AP Calculus AB
Limit and Continuity Test

1. The function f is differentiable at $x = b$. Which of the following statements could be false?

a) $\lim_{x \to b} f(x)$ exists b) $\lim_{x \to b} f(x) = f(b)$ c) $\lim_{x \to b^-} f(x) = \lim_{x \to b^+} f(x)$

d) $\lim_{x \to b^-} f'(x) = \lim_{x \to b^+} f'(x)$ e) None of these

a) b) & c) ARE PARTS OF THE CONTINUITY DEFN

d) IS REQUIRED FOR DIFFERENTIABILITY.

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2. The function f is defined for all Reals such that $f(x) = \begin{cases} x^2 + kx & \text{for } x < 5 \\ 5\sin\frac{\pi}{2}x & \text{for } x \ge 5 \end{cases}$.

For which value of k will the function be continuous throughout its domain?

a) -2 b) -1 c) $\frac{2}{3}$ d) 1 e) None of these

$X = 5 \Rightarrow 25 + 5k = 5\sin 5\pi/2 = 5$

$5k = -20$

$k = -4$

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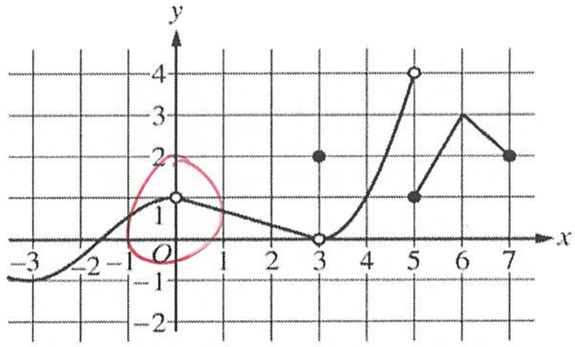
3. The graph of the function f is shown below. At which value of x is f continuous, but not differentiable?



Graph of f

a) 0 b) 2 c) 3 d) 5 e) 6

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4. If $f(x)=\begin{cases} x+2 & \text{for } x \le 3 \\ 4x-7 & \text{for } x>3 \end{cases}$, which of the following statements are true?

I. $\lim_{x\to 3} f(x)$ exists II. f is continuous at $x=3$ III. f is differentiable at $x=3$

a) None b) I only c) II only

d) I and II only e) I, II, and III

$\lim_{x\to 3^-} = 5 = \lim_{x\to 3^+}$

$f' = \begin{cases} 1 \\ 4 \end{cases}$ NOT DIFF

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5. Which of the following functions is differentiable at $x=0$?

a) $f(x)=\sqrt{1+|x|}$ ~~b)~~ $f(x)=|x|$ c) $f(x)=\begin{cases} x^2\sin\frac{1}{x} & \text{for } x\neq 0 \\ 0 & \text{for } x=0 \end{cases}$

~~d)~~ $f(x)=\begin{cases} \frac{1}{x} & \text{for } x\neq 0 \\ 0 & \text{for } x=0 \end{cases}$ e) $f(x)=\begin{cases} \cos x & \text{for } x<0 \\ \sin x & \text{for } x\geq 0 \end{cases}$

$\cos 0 = 1 \neq \sin 0 = 0$

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6. $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}=$ $\left.\frac{d}{dx}(\sin x)\right|_{x=\pi/2} = \cos\frac{\pi}{2} = 0$

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) 0 d) $-\frac{\pi}{4}$ e) DNE

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7. $\lim_{x\to 0} \frac{\int_0^x \sin t^2 dt}{x^3}$ $\overset{L'H}{=}$ $\lim_{x\to 0} \frac{\sin x^2}{3x^2}$ $\overset{L'H}{=}$ $\lim_{x\to 0} \frac{\cos x^2 (2\cancel{x})}{6\cancel{x}} = \frac{2}{6}$

a) 0 b) 1 c) $\frac{1}{3}$ d) 3 e) DNE

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8. $\lim_{x\to\infty} \frac{4x^5 + 3x^4 + 2x^3 + x^2 + 1}{3x^5 - 9x^4 + 4x^3 + 15} =$ $\frac{4}{3}$

