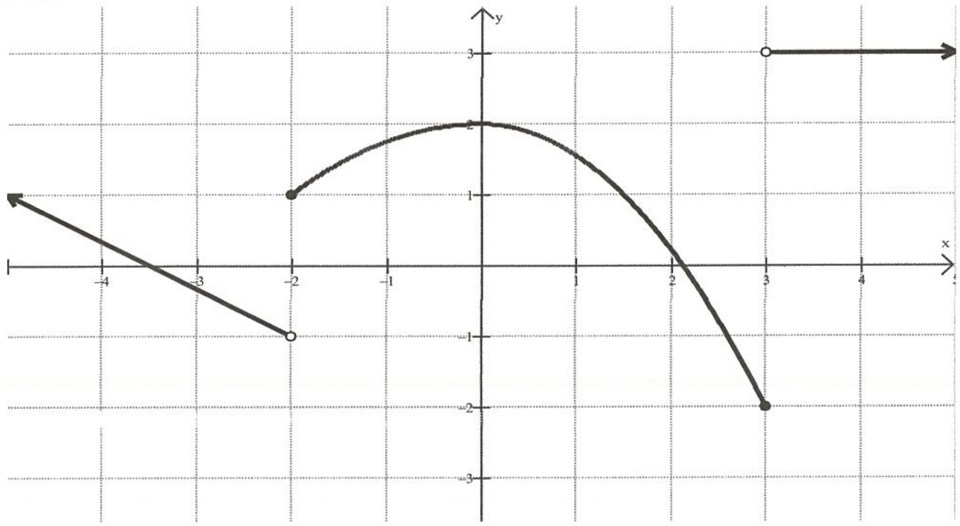
AB Calculus ’19-20
Limit Take-Home Test
No Calculator

Name Solution Key

1. The function f is defined on the interval $x \in (-4, 7)$ and has the graph shown below.



For which of the following statements are **false**?

I. $\lim_{x \to 5^+} f(x) = 4$. F

II. f is differentiable at $x = 0$. T

III. f has a local minimum at $x = 3$. T

(a) I only (b) II only (c) III only

(d) I and II only (e) I and III only

2. The end behavior of $g(x)=\sqrt{\frac{x^3-9}{x^2+4}}$

$n>m$ SO LEFT: $\sqrt{-\infty}$ DNE

RIGHT UP

a) $y=0$ on both ends
b) None on the left and up on the right
c) Up on the left and none on the right
d) $y=1$ on both ends
e) None on the left and $y=0$ on the right

---

3. Let m and b be real numbers and let the function f be defined by

$$f(x)=\begin{cases} 3x^2-mx+5 & \text{for } x\le 1 \\ mx+b & \text{for } x>1 \end{cases}$$

If f is both continuous and differentiable at $x=1$, then

(a) $m=3,\ b=2$
(b) $m=3,\ b=-2$
~~(c)~~ $m=-3,\ b=2$
~~(d)~~ $m=-3,\ b=-2$
(e) None of these

$3-m+5=m+b \Rightarrow 2m+b=8$

$m=3 \Rightarrow b=2$

$f'=\begin{cases} 6x-m \\ m \end{cases}$

$6-m=m$

$6=2m$

$3=m$

---

4. Let $f(x)=\begin{cases} -x+5, & \text{if } x<-2 \\ x^2+3, & \text{if } -2\le x\le 1 \\ 2x^3, & \text{if } 1<x \end{cases}$. Which of the following statements is **true** about f?

I. f is continuous at x = -2. T

II. f is differentiable at x = 1. F

III. f has a local minimum at x = -2. F

(a) I only (b) II only (c) III only

(d) I and III only (e) II and III only

---

5. $\lim_{x\to\infty} \dfrac{3x^5+3x^4+2x^3+x^2+1}{4x^6-9x^4+4x^3+15} =$

(a) 0 (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 3 (e) DNE

---

6. Let f be defined by $f(x)=\begin{cases} x^2+kx & \text{for } x<4 \\ 4\cos\left(\frac{\pi}{2}x\right) & \text{for } x\ge 4 \end{cases}$. Determine the value of k for which is continuous for all real x.

