AP Calculus AB '19-20
Anti-Derivative Test

Name Solution Key

Score___________

1. Which of the following statements are true?

I. $\int (\tan u)\, d\cancel{x}^{u} = \ln|\sec u| + c$

II. $\int \csc^2 u\, du = \cot u + c$ $= -\cot u + c$

III. $\int \left(\frac{1}{\sqrt{4-x^2}}\right) dx = \frac{1}{2}(4-x^2)^{1/2} + c$

$= \sin^{-1}\frac{x}{2} + c$

a) I only b) II only c) III only d) I and II e) II and III only

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2. $\int e^x \cos(e^x+1)\,dx$ $= \int \cos u\, du = \sin u + c$

a) $e^x \sin(e^x+1)+c$ b) $\sin(e^x+1)+c$ c) $e^x \sin(e^x+x)+c$

d) $\frac{1}{2}\cos^2(e^x+1)+c$ e) None of these

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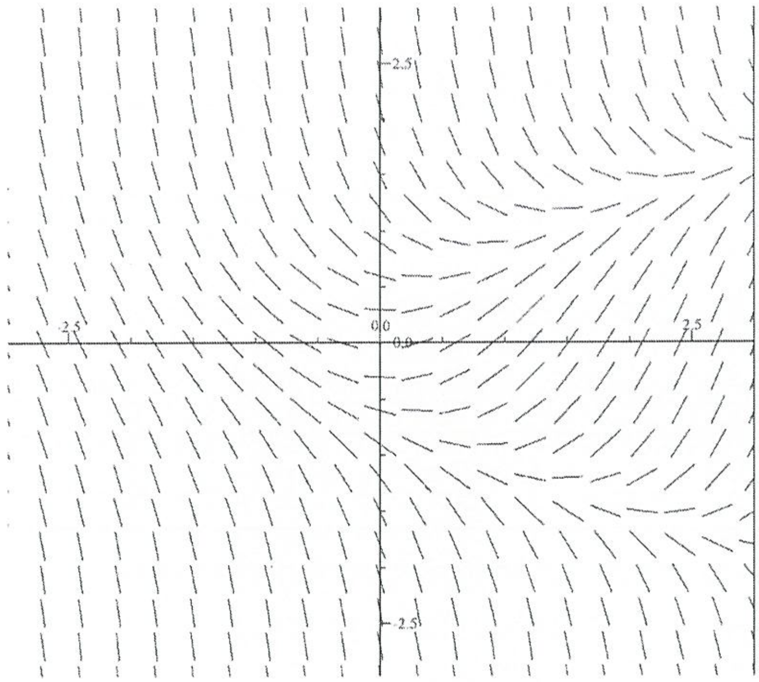
3. Which of the following differential equations corresponds to the slope field shown in the figure below?



a) $\dfrac{dy}{dx} = x - y^2$

b) $\dfrac{dy}{dx} = 1 - \dfrac{y}{x}$

NO SLOPE @x=0

c) $\dfrac{dy}{dx} = -y^3$

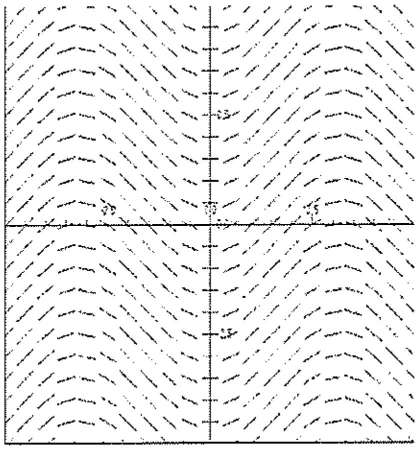
NO // ROWS

d) $\dfrac{dy}{dx} = x - \dfrac{1}{2}x^3$

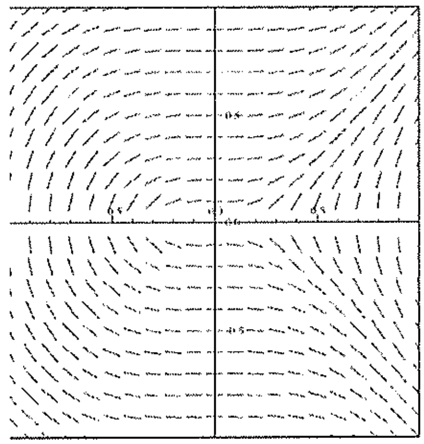
NO // COLUMNS

e) $\dfrac{dy}{dx} = x + y$

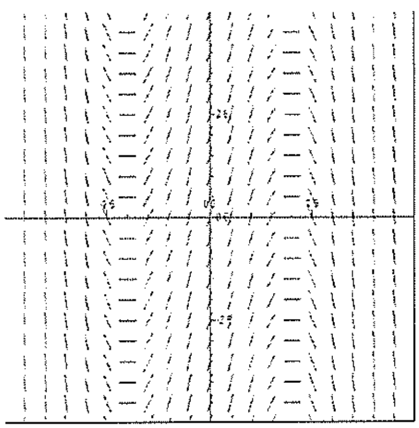
4. Which of the slope field shown below corresponds to $y = \sin\left(x - \frac{\pi}{2}\right)$?