a) 0 b) $\frac{3}{4}$ c) $\frac{4}{3}$ d) 3 e) DNE

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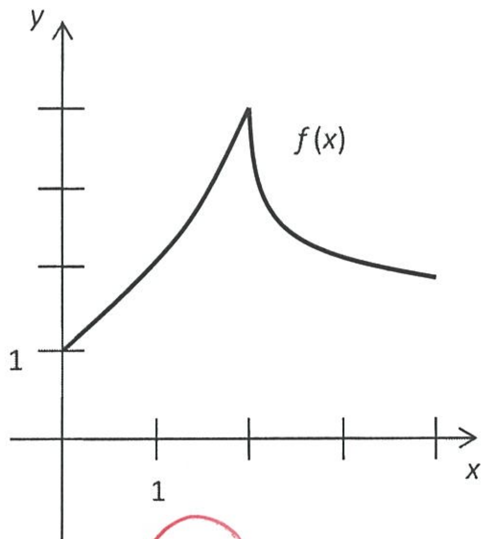
10. The graph of a function f is given below. Which of the following statements are true?

$\frac{dy}{dx} = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$ dne T

$\lim_{x\to 2} f(x) = 4$ T

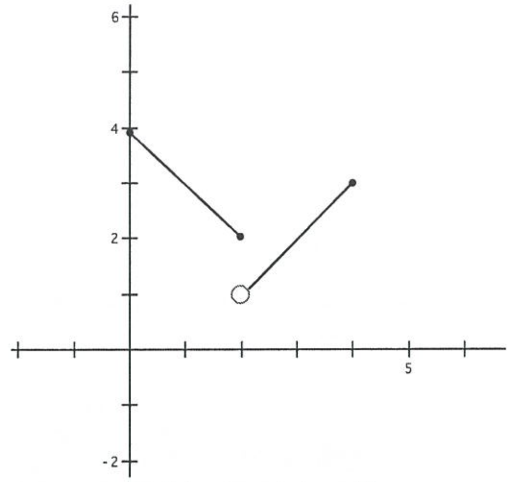
$\lim_{x\to 2} f(x)$ dne F



a) I only b) II only c) I and II only

d) I, II, and III e) II and III only

11. The graph of a function is shown below.



Which of the following statement(s) is (are) true?

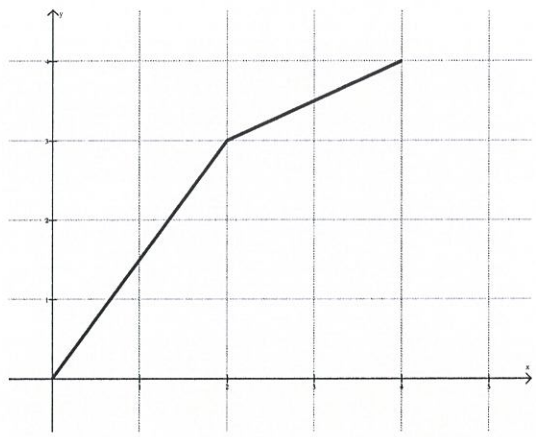
T I. $\lim\limits_{x \to 2^-} f(x)$ exists $= 2$

T II. $\lim\limits_{x \to 2^+} f(x)$ exists $= 1$

F III. $\lim\limits_{x \to 2} f(x)$ exists

a) I only b) II only c) I and II only

d) I and III only e) I, II, and I



12. At which x-value is f (graphed above) differentiable but not continuous?

a) 0 b) 1 c) 2 d) 4 e) nowhere

IT IS NOT ~~TO~~ FOR A FUNCTION TO BE DIFFERENTIABLE BUT NOT CONTINUOUS

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13. $f(x)=\begin{cases} \sin^{-1}\left[\pi(x-1)\right], & \text{if } x<1 \\ 0, & \text{if } x=1 \\ Ln\, x^2, & \text{if } x>1 \end{cases}$

a) Is $f(x)$ continuous? Why/Why not?

