AB Calculus '19-20 Name Solution Key

Dx Apps II Practice Test

No Calculator

1. A particle moves along a straight line with velocity given by $v(t) = 5 + 4t - t^2$. When is the particle *slowing down*?

a) $t \in (-\infty, 1)$

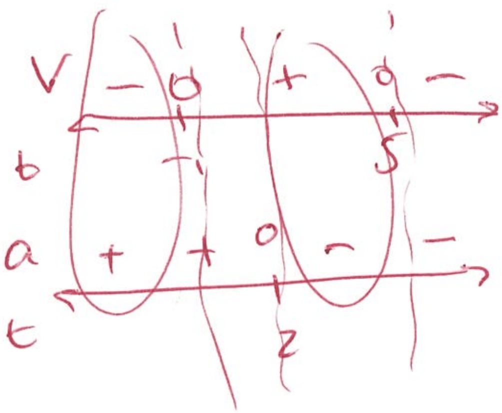
b) $t \in (-\infty, -1) \cup (5, \infty)$

c) $t \in (-1, 2) \cup (5, \infty)$

d) $t \in (-\infty, -1) \cup (2, 5)$

e) $t \in (5, \infty)$

$v = (5-t)(1+t)$



$a = 4 - 2t$

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2. A particle is moving along the x-axis in such a way that its velocity at time $t > 0$ is given by $v(t) = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

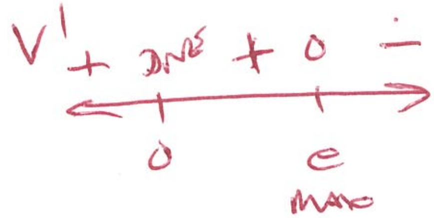
(a) 1 (b) $e^{1/2}$ (c) e (d) $e^{3/2}$

(e) There is no maximum value of v.

$$v' = \frac{t(1/t) - \ln t}{t^2} = \frac{1 - \ln t}{t^2} = 0$$

$\ln t = 1$

$t = e$



3. Consider the closed curve in the *x*-*y* plane given by $2x^2+5x+y^2+y=8$. Which of the following is correct?

(a) $\frac{dy}{dx}=-\frac{4x+5}{8x+2y+1}$ (b) $\frac{dy}{dx}=\frac{4x+5}{2y+1}$

(c) $\frac{dy}{dx}=-\frac{4x+5}{8x+2y}$ (d) $\frac{dy}{dx}=\frac{4x+5}{8x+2y}$

(d) $\frac{dy}{dx}=\frac{-4x-5}{1+2y}$

$4x+5+2y\frac{dy}{dx}+\frac{dy}{dx}=0$

$(2y+1)\frac{dy}{dx}=-5-4x$

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4. Let *H* represent a circle with diameter *k*. The area of *H* decreases at a rate of 2π cm/sec. When the radius is 3cm, what is $\frac{dk}{dt}$ in cm/sec?

a) $-\frac{2}{3}$ b) $-\frac{1}{3}$ c) $\frac{1}{3}$ d) $\frac{2}{3}$ e) 2

$\frac{dA}{dt}=2\pi$

$A=\pi\left(\frac{k}{2}\right)^2=\frac{\pi}{4}k^2$

$\frac{dA}{dt}=\frac{\pi}{2}k\frac{dk}{dt}=\frac{\pi}{2}(3)k'$

$=2\pi$

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5. A particle's acceleration function is $a(t)=\sin 2t$, and its velocity is 0 and position is 1 at $t=0$. Which of these represents the particle's position function?

a) $x(t)=-\sin 2t+1$

b) $x(t)=-\sin 2t-t+1$

c) $x(t)=-\frac{1}{2}\cos 2t+\frac{1}{2}$

d) $x(t)=\frac{1}{2}\cos 2t-\frac{1}{2}$

e) $x(t)=-\frac{1}{4}\sin 2t+\frac{1}{2}t+1$

$v=\int \sin 2t\,dt$

$=-\frac{1}{2}\cos 2t+C$

$(0,0)\Rightarrow 0=-\frac{1}{2}+C \Rightarrow C=1/2$

$x=\int -\frac{1}{2}\cos 2t+\frac{1}{2}\,dt$

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6. A particle is moving upward along the y-axis until it reaches the origin and then it moves downward such that $v(t) = 8 - 2t$ for $t \geq 0$. The position of the particle at time t is given by

a) $y(t) = -t^2 + 8t - 16$ b) $y(t) = -t^2 + 8t + 16$
c) $y(t) = 2t^2 - 8t - 16$ d) $y(t) = 8t - t^2$
e) $y(t) = 8t - 2t^2$

$y = \int 8 - 2t = 8t - t^2 + c$

$0 = 8(0) - 0^2 + c \Rightarrow c = 0$

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7. Gravel is being dumped from a conveyor belt at a rate of 35 ft^3/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 15ft high?