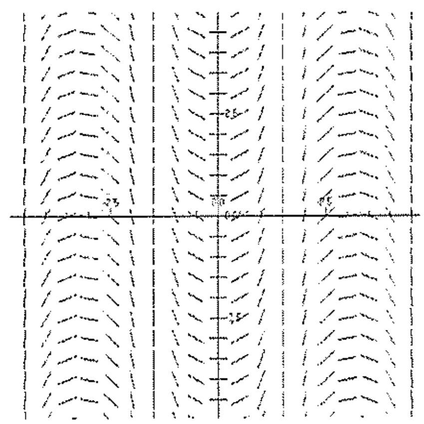
a)



b)



c)



d)

5. A function $y = f(x)$ has derivative $\frac{dy}{dx} = 4x\sqrt{y}$, with $f(0) = 1$. Which of these is $f(x)$?

a) $y = e^{x^2} + 1$

b) $y = x^2 + 1$

c) $y = \left(x^2 + 1\right)^2$

d) $y = x^4 + 1$

e) $y = \left(x + 1\right)^{2/3}$

$y^{-1/2} dy = 4x\,dx$

$2y^{1/2} = 2x^2 + C \to (0,1) \Rightarrow 2 = 0 + C$

$2 = C$

$2y^{1/2} = 2x^2 + 2$

$y^{1/2} = x^2 + 1$

$y = (x^2+1)^2$

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6. A particle is moving upward along the y-axis until it reaches the origin and then it moves downward such that $v(t) = 8 - 2t$ for $t \geq 0$. The position of the particle at time t is given by

a)	$y(t) = -t^2 + 8t - 16$	b)	$y(t) = -t^2 + 8t + 16$
c)	$y(t) = 2t^2 - 8t - 16$	d)	$y(t) = 8t - t^2$
e)	$y(t) = 8t - 2t^2$		

$V = 0 \to t = 4$

$y(t) = 8t - t^2 + C$

$0 = 8(4) - 4^2 + C \Rightarrow C = -16$

~~$(0,0) \Rightarrow C = 0$~~

$y(t) = -t^2 + 8t - 16$

7. $\int \frac{x}{x^2-1}\,dx =$ $\frac{1}{2}\int \frac{2x}{x^2-1}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u|+c$

a) $\ln|x^2-1|+c$

b) $\frac{1}{2}\ln|x^2-1|+c$

c) $2\ln|x^2-1|+c$

d) $\ln\left|\frac{x^2-1}{x}\right|+c$

e) $\ln\left|\frac{x^2-1}{x}\right|+c$

X 8. If $\frac{dy}{dx} = \sin x \cos^3 x$ and if $y = 1$ when $x = \pi$, what is the value of y when $x = 0$?

$\int dy = -\int \cos^3 x\,(-\sin x\,dx)$

$y = -\int u^3\,du$

$y = -\frac{1}{2}\frac{u^4}{4} + c$ $(\pi, 1) = 1 = \frac{-1}{8} + c$

$y = -\frac{1}{8}\cos^4 x + C$ $\frac{9}{8} = c$

a) –3
b) –2
c) 0
d) 2
e) 3

$y = \frac{-1}{8}\cos^4 x + \frac{9}{8}$

$y(0) = \frac{-1}{8} + \frac{9}{8} = 1$

9. State the step that has the first mistake in this process:

$$\int \frac{x}{3x^4+4}\, dx =$$

Step 1: $\dfrac{1}{12}\displaystyle\int \frac{12x}{3x^4+4}\, dx =$

Step 2: $\dfrac{1}{12}\displaystyle\int \left(\frac{12x}{3x^4}+\frac{12x}{4}\right) dx =$

Step 3: $\dfrac{1}{12}\displaystyle\int \left(\frac{4}{x^3}+3x\right) dx =$

Step 4: $\dfrac{1}{12}\displaystyle\int \left(\frac{4}{x^3}\right) dx + \frac{1}{12}\int (3x)\, dx =$

Step 5: $-\dfrac{1}{3x^2}+\dfrac{1}{4}x^2+c$

a) Step 2
b) Step 3
c) Step 4
d) Step 5
e) There is no mistake.

