AB Calculus '19-20 Name Solution Key

Limit Practice Test

No Calculator

1. Let $f(x)=\begin{cases} \tan x, & \text{if } x\le 0 \\ -\ln|x+1|, & \text{if } 0<x \end{cases}$. Which of the following statements is true about f?

$\tan 0 = 0 = -\ln 1$

$f' = \begin{cases} \sec^2 x & \to \sec^2 0 = 1 \\ -1/(x+1) & \to \frac{-1}{0+1} = -1 \end{cases}$

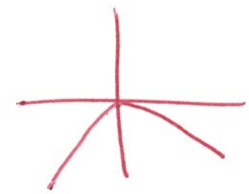
I. f is continuous at $x = 0$.

~~II.~~ f is differentiable at $x = 0$.

III. f has a local maximum at $x = 0$.

a) I only b) II only c) III only

d) I and II only e) I and III only



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2. The function f is not continuous at $x = b$. Which of the following statements **must** be true?

(a) $\lim_{x\to b} f(x)$ dne (b) $\lim_{x\to b} f(x) \ne f(b)$ (c) $\lim_{x\to b^-} f(x) = \lim_{x\to b^+} f(x)$

(d) $\lim_{x\to b^-} f'(x) \ne \lim_{x\to b^+} f'(x)$ (e) ~~All of these~~

None of these

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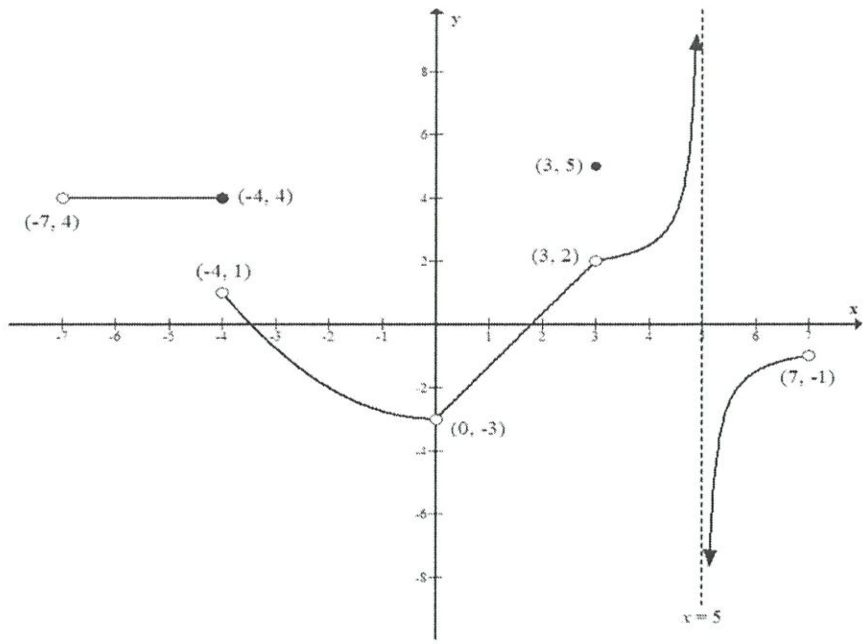
3. The function f is defined on the interval $x \in [-1, 5]$ and has the graph shown below.



Which of the following is (are) true?

I. $\lim_{x\to 0} f(x) = -3$

II. $\lim_{x\to 3} \dfrac{f(x) - f(3)}{x-3} = 5$

III. $\lim_{x\to -4^-} [1 + f(x)] = f(3)$

a) I only b) II only c) III only

d) I and II only e) I and III only

4. $\lim_{h\to 0}\dfrac{\cos^2\left(\dfrac{\pi}{2}+h\right)}{h} =$ $\overset{L'H}{=}$ $\lim_{h\to 0} \dfrac{2\cos\left(\frac{\pi}{2}+h\right)\left(-\sin\left(\frac{\pi}{2}+h\right)\right)}{1} = 0$

(a) e^2 (b) 1 (c) $\frac{1}{2}$ (d) 0 (e) DNE

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5. $\lim_{x\to\infty}\dfrac{4x^5+3x^4+2x^3+x^2+1}{3x^3-9x^2+4x+15} =$

