AP Calculus AB '19-20

Practice Fall Final
Part II

Calculator Allowed

Name:

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AP Calculus BC '18-19

Fall Final
Part IIa

Calculator Required

Name:

Solution Key

1. At time $t = 0$, there are 120 gallons of oil in a tank. During the time interval $0 \le t \le 10$ hours, oil flows into the tank at a rate of $h(t) = 10 - \frac{t\cos(t)}{2}$ and out of the tank at a rate given by $g(t) = 6 + \frac{e^{0.52t}}{t+1}$. Both h and g are measured in gallons per hour.

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a) How much oil flows out of the tank during this 10-hour time period?

1

$$\int_0^{10} 6 + \frac{e^{.52t}}{t+1}\,dt = 100.827 \text{ GALLONS}$$

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b) Find the value of $h(4.3) - g(4.3)$. Using correct units, explain what this value represents in the context of this problem.

2

$h(4.3) - g(4.3) = \cancel{6.627}\ 3.096\ \frac{\text{GAL}}{\text{HR}}$

AT $t = 4.3$ HOURS, THE OIL IS FLOWING INTO THE TANK AT A ~~6.627~~ 3.096 GAL/HR FASTER THAN IT IS FLOWING OUT.

THE RATE AT WHICH THE AMOUNT OF OIL IN THE TANK IS CHANGING

c) Write an expression for $A(t)$, the total amount of oil in the tank at time t.

$$A(t) = 120 + \int_0^{t}\left(\left(10 - \frac{t}{2}\cos t\right) - \left(6 + \frac{e^{.52t}}{t+1}\right)\right)dt$$

d) Find the absolute maximum and minimum amount of oil in the tank during $0 \le t \le 10$ hours.

$h(t) - g(t) = 0 \rightarrow t = 5.309$

t	A
0	120 GAL
5.309	136.824 GAL
10	122.812 GAL

ABS MAX = 136.824 GAL

ABS MIN = 120 GAL

2. A tree trunk has circular horizontal cross sections. The radius $r(h)$ of the tree trunk is a differentiable function of the height h of the tree measured from the ground. Both r and h are measured in feet. Selected values of r and h are given in the table below.

h height from the ground (feet)	0	1	3	5	8
$r(h)$ radius of the tree trunk (feet)	3	2.5	2	2.5	1.5

a) Use a Right Riemann sum with subintervals indicated by the table to approximate the value of $\frac{1}{8}\int_0^8 r(h)\,dh$. Using correct units, explain the meaning of this value in the context of this problem.

$\frac{1}{8}\int_0^8 r(h)\,dh = \frac{1}{8}\left[2.5 + 2(2) + 2(2.5) + 3(1.5)\right) = 2$

2 IS THE APPROXIMATE AVERAGE VALUE OF THE RADIUS OF THE TREE OVER THE FIRST 8 FEET.

b) Must there be a height within the first eight feet of tree trunk where $\frac{dr}{dh} = 0$? Explain.

YES BECAUSE $r(1) = r(5)$ AND THE FUNCTION IS DIFFERENTIABLE, SO THE MVT HOLDS

c) Write an expression involving one or more integrals to calculate the volume of the tree trunk from $h=0$ to $h=8$. Use a Left Riemann sum to approximate the value of this expression.

$$V = \pi \int_0^8 r^2\,dh$$

$$= \pi \left[3^2 + 2(2.5)^2 + 2(2)^2 + 3(2.5)^2\right]$$

$$= 42\pi$$

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d) For heights above 8 feet, the radius is given by $g(h) = \frac{1}{h^2} + \frac{95}{64}$. A squirrel climbs up the tree at a rate of $\frac{dh}{dt} = 3\ {}^{\text{ft}}/_{\text{sec}}$. How quickly is the radius of the tree changing when the squirrel is 9 feet above the ground?

