AP Calculus AB '19-20

Fall Final Part I

Calculator Allowed

Name: Solution Key

1. Which of the following statements are **false**?

I. $\int(\csc u)\,dx = \ln|\csc u \cancel{+} \cot u| + c$

II. $\int a^u\,du = \dfrac{a^{u+1}}{u+1} + c,\ u \neq -1$ $= a^u/\ln a + c$

III. $\int\left(\dfrac{1}{\sqrt{9-x^2}}\right)dx = \sin^{-1}\dfrac{x}{3} + c$ TRUE

(A) I only (B) I and II only (C) III only

(D) I and III (E) All these

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2. $\int(t-4)\left(t^2-8t\right)^5 dt =$ $\frac{1}{2}\int(t^2-8t)^5(2(t-4))\,dt$

$u = t^2-8t$

$du = 2t-8 = 2(t-4)$

$= \frac{1}{2}\int u^5\,du = \frac{1}{12}u^6 + c$

(A) $\dfrac{\left(t^2-8t\right)^6}{6}+C$

(B) $\dfrac{\left(t^2-8t\right)^6}{12}+C$

(C) $\dfrac{\left(t^2-8t\right)^6}{3}+C$

(D) $\dfrac{(t-4)^6}{6}+C$

(E) $\dfrac{(t-4)^6}{3}+C$

3.

h(t) (°C)	10.5	11.4	12.5	11.3
t (hours)	2	3	5	8

The continuous function h(t) gives the temperature, in °C, of a small town in Finland. Using a trapezoidal sum with subintervals indicated by the table, approximate the average temperature of the town over the interval 2 to 8 hours.

(A) 66.55
(B) 21.85
(C) 11.425
(D) 10.925
(E) 4.50

$$\frac{10.5+11.4}{2}(1) + \frac{11.4+12.5}{2}(2) + \frac{12.5+11.3}{2}(3)$$

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4. The minimum value of $g(x) = -x^3 + 2x^2$ on $[-1, 3]$ occurs when $x=$

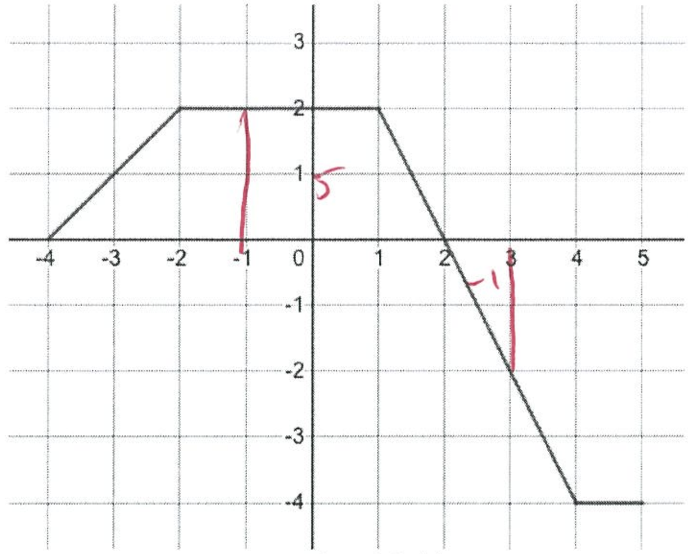
(A) -1 (B) 0 (C) $\frac{4}{3}$ (D) 2 (E) 3

$$g' = -3x^2 - 4x = -x(3x+4) = 0$$

$= \cancel{23x}$ $x = 0, \frac{4}{3}$

cv	g(x)	
−1	3	max
0	0	min
4/3		max
3	−9	min

5. The graph below gives the graph of $f'(x)$, the derivative of $f(x)$. If it is known that $f(-1)=3$, what is the value of $f(3)$?



graph of f'(x)

$3+\int_{-1}^{3} = 3+5-1$

(A) 3 (B) 4 (C) 6 (D) 7 (E) 9

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6. Using the line tangent to $y=\sqrt[4]{3x}$ at $x=27$, approximate $\sqrt[4]{90}$.