(a) 0 (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 3 (e) DNE

NUM > DENOM DEGREE

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6. If a and b are positive constants, then $\lim_{x\to\infty}\dfrac{\ln(bx+1)}{\ln(ax^2+3)} =$ $\overset{L'H}{=}$ $\lim_{x\to\infty} \dfrac{\frac{1}{(bx+1)}(b)}{\frac{1}{(ax^2+3)}(2ax)}$

$= \lim_{x\to\infty} \dfrac{b(ax^2+3)}{2ax(bx+1)}$

$= \lim_{x\to\infty} \dfrac{abx^2+3}{2abx^2+2ax}$

a) 0 b) $\frac{1}{2}$ c) $\frac{ab}{2}$ d) 2 e) ∞

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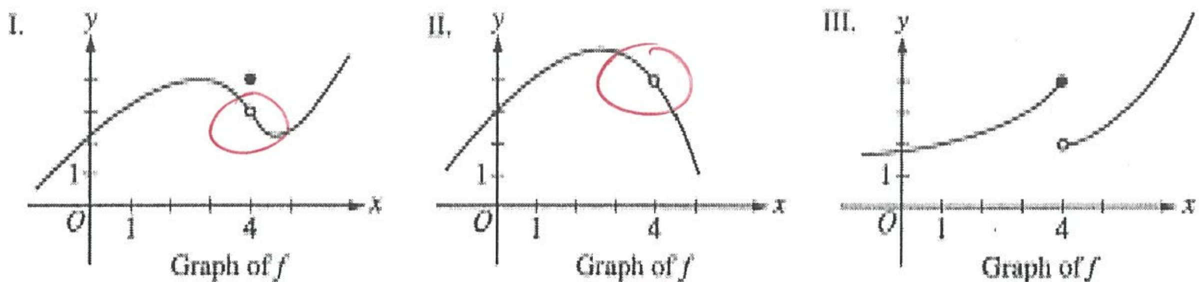
7. For which of the following does $\lim_{x\to 3} f(x)$ exist?



(a) I only (b) II only (c) III only (d) I and II only (e) All of these

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8. Let $f(x)=\begin{cases} \frac{\pi}{2}-\sin^{-1}(x-1), & \text{if } x<2 \\ 0, & \text{if } x=2 \\ \frac{\pi}{2}\sqrt{x-2}, & \text{if } x>2 \end{cases}$. Which of the following statements is true about f?

$f'(x) = \begin{cases} \frac{1}{\sqrt{1-(x-1)^2}} \\ \frac{\pi}{4}(x-2)^{-1/2} \end{cases}$

I. f is continuous at $x = 2$.
II. f is differentiable at $x = 2$.
III. f has a local minimum at $x = 2$.

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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9. Which of the following functions is NOT differentiable at $x=1$?

(a) $f(x)=x^2-1$ (b) $f(x)=e^{1-x}$ (c) $f(x)=\ln(1-x)$

(d) $f(x)=\begin{cases} \dfrac{1}{x+1} & \text{for } x\neq -1 \\ 0 & \text{for } x=-1 \end{cases}$ (e) $f(x)=\tan x$

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10. $\lim\limits_{x\to 0}\dfrac{\int_0^x \sin t^2\,dt}{x^3}=$ L'H $\lim\limits_{x\to 0}\dfrac{\sin x^2}{x^3}=\lim\limits_{x\to 0}\dfrac{\cos x^2(2x)}{3x^2}=\dfrac{1}{0}$

(a) 0 (b) 1 (c) $\frac{1}{3}$ (d) 3 (e) DNE

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11. A function $f(x)$ has a vertical asymptote at $x=2$. The derivative of $f(x)$ is negative for all $x\neq 2$. Which of the following statements are **false**?

