AB Calculus '19-20
Dx Apps II Test
Calculator allowed.

Name Solution Key

1. If $y^2 - 3x = 7$, then $\frac{dy}{dx} =$

a) $-\frac{6}{7y^3}$ b) $-\frac{3}{y^3}$ c) 3 d) $\frac{3}{2y}$ e) $-\frac{9}{4y^3}$

$2y\frac{dy}{dx} - 3 = 0$

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2. A particle moves along the x-axis so that its velocity at time $t \geq 0$ is given by $v(t) = \frac{t^2 - 1}{t^2 + 1}$. What is the total distance traveled by the particle from $t = 0$ to $t = 2$?

a) 0.214 b) 0.320 c) 0.600 d) 0.927 e) 1.600

$\int_0^2 \left| \frac{t^2 - 1}{t^2 + 1} \right| dt =$ (MATH 8)

3. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 cm, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$.)

a) -108π b) -72π c) -48π d) -24π e) -16π

$\frac{dr}{dt} = -2$

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 8\pi (3)(-2)$

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4. The acceleration function for a particle moving along a line is $a(t) = t + 4$ with initial conditions $v(0) = 3$ and $x(0) = 1$. Find the velocity at time t.

a) $x(t) = \frac{t^2}{2} + 4t + 3$

b) $x(t) = \frac{t^3}{6} + 2t^2 + 3t + 1$

c) $x(t) = \frac{t^3}{6} + 2t^2 + 1$

d) $x(t) = \frac{t^3}{6} + 2t^2 + t + 3$

e) $x(t) = \frac{t^3}{6} + 4t^2 + 4$

$v = \frac{t^2}{2} + 4t + C_1$

$(0, 3) \rightarrow C_1 = 3$

$a = \int \left(\frac{1}{2}t^2 + 4t + 3\right) dt$

$= \frac{1}{6}t^3 + 2t^2 + 3t + C_2$

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5. Consider the closed curve in the x-y plane given by $x^2+xy+y^3=-1$. Which of the following is correct?

(a) $\frac{dy}{dx}=-\frac{2x+y}{x+3y^2}$

(b) $\frac{dy}{dx}=-\frac{x+3y^2}{2x+y}$

(c) $\frac{dy}{dx}=\frac{-2x}{1+3y^2}$

(d) $\frac{dy}{dx}=\frac{-2x}{x+3y^2}$

(e) $\frac{dy}{dx}=-\frac{2x+y}{x+3y^2-1}$

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6. An object moves with velocity $v(t)=\sec^2(2t)$. It is known that the particle's position at time 0 is 2. What is the particle's position function?

a) $s(t)=\tan(2t)+2$

b) $s(t)=\frac{1}{2}\tan(2t)+2$

c) $s(t)=\sec^2(2t)\tan^2(2t)+2$

d) $s(t)=\ln|\sec(2t)|+2$

e) $s(t)=\frac{1}{2}\ln|\sec(2t)|+2$

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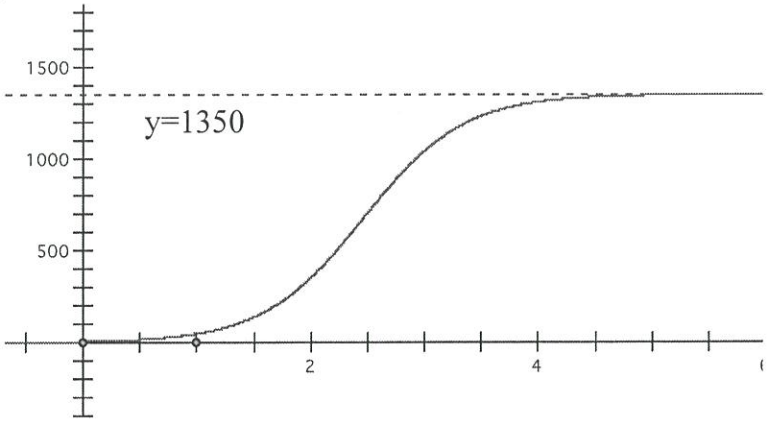
7. The rate at which a rumor spreads at school of 1350 students can be modeled by the graph below, where R is the number of students that have heard the rumor t hours after 9am.



Which of the following statements is false?

a) $\lim_{t\to\infty} y(t) = 1350$

b) A solution equation to the differential might be $\frac{dR}{dt} = 2.272(1350 - R)$

NEEDS R HERE

c) $\frac{d^2y}{dt^2} = 0$ when $y = 650$. F

d) When $y < 675$, $\frac{dR}{dt} > 0$ and $\frac{d^2R}{dt^2} > 0$

8. A particle is moving along the x-axis and its position at time $t \geq 0$ is given by $S(t) = (t-2)^2(t-6)$. Which of the following is (are) true?

