

Multiple choice – Circle correct answer.

1. A normal line to the graph of a function f at the point $(x, f(x))$ is defined to be the line perpendicular to the tangent line at that point. An equation of the **normal** line to the curve $y = \sqrt[3]{x^2 - 1}$ at the point where $x = 3$ is

- a) $y + 12x = 38$
- b) $y - 4x = 10$
- c) $y + 2x = 4$
- d) $y + 2x = 8$
- e) $y - 2x = -4$

$$y(3) = 2 \quad y'(x) = \frac{1}{3}(x^2 - 1)^{-2/3}(2x)$$
$$m = y'(3) = \frac{1}{3} \left(\frac{8}{1} \right)^{-2/3} (6) = \frac{1}{3} \left(\frac{1}{4} \right) 6 = \frac{1}{2}$$
$$y - 2 = \frac{1}{2} - 2(x - 3)$$

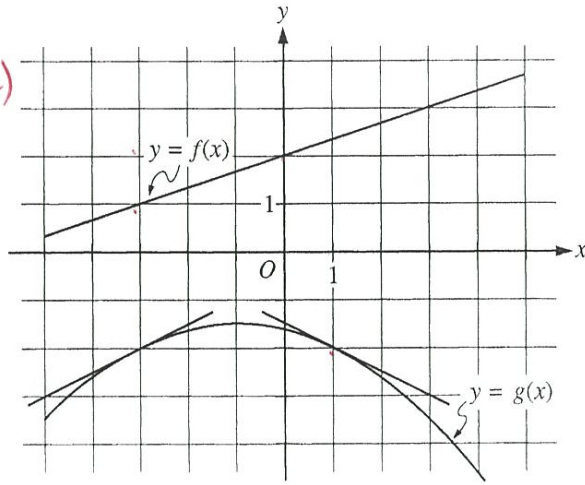
2. If $f(x) = \sin^3(3-x)$, then $f'(x) = 3 \sin^2(3-x) \cos(3-x) (-1)$

- a) $3\sin^2(3-x)$
 - b) $-3\sin^2(3-x)$
 - c) $-3\cos^2(3-x)$
 - d) $3\sin^2(3-x)\cos(3-x)$
 - e) $-3\sin^2(3-x)\cos(3-x)$
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3. The figure below shows the graph of the functions f and g . The graphs of the lines tangent to the graph of g at $x = -3$ and $x = 1$ are also shown. If $B(x) = f(g(x))$, what is $B'(1)$?

- a) $-\frac{1}{2}$
- b) $-\frac{1}{6}$**
- c) $\frac{1}{6}$
- d) $\frac{1}{3}$
- e) $\frac{1}{2}$

$$\begin{aligned}
 B' &= f'(g(x)) \cdot g'(x) \\
 &= f'(-2) \cdot g'(1) \\
 &= \left(\frac{1}{3}\right) \left(-\frac{1}{2}\right)
 \end{aligned}$$



4. Which of the following statements must be true?

I. $\frac{d}{dx}(x \sec^{-1} x) = \sec^{-1} x + \frac{1}{\sqrt{x^2 - 1}}$ ~~II.~~ $\frac{d}{dx} \left(\frac{3-2x}{3x+2} \right) = \frac{13}{(3x+2)^2}$

III. $\frac{d}{dx} \ln(1-x) = \frac{1}{x-1}$

- a) I only
- b) II only
- c) II and III only

- d) I and III only**
- e) I, II, and III

~~I~~ $\times \left(\frac{1}{|x| \sqrt{x^2 - 1}} \right) + \sec^{-1} x$ TRUE

~~II~~ $\frac{(3x+2)(-2) - (3-2x)(3)}{(3x+2)^2} = \frac{-13}{(3x+2)^2}$ FALSE

~~III~~ $= \frac{1}{1-x} (-1) = \frac{1}{x-1}$ TRUE

CHAIN RULE, NOT PRODUCT RULE

5. If $f(x) = \sin[g(x)]$, then $\frac{d}{dx}[f(x)]$ is $= \cos(g(x)) \cdot g'(x)$

a) $\sin x \cdot g'(x) + g(x) \cdot \cos x$ b) $\sin x \cdot g'(x) - g(x) \cdot \cos x$

c) $\cos x \cdot g'(x)$ d) $\cos[g(x)] \cdot g'(x)$

e) None of these

6.

x	f(x)	f'(x)	g(x)	g'(x)
1	2	3	4	-1
2	4	-1	7	8
4	1	2	2	1

Selected values of f , g , and their derivatives are indicated in the table above. Let $h(x) = g(f(x^2))$. What is the value of $h'(2)$?