~~I.~~ $\lim\limits_{x\to 2}f(x)=+\infty$ II. $\lim\limits_{x\to 2^-}f(x)=-\infty$ III. $\lim\limits_{x\to 2^+}f(x)=+\infty$

(a) I only (b) II only (c) III only

(d) I and II only (e) I, II and III

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12. At $x = -3$, the function given by $f(x) = \begin{cases} -x^2, & \text{if } x < -3 \\ -9 - 6x, & \text{if } -3 \le x \end{cases}$ is

(A) Undefined

(B) Continuous but not differentiable

(C) Differentiable but not continuous

(D) Neither continuous nor differentiable

(E) Both continuous and differentiable

$\lim_{x \to -3^-} f(x) = -9$

$\lim_{x \to -3^+} f(x) = 9$

AB Calculus '19-20
Limit Practice Test
Calculator allowed

Name SOLUTION KEY

Score__________

Directions: Show all work.

1. $$f(x)=\begin{cases} \frac{4}{\pi}\tan^{-1}(1-x), \text{ if } x<0 \\ 1, \quad \text{if } x=0 \\ e^{-x}, \text{ if } 0<x \end{cases}$$

a) Is $f(x)$ continuous at $x=0$? Why/Why not?

i) $f(0)$ EXISTS

ii) $\lim_{x\to 0^+} \frac{4}{\pi}\tan^{-1}(1-x) = \frac{4}{\pi}\cdot\frac{\pi}{4} = 1$

$\lim_{x\to 0^-} e^{-x} = e^0 = 1$

$\lim_{x\to 0} f(x)$ EXISTS

iii) $\lim_{x\to 0} f(x) = f(0)$

CONTINUOUS

b) Is $f(x)$ differentiable at $x=0$? Why/Why not?

i) f IS CONTINUOUS

ii) $f'(x) = \begin{cases} \frac{4}{\pi}\cdot\frac{1}{1+(1-x)^2} \\ -e^{-x} \end{cases}$

$\lim_{x\to 0^-} = \frac{4}{2\pi} \neq \lim_{x\to 0^+} = -e^0 = -1$

$\therefore$ NOT DIFFERENTIABLE

2. $h(x)=\begin{cases} \dfrac{4-x^2}{4x^2-25}, & \text{if } x<-2 \\ 0, & \text{if } x=-2 \\ -\dfrac{1}{2}x+1, & \text{if } -2<x \end{cases}$

a) Is $h(x)$ continuous at $x=-2$? Why/Why not?

i) $h(-2)=0$

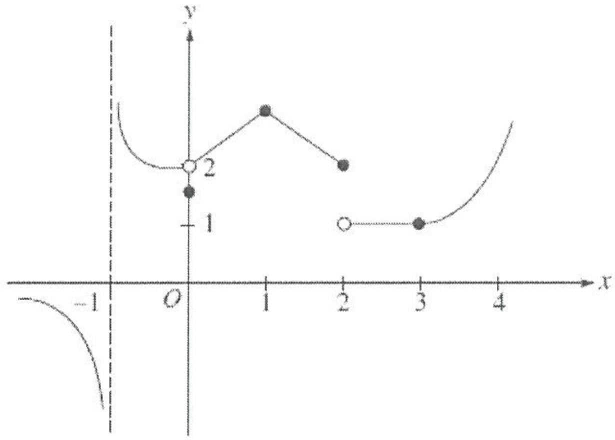
ii) $\lim_{x\to -2^-} h(x) = 0$

$\lim_{x\to -2^+} h(x) = 1+1=2$

$\therefore$ NOT CONTINUOUS

b) Is $h(x)$ differentiable at $x=-2$? Why/Why not?

NOT DIFF BECAUSE $h(x)$ IS NOT CONTINUOUS



3. For this graph, find

(a) $\lim_{x\to -1^-} f(x) =$ $-\infty$ (b) $\lim_{x\to 2^-} f(x) =$ 2 (c) $\lim_{x\to 3} f(x) =$ 1

(d) $\lim_{x\to -1} f(x) =$ D.N.E (e) $\lim_{x\to 2^+} f(x) =$ 1 (f) $\lim_{x\to 0} f(x) =$ 2

(g) $f(-1) =$ DNE (h) $f(0) =$ 1.5 (i) $f(2) =$ 2 (j) $f(3) =$ 1