(A) 3.070

(B) 3.078

(C) 3.080

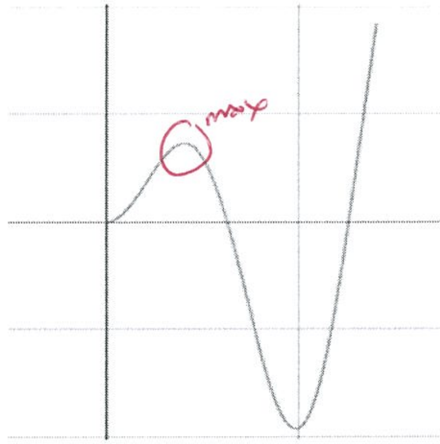
(D) 3.083

(E) 3.105

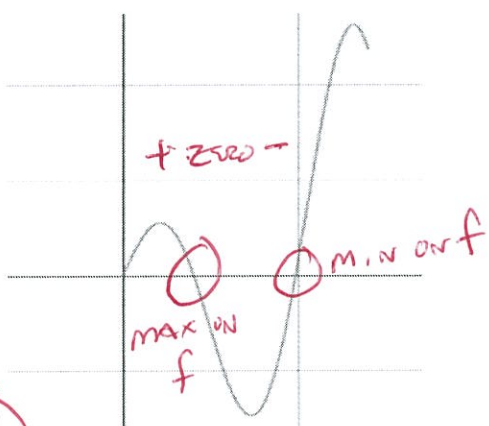
$\frac{dy}{dx} = \frac{1}{4}(3x)^{-3/4}(4)$ $m = \frac{3}{4}\left(\frac{1}{27}\right) = \frac{1}{36}$

$y - 3 = \frac{1}{36}(x-27)$

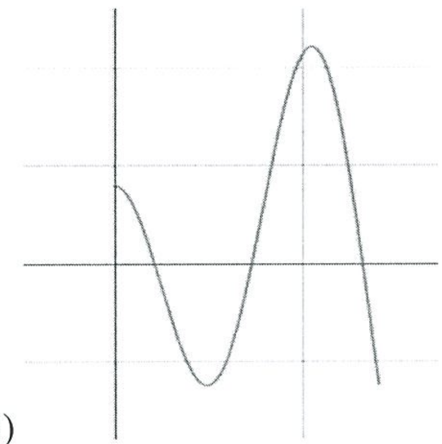
$y(30) = \frac{1}{12} + 3$



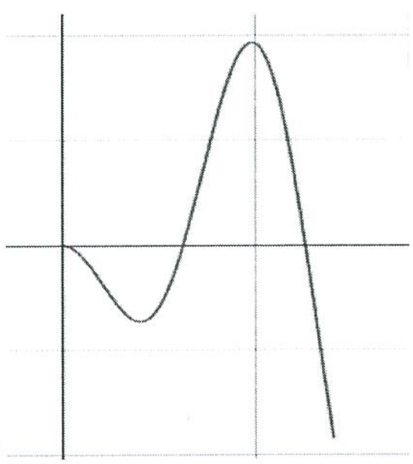
7. Which of the graphs below represents the derivative of the function graphed above?



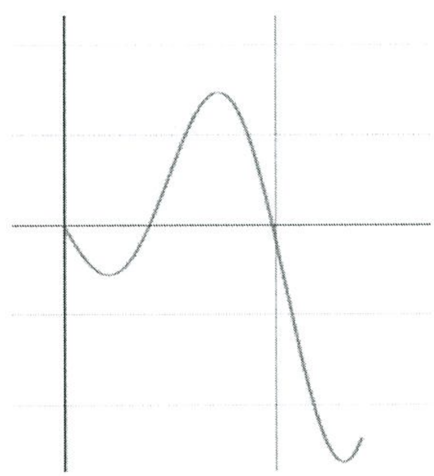
(A)



(B)



(C)



(D)

8. Let $y = g(x)$ be a twice-differentiable function and let $y = t(x)$ represent the line tangent to $g(x)$ at $x = 1$. If $t(x) < g(x)$ for all x-values except $x = 1$, which of the following must be true?

MEANS THE TANGENT LINE IS BELOW THE CURVE, SO $g(x)$ IS CONCAVE UP

(A) $g(1) \geq 0$
(B) $g'(1) \geq 0$
(C) $g'(1) \leq 0$
(D) $g''(1) \geq 0$
(E) $g''(1) \leq 0$

9.

x	1	2	4	8
$f(x)$	-3	4	9	-1
$g(x)$	0	6	2	1
$f'(x)$	9	-4	3	2
$g'(x)$	10	1	3	5

Let $h(x) = g(x) \cdot f(x^3)$. What is the value of $h'(2)$?