a) -8

b) -4

c) 1

d) 2

e) 16

$$h' = g'(f(x^2)) \cdot f'(x^2) (2x)$$

$$h'(1) = g'(f(2)) \cdot f'(2) (4)$$

$$= g'(4) f'(2) (4)$$

$$= 1 \cdot (-1) (4) = -4$$

$$= 1 \cdot (-1) (4) = -4$$

7. Find the approximate value of $\sqrt[3]{7}$ using the tangent approximation for $\sqrt[3]{x}$ at $x=8$.

- a) $\frac{25}{12}$ **b)** $\frac{23}{12}$ c) $\frac{15}{8}$ d) $\frac{7}{4}$ e) $\frac{13}{8}$

$$f(x) = x^{1/3}$$

$$f(8) = 2$$

$$f'(x) = \frac{1}{3}x^{-2/3}$$

$$f'(8) = \frac{1}{12}$$

$$y - 2 = \frac{1}{12}(x - 8)$$

$$\begin{aligned}\sqrt[3]{7} = f(7) &\approx y(7) = \frac{1}{12}(7 - 8) + 2 = \\ &= \frac{23}{12}\end{aligned}$$

$$8a. \quad \frac{d}{dx}(\sqrt[3]{16+x^3}) = \frac{1}{3} (16+x^3)^{-2/3} (3x^2)$$

$$= \frac{x^2}{(16+x^3)^{2/3}}$$

$$b. \quad \frac{d}{dx}(5e^{\cos(7x)}) = 5e^{\cos 7x} (-\sin 7x)(7)$$

$$= -35 \sin 7x e^{\cos 7x}$$

$$c. \quad \frac{d}{dx} \left[x^4 - 14 \sqrt{x^9} + 8^x - \frac{1}{\sqrt[3]{x^7}} + \frac{1}{8x} \right] = \frac{d}{dx} \left[x^4 - 14x^{9/2} + 8^x - x^{-7/3} + \frac{1}{8}x^{-1} \right]$$

$$= 4x^3 - 18x^{7/2} + 8^x \ln 8 + \frac{7}{3}x^{-10/3} - \frac{1}{8}x^{-2}$$

$$d. \quad \frac{d}{dx}(\sec^{-1}(2x^2)) = \frac{1}{2x^2 \sqrt{(2x^2)^2 - 1}} (4x)$$

$$= \frac{2}{x \sqrt{4x^4 - 1}}$$

$$\begin{aligned}
 9. \quad \frac{d}{dx}(\sqrt{\cos(1-x^2)}) &= \frac{1}{\cos} (\cos(1-x^2))^{1/2} \\
 &= \frac{1}{\cancel{2}} (\cos(1-x^2))^{1/2} (-\sin(1-x^2)) (-2x) \\
 &= \frac{x \sin(1-x^2)}{\cos^{1/2}(1-x^2)}
 \end{aligned}$$

10. If $f(x) = e^{\cot 3x}$, find $f''(x)$.

$$\begin{aligned}
 f' &= e^{\cot 3x} (-\csc^2 3x) \\
 f'' &= e^{\cot 3x} (-2 \csc 3x (-\csc 3x \cot 3x)(3) + \\
 &\quad (\csc^2 3x)(e^{\cot 3x} (-\csc^2 3x)(3))) \\
 &= 6 e^{\cot 3x} \csc^2 3x \cot 3x + 3 e^{\cot 3x} \csc^4 3x \\
 &= 3 \csc^2 3x e^{\cot 3x} [2 \cot 3x + \csc^2 3x]
 \end{aligned}$$

11. If $f(x) = \tan^2(3x)$, find $f'(x)$ and $f''(x)$. Write answers in simplified, factored form.

$$f'(x) = 2 \tan 3x \sec^2 3x \quad (3)$$

$$= 6 \tan 3x \sec^2 3x$$

$$u = 6 \tan 3x$$

$$v = \sec^2 3x$$

$$Du = 6 \sec^2 3x \quad (3)$$

$$Dv = 2 \sec 3x \sec 3x \tan 3x \quad (3)$$

$$= 18 \sec^2 3x$$

$$= 6 \sec^2 3x \tan 3x$$

$$f''(x) = 6 \tan 3x (6 \sec^2 3x \tan 3x) + \sec^2 3x (18 \sec^2 3x)$$

$$= 36 \sec^2 3x \tan^2 3x + 18 \sec^4 3x$$

$$= 18 \sec^2 3x (2 \tan^2 3x + \sec^2 3x)$$

12. $y = (4x^5 - 3)^7 (7x^2 + 1)^5$. Find $\frac{dy}{dx}$ in factored form.

$$u = (4x^5 - 3)^7$$

$$v = (7x^2 + 1)^5$$

$$Du = 7(4x^5 - 3)^6 (20x^4)$$

$$Dv = 5(7x^2 + 1)^4 (14x)$$

$$= 140x^4 (4x^5 - 3)^6$$

$$= 70x(7x^2 + 1)^4$$

$$\frac{dy}{dx} = \overset{u}{(4x^5 - 3)^7} \left(\overset{Dv}{\cancel{140x^4(4x^5 - 3)^6}} \right) + \overset{v}{(7x^2 + 1)^5} \left(\overset{Du}{140x^4} \right) (4x^5 - 3)^6$$

$$= 70x (4x^5 - 3)^6 (7x^2 + 1)^4 \left[(4x^5 - 3) + (7x^2 + 1)(2x^3) \right]$$

$$= 70x (4x^5 - 3)^6 (7x^2 + 1)^4 \left[4x^5 - 3 + 14x^5 + 2x^3 \right]$$

$$= 70x (4x^5 - 3)^6 (7x^2 + 1)^4 (18x^5 + 2x^3 - 3)$$