Substitution	252
4-6: Sign Patterns	262
PreCalculus Basics Practice Test	274
PreCalculus Basics Answer Key	278
Chapter 5: Limits and Derivatives	
Chapter 5 Overview: Limits and Derivatives	289
5-1: The Indeterminate Forms and Limits	290
5-2: Slope of a Tangent Line	296
5-3: The Power Rule	305
5-4: Local Linearity and Approximations	312
5-5: Rectilinear Motion	320
5-6: Parametric Motion	329
Limits and Derivatives Practice Test	336
Limits and Derivatives Answer Key	341

Chapter 6: Polynomial Functions	
Chapter 6 Overview: Polynomial Functions	345
6-1: Critical Values and Extremes of a Polynomial Function	347
6-2: Interpreting the Sign Pattern of the First Derivative	355
6-3: Optimization Problems	367
6-4: General Polynomial Sketching	381
6-5: Multiple Sign Patterns	391
Polynomial Functions Practice Test	401
Polynomial Functions Answer Key	409
Chapter 7: Rational Functions	
Chapter 7 Overview: Types and Traits of Rational Functions	429
7-1: Zeros, Vertical Asymptotes, and POEs	431
7-2: Sign Patterns and Rational Functions	442
7-3: End Behavior: Horizontal Asymptotes	455
7-4: End Behavior: Slant Asymptotes	464
7-5: Derivatives: The Quotient Rule	471
7-6: General Rational Curve Sketching	484
7-7: Sign Pattern of the Derivative of a Rational Function	500
Rational Functions Practice Test	508
Rational Functions Answer Key	518
Chapter 8: Radical Functions	
Chapter 8 Overview: Types and Traits of Radical Functions	541
8-1: Zeros and Domain	543
8-2: The Chain Rule	553
8-3: Extreme Points of Radical Functions	567
8-4: General Radical Curve Sketching	581
8-5: Implicit Differentiation	596
8-6: Related Rates	602
Radical Functions Practice Test	617
Radical Functions Answer Key	619

Chapter 9: Piece-wise Defined Functions	
Chapter 9 Overview: Continuity and Piece-wise Defined Functions	631
9-1: One-Sided Limits	633
9-2: Continuity and Discontinuity	643
9-3: Differentiability and Smoothness	654
9-4: Sketching Piece-wise Defined Functions	665
Piece-wise Defined Functions Practice Test	683
Piece-wise Defined Functions Answer Key	691
Chapter 10: Exponential and Logarithmic Functions	
Chapter 10 Overview: Traits of Transcendental Functions	701
10-1: Review of Exponential and Logarithmic Operations	702
10-2: Financial Applications	712
10-3: Zeros, VAs, and Domains of Log and Exponential Functions	718
10-4: Limit and Derivative Rules	728
10-5: The Product Rule	737
10-6: Limits and L'Hopital's Rule	748
10-7: End Behavior of Exponential, Logarithmic, and Combination Functions	754
10-8: Exponential and Product Curve Sketching	763
10-9: Logarithmic Curve Sketching	779
Exponential and Logarithmic Functions Practice Test	793
Exponential and Logarithmic Functions Answer Key	799
Chapter 11: General Trigonometric Functions	
Chapter 11 Overview: Types and Traits of General Trigonometric Functions	831
11-1: Review of Parent Trigonometric Graphs	833
11-2: Trigonometric Derivatives and the Chain Rule	847
11-3: The Product and Quotient Rules Revisited	855
11-4: General Trigonometric Curve Sketching	861
11-5: Inverse Trigonometric Functions and Their Derivatives	876
General Trigonometric Functions Practice Test	886
General Trigonometric Functions Answer Key	891

Chapter 12: The Second Derivative	
Chapter 12 Overview	905
12-1: Second Derivatives	907
12-2: Concavity and Points of Inflection	926
12-3: Advanced Curve Sketching	938
12-4: Graphical Analysis	951
The Second Derivative Practice Test	980
The Second Derivative Answer Key	992
Chapter 13: Integrals	
Chapter 13 Overview: The Integral	1025
13-1: Anti-Derivatives: The Power Rule	1026
13-2: Integration by Substitution: The Chain Rule	1034
13-3: Anti-Derivatives: The Transcendental Rules	1041
13-4: Definite Integrals	1050
13-5: Definite Integrals and Area	1058
Integrals Practice Test	1067
Integrals Answer Key	1071

Introduction: Analytic Geometry and Calculus

Analytic Geometry and Calculus are closely related subjects. Analytic Geometry is the study of functions and relations as to how their graphs on the Cartesian Coordinate System relate to the algebra of their equations. Traditionally, Calculus has been the study of functions with a particular interest in tangent lines, maximum and minimum points, and area under the curve. (The Reform Calculus movement places an emphasis on how functions change, rather than the static graph, and consider Calculus to be a study of change and of motion.) Consequently, there is a great deal of overlap between the subjects. The advent of graphing calculators has blurred the distinctions between these fields and made subjects that had previously been strictly Calculus topics easily accessible at the lower level. The point of this course is to thoroughly discuss the subjects of Analytic Geometry that directly pertain to entry-level Calculus and to introduce the concepts and algebraic processes of first semester Calculus.