$h'(x) = g(x) \cdot f'(x^3)(3x^2) + f(x^3) \cdot g'(x)$

$= g(2) \cdot f'(8) \cdot 12 + f(8) \cdot g'(2)$

(A) -6

(B) 2

(C) 11

(D) 24

(E) 143

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10. If $j(x)$ is a continuous function where $\int_4^0 j(x)\,dx = 5$, then $\int_4^0 j(4-x)dx =$

(A) 5
(B) -5
(C) 11
(D) 21
(E) -4

$u = (4-x) \quad u(0) = 4$
$du = -dx \quad u(4) = 0$

$-\int_0^4 j(u)\,du = \int_4^0 j(u)\,du = 5$

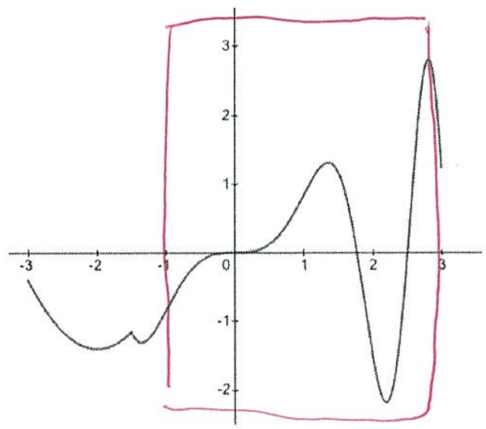
11.] To which of these functions does the Mean Value Theorem apply on the interval $[-1,3]$?

$$f(x) = \begin{cases} 4x-1, & -1 \le x < 2 \\ x^2, & 2 \le x \le 3 \end{cases} \qquad g(x) = \begin{cases} 3x^4 - x, & x \le 1 \\ x+1, & x > 1 \end{cases}$$

NOT CONTINUOUS

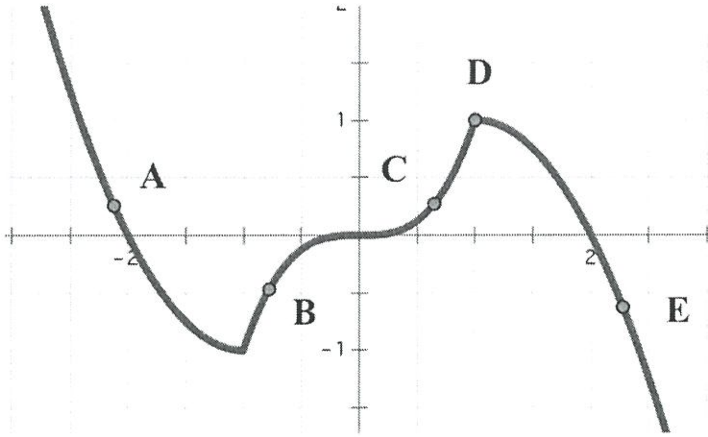
NOT DIFFERENTIABLE

$h(x)$, shown in the graph below



(A) $f(x)$ only (B) $g(x)$ only (C) $h(x)$ only

(D) $g(x)$ and $h(x)$ only (E) None of these



DECREASING & CONCAVE UP

12. At what point on the above curve is $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$

(A) A (B) B (C) C (D) D (E) E

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13. For $t \geq 0$ hours, H is a differentiable function of t that gives the change in temperature, in degrees Celsius per hour, at an Arctic weather station. In what units would $\frac{1}{t}\int_0^t H(x)\,dx$ be measured?