10. $\int\left(-3x^4+7^x-\frac{3}{\sqrt[5]{x^6}}-\frac{1}{12x}\right)dx$ $= \int\left(-3x^4+7^x-3x^{-6/5}-\frac{1}{12}x^{-1}\right)dx$

$= \frac{-3x^5}{5}+\frac{7^x}{\ln 7}-15x^{-1/5}-\frac{1}{12}\ln|x|+C$

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11. $\int\frac{5t^3\,dt}{\sqrt{t^4+5}}$

$u=t^4+5 \qquad du=4t^3\,dt$

$= 5\int\frac{t^3\,dt}{\sqrt{t^4+5}}$

$= \frac{5}{4}\int(t^4+5)^{-1/2}(4t^3\,dt)$

$= \frac{5}{4}\frac{(t^4+5)^{1/2}}{1/2}+C$

$= \frac{5}{2}(t^4+5)^{1/2}+C$

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12. A particle's acceleration is given by $a(t) = 3t^2 + 6t - 1$ meters per second squared. At time $t = 1$, the particle's velocity is 4 meters per second and its position is 0 meters. Find the particle's position at time $t = 2$.

$$V = \int (3t^2 + 6t - 1)\, dt$$

$$V = t^3 + 3t^2 - t + C_1 \rightarrow (1, 4) \rightarrow 4 = 1 + 3 - 1 + C_1$$

$$C_1 = 1$$

$$V = t^3 + 3t^2 - t + 1$$

$$x(t) = \int (t^3 + 3t^2 - t + 1)\, dt$$

$$= \frac{1}{4}t^4 + t^3 - \frac{1}{2}t^2 + t + C_2 \qquad (1, 0) \rightarrow 0 = \frac{1}{4} + 1 - \frac{1}{2} + 1 + C_2$$

$$-\frac{7}{4} = C_2$$

$$x(t) = \frac{1}{4}t^4 + t^3 - \frac{1}{2}t^2 + t - \frac{7}{4}$$

$$x(2) = 41/4$$

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13. $\int \left(x^3 - x^2 \sec\left(2x^3\right) + x^3 e^{x^4}\right) dx$

$$\int x^3\, dx - \frac{1}{6}\int \sec 2x^3 \,(6x^2\, dx) + \frac{1}{4}\int e^{x^4} (4x^3\, dx)$$

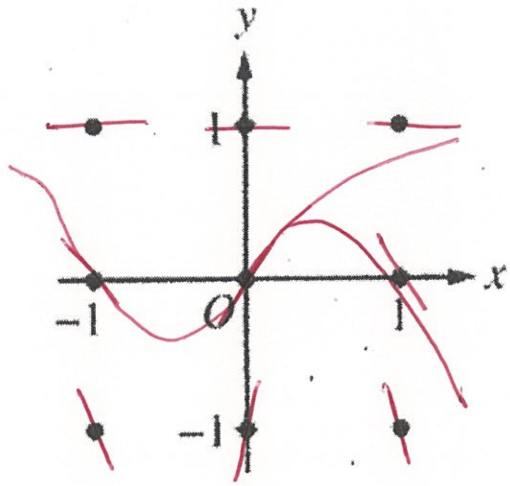
$$= \int x^3\, dx - \frac{1}{6}\int \sec u_1\, du_1 + \frac{1}{4}\int e^{u_2}\, du_2$$

$$= \frac{1}{4}x^4 - \frac{1}{6}\ln\left|\sec u_1 + \tan u_1\right| + \frac{1}{4}e^{u_2} + C$$

$$= \frac{1}{4}x^4 - \frac{1}{6}\ln\left|\sec 2x^3 + \tan 2x^3\right| + \frac{1}{4}e^{x^4} + C$$

14. Given the differential equation, $\frac{dy}{dx}=(y-1)^2\cos(\pi x)$

a. On the axis system provided, sketch the slope field for the $\frac{dy}{dx}$ at all points plotted on the graph.



b. If the solution curve passes through the point (0, 0), sketch the solution curve on the same set of axes as your slope field.

c. Find the particular solution to $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$ with the initial condition $f(\cancel{1}\,2) = 0$.

$$\int (y-1)^{-2}\,dy = \int \cos \pi x \, dx$$

$$\frac{(y-1)^{-1}}{-1} = \frac{1}{\pi} \sin \pi x + C$$

$$(2,0) \rightarrow -1 = \frac{1}{\pi} \sin 0 + C$$

$$-1 = C$$

$$\frac{-1}{y-1} = \frac{\sin \pi x}{\pi} - 1$$

$$= \frac{\sin \pi x - \pi}{\pi}$$

$$\frac{1}{y-1} = \frac{\pi - \sin \pi x}{\pi}$$

$$y - 1 = \frac{\pi}{\pi - \sin \pi x}$$

$$y = 1 + \frac{\pi}{\pi - \sin \pi x}$$