a) 0.27 ft/min b) 1.24 ft/min c) 0.14 ft/min

d) 0.2 ft/min e) 0.6 ft/min

$\frac{dv}{dt} = 35$ $V = \frac{\pi}{3} r^2 h$ $h = 2r$

$V = \frac{\pi}{3} r^2 (2r) = \frac{2\pi}{3} r^3$

$\frac{dv}{dt} = 2\pi r^2 \frac{dr}{dt}$

$35 = 2\pi (7.5)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = .099$

$\frac{dh}{dt} = 2\frac{dr}{dt} = .198$

8. The population P(t) of a species satisfies the logistic differential equation $\frac{dP}{dt} = \frac{1}{4000}P(400-P)$, where P(0) = 100. What is the maximum rate of change of $P(t)$?

a) 10

b) 100

c) 200

d) 400

e) 4000

$A = 400$

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9. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

a) $-\frac{7}{2}$ b) -2 c) $\frac{2}{7}$ d) $\frac{3}{2}$ e) $\frac{7}{2}$

$2x + x\frac{dy}{dx} + y = 0$

$x = 2 \Rightarrow 4 + 2y = 10$

$y = 3$

$4 + 2\frac{dy}{dx} + 3 = 0$

$2\frac{dy}{dx} = -7$

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Score__________

Directions: Show all work.

1. Below is a chart of your speed driving to school in meters/second. Use the information below to find the values in a) and b) below.

t (in seconds)	0	30	90	120	220	300	360
$v(t)$ (in m/sec)	0	21	43	38	30	24	0

a. Approximate your acceleration at $t = 100$.

$$A = v'(100) = \frac{38-43}{120-90} = \frac{-5}{30} = -\frac{1}{6} \text{ m/sec}^2$$

b. Given you result in (a), are you speeding up or slowing down at $t = 100$? Explain, using the correct units.

$V > 0$ & $a < 0$ $\therefore$ The car is slowing down

c. Find an approximation for $\int_0^{360} v(t)\,dt$ using left Riemann rectangles. Express your answer in correct units. Using the correct units, explain the meaning of your result.

$$\int_0^{360} v(t)\,dt \approx 0(\overset{0}{\cancel{30}}) + 60(21) + 30(43) + 100(38)$$
$$+ 80(30) + 60(24)$$

$$= 13190 \text{ m}$$

YOU TRAVELED APPROX 13,190 METERS BETWEEN $t = 0$ AND $t = 360$ SECONDS

2. The fishing industry is a major part of California's economy. A catch-and-release study of Chinook salmon on the Sacramento Delta near Rio Vista was undertaken in 2008. Over 60 days, the rate at which new fish were caught and released followed the equation $\frac{dF}{dt} = .004F(100 - F)$, where $\frac{dF}{dt}$ was measured in number of smolt (young salmon) caught per day.

a) If $F(0) = 10$, what is $\lim_{t \to \infty} F(t)$? $= A$

$A = 100$

b) Using the correct units, explain $\lim_{t \to \infty} F(t)$.

100 SALMON WHERE THE MAXIMUM NUMBER OF SALMON CAUGHT & RELEASED

c) If $F(0) = 25$, how many smelt are captured and release when $\frac{dF}{dt}$ is at its greatest?

$\frac{A}{2} = 50$

d) Data from a different study showed $\frac{dF}{dt} = .004(100 - F)$, where $F(0) = 10$.

Use separation of variables to solve the differential equation.

$$-\int \frac{(-dF)}{100-F} = \int .004\, dt$$

$$-\ln|100-F| = .004t + C$$

$$\ln|100-F| = -.004t + C$$

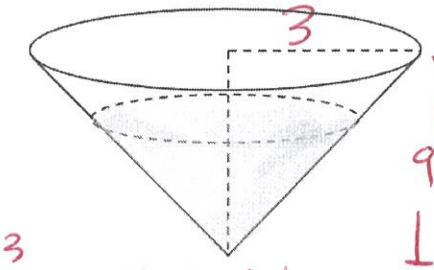
$$|100-F| = e^{-.004t+C} = Ke^{-.004t}$$

$$(0,10) \rightarrow 90 = Ke^{0} \rightarrow K = 90$$

$$100 - F = 90e^{-.004t}$$

$$F = 100 - 90e^{-.004t}$$

3. You are designing a manufacturing process where you are draining water from an inverted conical tank. The radius of the tank 6 meters, and the tank is 9 meters tall. You know that the water drains from the tank at a rate of 23 m^3/hour. [Note: $V = \frac{1}{3}\pi r^2 h$]



$$\frac{r}{h} = \frac{3}{9} \rightarrow r = \frac{1}{3}h$$

a. Find an equation for the volume of the cone depending on the height only.

$$V = \frac{\pi}{3} \cdot \left(\frac{1}{3}h\right)^2 h$$

$$= \frac{\pi}{27} h^3$$

b. Find $\frac{dh}{dt}$ when $h = 6$.

$$\frac{dv}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$$

$$23 = \frac{\pi}{9}(6)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{23}{4\pi}$$

c. How fast is the surface area of the water changing?

$$A = \pi r^2 \qquad \frac{dr}{dt} = \frac{1}{3}\frac{dh}{dt} = \frac{23}{12\pi}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(3)\frac{23}{12\pi} = \frac{23}{2} \ m^2/\text{HR}$$