To this end, we have determined the following six Enduring Understandings are at the heart of this course:

- Understanding the relationship between a function and the derivative of that function.
- Understanding the similarities among and differences between the various members of the families of functions.
- Understanding the nature and purpose of the equations for a function and its derivative (plus, in Honors, the 2nd Derivative)
- Understanding word problems and mathematical modeling as a context for mathematical skills and thinking rather than as a separate topic.
- Understanding when it is appropriate to use a calculator and when it is not.
- Developing flexibility of thought by not getting locked into seeing things in a locked in, definite way.

A Note about the Text

This text is designed to study the various families of functions in light of the main characteristics—or traits—that the graphs of each family possess. Chapters 1 through 3 are really a single topic, Polynomial Functions. Each subsequent chapter will take a different family and a) review what is known about that family from Algebra 2, b) investigate the analytic traits, c) introduce the Calculus rule that most

applies to that family, d) put it all together in full sketches, and e) take one step beyond.

Basic Concepts and Definitions

A common vocabulary will be used throughout this course. Much of it comes directly from Algebra 1 and 2.

Domain – Defn: the set of values of the independent variable

Means: the set of x -values that can be substituted into the equation to get a real y -value (i.e., *no zero denominator and no negative under an even radical*)

Range – Defn: the set of values of the dependent variable

Means: the set of y -values that can come from the equation

Relation – Defn: a set of ordered pairs

Means: the equation that creates or defines the pairs

Function – Defn: a relation for which there is exactly one value of the dependent variable for each value of the independent variable

Means: an equation where every x gets only one y

Degree – Defn: the maximum number of variables that are multiplied together in any one term of the polynomial

Means: usually, the highest exponent

The Sets of Numbers

Natural Numbers – the counting numbers: 1, 2, 3, 4, ...

Whole Numbers – the natural numbers, as well as 0

Integers – positive and negative whole numbers

Rational Numbers – any number that can be expressed as a fraction of two integers

Irrational Numbers – any number that cannot be expressed as a fraction of two integers (e.g., π , $\sqrt{2}$, or $e = 2.718281828459\dots$)

Real Numbers – all rational and irrational numbers

Imaginary Numbers – a real number times the square root of -1

Complex Numbers – a number that has a real part and an imaginary part (e.g., $2 + 3i$)

Transfinite Numbers – numbers involving infinity (∞)

The Cartesian Coordinate System

As noted before, Analytic Geometry is the study of functions and relations as to how their graphs on the Cartesian Coordinate System relate to the algebra of their equations. Rene Descartes created this system in the 17th century by putting two real number lines perpendicular to each other and defining any point on the plane by their horizontal and vertical distances from the origin. These numbers are called ordered pairs, because there is a specific order—the x is the first number and the y is the second. This allowed for writing equations that represented sets of points that were related in a specific way. *Because these functions are on the Cartesian Coordinate System, assume that they are real numbers, unless the directions state otherwise.*

NB. *This is not the only graphing system.* There are polar and rectangular coordinate systems, as well as others, but the Cartesian Coordinate System is the one that will be primarily used.

Families of Functions and Relations

On the Cartesian Coordinate System, x -values can be randomly assigned to y -values, but that is not particularly interesting. The equations mentioned above that related the ordered pairs, along with the sets of x -values and y -values, form what are called relations. There is one kind of relation that is of particular interest—the function. A function is a specific kind of relation, where every x -value gets exactly one y -value. There are several groups of functions, similar to the kinds of sets of numbers.

Algebraic Functions

Polynomial – Defn: an expression containing no other operations than addition, subtraction, and multiplication performed on the variable

Means: any equation of the form $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$, where n is a non-negative integer

Rational – Defn: an expression that can be written as the ratio of one polynomial to another

Means: an equation with an x in the denominator

Irrational (radical) – Defn: an expression whose general equation contains a root of a variable and possibly addition, subtraction, multiplication, and/or division

Means: an equation with an x in a radical

Transcendental Functions

Exponential – Defn: a function whose general equation is of the form $y = a \cdot b^x$

Means: there is an x in the exponent

Logarithmic – Defn: the inverse of an exponential function

Means: there is a “log” or “ln” in the equation

Trigonometric – Defn: a function (sin, cos, tan, sec, csc, or cot) whose independent variable represents an angle measure

Means: an equation with sine, cosine, tangent, secant, cosecant or cotangent in it

Other Kinds of Functions and Relations

Piece-wise Defined – a function that is defined by different equations for different parts of its domain

Inverse – two functions or relations that cancel/reverse each other (e.g., $y = x^2$ and $y = \sqrt{x}$)

Conic Sections – Defn: shapes formed by the intersection of a plane and a cone of two knaps

Means: circles, ellipses, parabolas, or hyperbolas (and occasionally lines or points)

Set Notation and Interval Notation

Interval Notation

Closed interval: $x \in [a, b]$ means $a \leq x \leq b$

Open interval: $x \in (a, b)$ means $a < x < b$

Half-open interval: $x \in [a, b)$ means $a \leq x < b$

or $x \in (a, b]$ means $a < x \leq b$