$\frac{1}{HR} \int \frac{°C}{HR} \cdot HR = \frac{°C}{HR}$

(A) degrees Celsius
(B) degrees Celsius per hour
(C) degrees Celsius per hour per hour.
(D) hours per degrees Celsius
(E) hours

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14. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t+1) - 2t + 1$. The total distance traveled by the particle from 0 to 3 is

(A) 0.667 (B) 0.704 (C) 1.540 (D) 2.667 (E) 2.901

$\int_0^3 |v(t)|\,dt =$

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15. If $f(x) = \ln(\sec x)$, then $f'(x) =$ $\frac{1}{\sec x}$ $\sec x \tan x$

(A) $\tan x$

(B) $\sin x$

(C) $\cos x$

(D) $\sec x$

(E) $\cot x$

$\frac{1}{y}\,dy = \frac{1}{x^2+1}\,dx \Rightarrow \ln|y| = \tan^{-1}x + C$

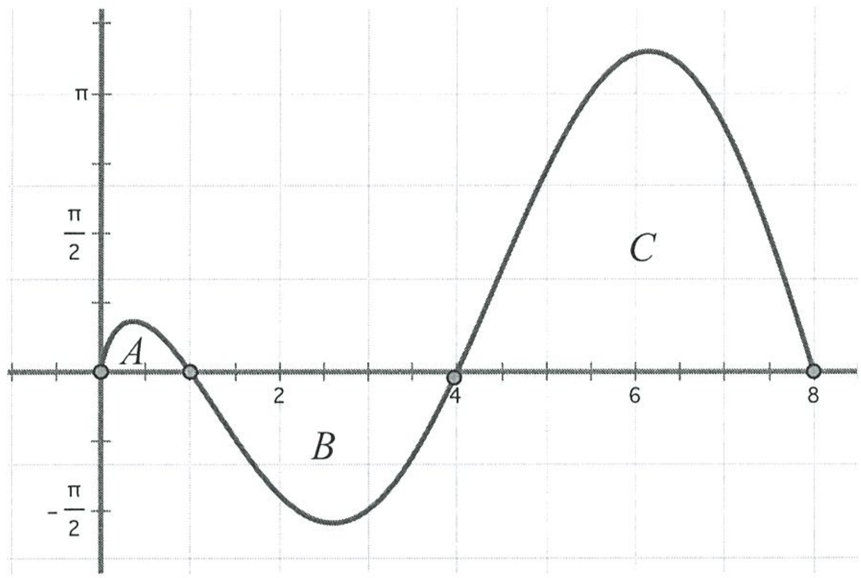
$\cancel{C = \ln 2}$

16. Find the particular solution to $(x^2+1)\dfrac{dy}{dx} = y$, where $y(0)=2$.

$y = Ke^{\tan^{-1}x}$

(A) $y = 2e^{\tan^{-1}x}$ (B) $y = e^{\tan^{-1}x}$ (C) $y = e^{(\tan^{-1}x)\ln 2}$ $K=2$

(D) $y = \sqrt{2\tan^{-1}x}$ (E) $y = \sqrt{2\tan^{-1}x + 4}$



17. In the figure above, A, B, and C are areas between the curve $f(x)$ and the x-axis. If $A=3$, $B=7$, and $C=21$, then $\int_0^8 f(x)\,dx =$ $A-B+C=$

(A) 31 (B) 17 (C) 25 (D) 28 (E) 42

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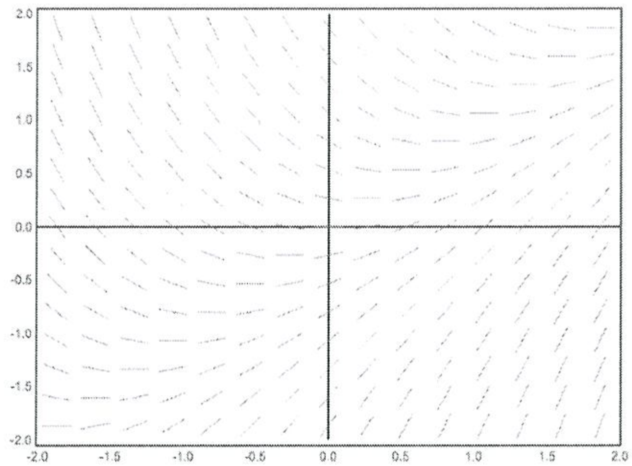
18. Which of the following differential equations corresponds to this slope field?



(A) $\frac{dy}{dx} = x$ (B) $\frac{dy}{dx} = xy$ (C) $\frac{dy}{dx} = y - x$

(D) $\frac{dy}{dx} = x + y$ (E) $\frac{dy}{dx} = x - y$

AT (0,1) THE SLOPE IS NEGATIVE 1

End of

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