I. The particle changes direction at $x = 2$ and $x = 6$.
II. The particle is slowing down on [0, 2].
III. The particle is speeding up on [2, 6].

a) I, II and III b) II and III only c) I and III only

d) II only e) I only

$a = 6t + 4$

a: − 0 + (at $-2/3$)

v: + 0 − 0 + (at $t = -10/3$ and 2)

$V = (t-2)^2(1) + (t-6)2(t-2)'$

$= (t-2)(t-2+2t+6)) = (t-2)(3t+10) = 3t^2+4t-20$

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9. Water is flowing into a spherical tank with 6-foot radius at the constant rate of 30π cu ft per hour. When the water is h feet deep, the volume of the water in the tank is given by $V = \frac{\pi h^2}{3}(18-h)$. What is the rate at which the depth of the water in the tank is increasing the moment when the water is 2 feet deep?

a) 0.5 ft/hour b) 1.0 ft/hour c) 1.5 ft/hour

d) 2.0 ft/hour e) 2.5 ft/hour

$\frac{dv}{dt} = 30\pi$

$V = 6\pi h^2 - \frac{\pi}{3}h^3$

$\frac{dv}{dt} = 12\pi h \frac{dh}{dt} - \pi h^2 \frac{dh}{dt}$

$= \pi h(12-h)\frac{dh}{dt}$

$30\pi = \pi(2)(10)\frac{dh}{dt}$

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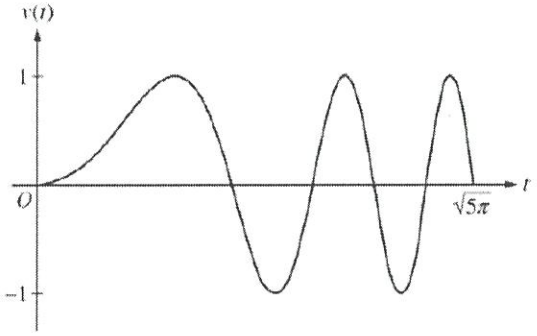
Score_____

Directions: Show all work.

1. A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of $v(t)$ for $0 \leq t \leq \sqrt{5\pi}$ is shown below.



The position of the particle at $t = 0$ is $x(0) = 5$.

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(1) a. Find the acceleration of the particle at $t = 3$.

$a = \frac{d}{dt}(\sin t^2) = \cos t^2 \,(2t)$

$a(3) = 6\cos 9 = -5.467$

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(2) b. Find the total distance traveled by the particle from $t = 0$ to $t = 3$.

TOTAL DISTANCE $= \int_0^3 |\sin t^2| \, dt = 1.702$

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(2) c. Find the position of the particle at $t = 3$.

$$x(3) = x(0) + \int_0^3 \sin t^2 \, dt$$

$$= 5 + .774$$

$$= 5.774$$

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d) For $0 \le t \le \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

(4) MAX DISTANCE MEANS $x' = 0$ OR THE ENDPOINTS

FIND t BY GRAPHING

t	$x(t)$
0	5
1.772	5.895
2.506	5.430
3.069	5.788
3.545	5.486
$\sqrt{5\pi}$	5.752

FURTHEST RIGHT AT $t = 1.772$

2. Research at the University of Tennessee Anthropological Research Facility, (aka The Body Farm) shows that a 233 lb. male body buried six feet underground without a coffin will decompose to a 33 lb. skeleton in 12 days. The table below shows $W(t)$, the rate of decomposition of the flesh in pounds per day, between $t=0$ and $t=12$ days.

t	0	1	2	3	4	5	6	7	8	9	10	11	12
$W(t)$	0	2.4	5.7	13.2	22.0	36.5	44.1	36.5	22.5	10.9	4.9	2.1	0.1

a) Approximate $W'(7)$ and explain thr result using the appropriate units.

(3) (4)

$$W'(7) = \frac{22.5 - 44.1}{8-6} = \frac{-21.6}{2} = -10.8$$

THE DECOMPOSITION RATE IS DECREASING BY 10.8 $\frac{LBS}{Day^2}$

b) Use a trapezoidal approximation to find the total weight of the body which decomposed between $t=0$ and $t=8$ days.

(2)

$$\int_0^8 W(t)\,dt \approx \frac{2.4}{2} + \frac{2.4+5.7}{2} + \frac{5.7+13.2}{2} + \frac{13.2+22}{2} + \frac{22+36.5}{2}$$

$$+ \frac{36.5+44.1}{2} + \frac{44.1+36.5}{2} + \frac{36.5+22.5}{2}$$

$$= 171.65 \text{ LBS}$$

c) Body decomposition rate depends on both the amount of material that has decomposed and the amount not yet decomposed. Thus, body decomposition is modeled by a logistic growth equation. In this case, the differential equation is $\frac{dN}{dt} = .2N(200 - N)$. At what weight of deposed flesh is the rate the highest?

(1) $= ky(A - y) \rightarrow A = 200$

THEREFORE THE FASTEST RATE IS AT $N = 100$

d) The data on the table can be modeled by $R(t) = 50e^{-.2(t-6)^2}$, where $R(t)$ is the rate, in pounds per day, at which the body is decomposing. Write an equation for $0 \le t \le 12$ which would determine the weight of the body at any time t.

(2)

$W(t) = \int_0^t 50e^{-.2(x-6)^2} dx$ lbs