$x=1 \rightarrow \begin{cases} \sin^{-1} 0 = 0 \\ \ln 1 = 0 \end{cases}$ $\therefore$

i) $f(1)$ exists

ii) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

iii) $\lim_{x\to 1} f(x) = f(1)$ So $f(x)$ is continuous

b) Is $f(x)$ differentiable? Why/Why not?

i) $f(x)$ is continuous.

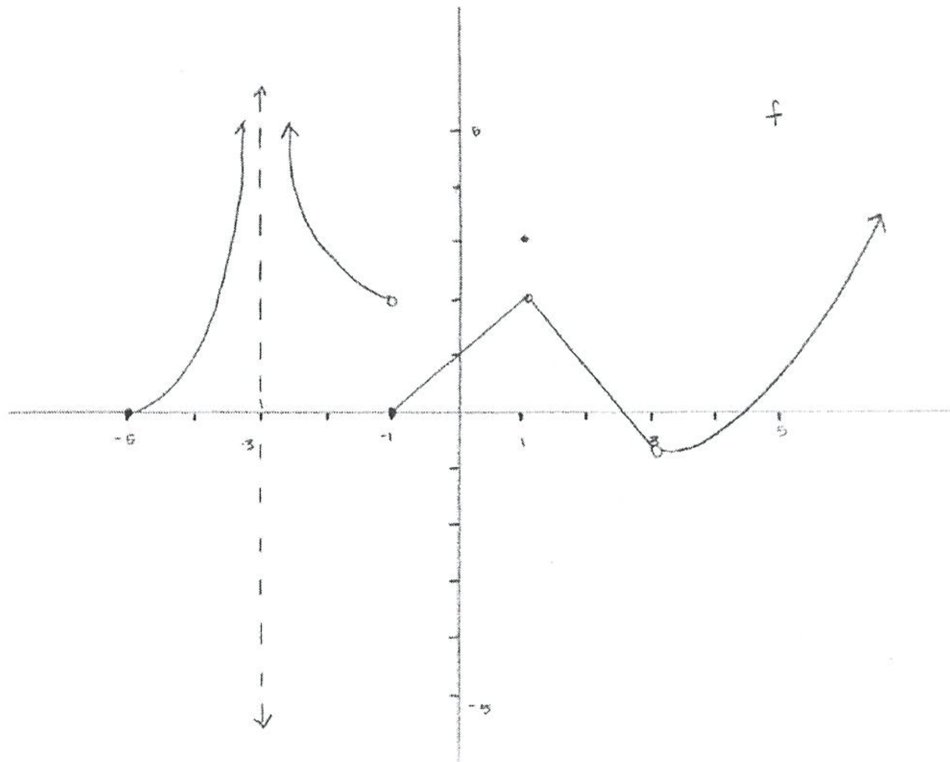
~~ii) Lim~~ $f'(x) = \begin{cases} \dfrac{\pi}{\sqrt{1-(\pi(x-1))^2}} \\ \dfrac{2}{x} \end{cases}$

$\lim_{x\to 1^-} f'(x) = \pi$

$\lim_{x\to 1^+} f'(x) = 2$

$\lim_{x\to 1^-} f'(x) \neq \lim_{x\to 1^+} f'(x)$

15. Given the graph of $f(x)$ below, find the values of the following:



a. $\lim_{x \to -3^-} f(x) = +\infty$

b. $\lim_{x \to -3^+} f(x) = +\infty$

c. $\lim_{x \to -3} f(x) = \infty$

d. $f(-3) =$ DNE

e. $\lim_{x \to -1^-} f(x) = 2$

f. $\lim_{x \to -1^+} f(x) = 1$

g. $\lim_{x \to -1} f(x) =$ DNE

h. $f(-1) = 0$

i. $\lim_{x \to 1^-} f(x) = 2$

j. $\lim_{x \to 1^+} f(x) = 2$

k. $\lim_{x \to 1} f(x) = 2$

l. $f(1) = 3$

m. $\lim_{x \to 3^-} f(x) = -1/2$

n. $\lim_{x \to 3^+} f(x) = -1/2$

o. $\lim_{x \to 3} f(x) = -1/2$

p. $f(3) = -1/2$