$$g(h) = h^{-2} + \frac{95}{64}$$

$$\frac{dr}{dt} =$$

$$\frac{dr}{dt} = -2h^{-3}\frac{dh}{dt} = \frac{-2}{9^3}(3) = \frac{-2}{243}$$

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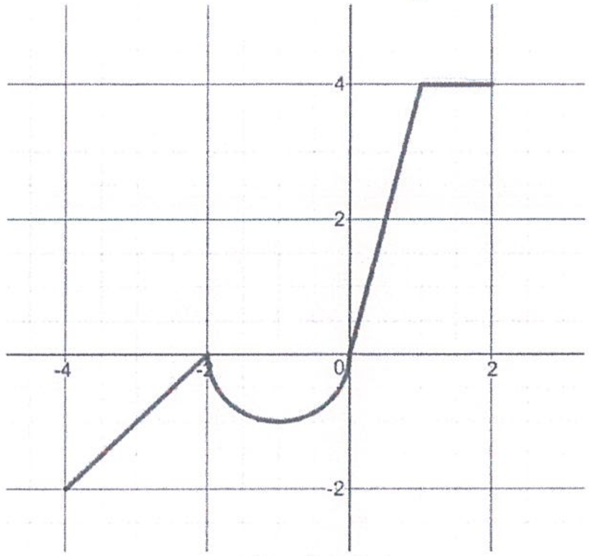
3. Below is the graph of $g(x)$, which is defined on $-4 \le x \le 2$ and consists of three line segments and a semicircle. Let $f(x) = \int_1^x g(t)\,dt$.



graph of $g(x)$

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a) Find $f(-2)$, $f'(-2)$, and $f''(-2)$

$f(-2) = -2 + \frac{\pi}{2}$ $\qquad g = f'$

$f'(-2) = 0$

$f''(-2) = $ DNE

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b) On what intervals, if any, is f(x) both decreasing and concave up? Justify your answer.

f' IS NEGATIVE AND INCREASING

$x \in (-4, -2) \cup (-1, 0)$

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c) Find the average rate of change of $f(x)$ from $x=-2$ to $x=2$.

$$\frac{f(2)-f(-2)}{2-(-2)} = \frac{4-(-2+\pi/2)}{4}$$

$$= \frac{6-\pi/2}{4}$$

$$= \frac{3}{2} - \pi/8$$

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d) Identify all x-values in the open interval $-4 < x < 2$ at which $f(x)$ has a critical point, and classify each critical point as a local minimum, local maximum, or neither. Justify your answers.

$f' = g = 0$ @ $x = -2$ & 0

$X = -2$ IS NEITHER A MAX OR MIN

$X = 0$ IS AT A MIN BECAUSE f' SWITCHES FROM $-$ TO $+$

6. A twice-differentiable function $y = f(x)$ has derivative given by the differential equation $\frac{dy}{dx} = \frac{x(y^2-1)}{y}$ and satisfies $f(1) = -\sqrt{2}$.

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a) Write the equation of the line **normal** to $y = f(x)$ at the point $(1, -\sqrt{2})$.

$m_{TAN} = \frac{1(2-1)}{-\sqrt{2}} = -\frac{1}{\sqrt{2}}$

$m_{NORMAL} = +\sqrt{2}$

$y + \sqrt{2} = \sqrt{2}(x-1)$

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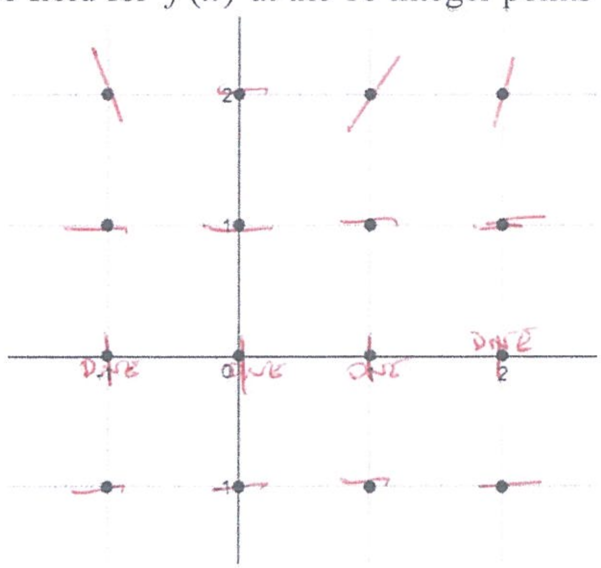
b) Sketch the slope field for $f(x)$ at the 16 integer points indicated.



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