$4^2+4k=4$

$k=-3$

a) -6 b) -2 c) 8 d) 14 e) None of these

---

7. The graph of the function $f(x)$ is shown below. Which of the following statements **must** be false?

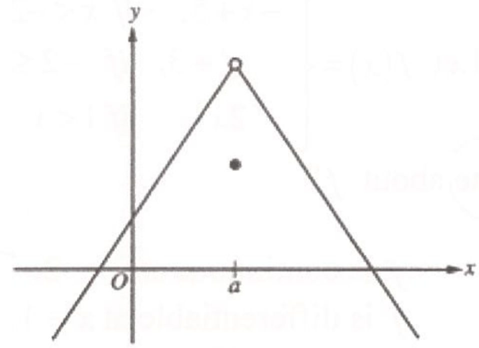
a) $f(a)$ does not exist. F

b) $f(x)$ is defined for $0<x<a$

c) $f(x)$ is not continuous at $x=a$.

d) $\lim_{x\to a} f(x)$ exists.

e) $\lim_{x\to a} f'(x)$ does exist.



Graph of f

---

8. A function $f(x)$ has a vertical asymptote at $x=2$. The derivative of $f(x)$ is negative for all $x\neq 2$. Which of the following statements are **false**?

$f'<0$

I. $\lim_{x\to 2} f(x)=+\infty$ F

II. $\lim_{x\to 2^-} f(x)=-\infty$ T

III. $\lim_{x\to 2^+} f(x)=+\infty$ T

(a) I only (b) II only (c) III only

(d) I and II only (e) I, II and III

---

9. Which of the following functions is both continuous and differentiable for all x in the interval $-4<x<4$

(a) $f(x)=\ln(x^2-4)$ (b) $f(x)=e^{(x^2-4)}$ (c) $f(x)=\sqrt{x^2-4}$

(d) $f(x)=\dfrac{1}{x^2-4}$ (e) $f(x)=|x^2-4|$

---

10. $\lim_{x\to 0}\frac{\int_0^{3x^2} e^t\,dt}{\tan x} =$ L'H $\lim_{x\to 0}\frac{e^{3x^2}(6x)}{\sec^2 x} = 0$

(a) 0 (b) 1 (c) $\frac{1}{3}$ (d) 6 (e) DNE

---

11. At $x = -3$, the function given by $f(x) = \begin{cases} 9 - x^2, & \text{if } x < -3 \\ 6x + 18, & \text{if } -3 \le x \end{cases}$ is

$\lim_{x\to -3} f(x) = 0$ CONT

(A) Undefined

(B) Continuous but not differentiable

(C) Differentiable but not continuous

(D) Neither continuous nor differentiable

(E) Both continuous and differentiable

$f' = \begin{cases} -2x \\ 6 \end{cases}$

$\lim_{x\to -3} f' = 6$

AB Calculus '19-20
Limit Take-Home Test
Calculator allowed

Name Solution Key

Score__________

Directions: Show all work.

1. $$f(x)=\begin{cases} -2\cos\left(\frac{\pi x}{2}\right), \text{ if } 0 \le x < 2 \\ 2 \ , \quad if\ x=2 \\ 2.25-.25e^{x-2}, \text{ if } 2 < x \le 5 \end{cases}$$

a) Is $f(x)$ continuous at $x=2$? Why/Why not?

i) $f(2)=2$

ii) $\lim_{x\to 2^-} f(x) = -2\cos\pi = 2$

$\lim_{x\to 2^+} f(x) = 2.25-.25e^0 = 2$

$\therefore \lim_{x\to 2} f(x)$ EXISTS

iii) $\lim_{x\to 2} f(x) = f(2) \therefore$ CONTINUOUS

b) Is $f(x)$ differentiable at $x=2$? Why/Why not?

$$f' = \begin{cases} \cancel{2}\sin\left(\frac{\pi}{2}x\right)\left(\frac{\pi}{\cancel{2}}\right) & \text{IF } x<2 \\ -.25e^{x-2} & \text{IF } 2<x \end{cases}$$

i) f IS CONTINUOUS

ii) $\lim_{x\to 2^-} f' = \pi\sin\pi = 0$

$\lim_{x\to 2^+} f'(x) = -.25e^0 = -.25$

$\therefore$ ~~DIS...~~ NOT DIFFERENTIABLE

3. $h(x)=\begin{cases} 1-2\sin x, \text{ if } 0 \le x \\ e^{-4x}, \text{ if } 0 < x \end{cases}$

a) Show that $h(x)$ is continuous at $x=0$.

i) $h(0)$ EXISTS $h(0)=1-2\sin 0=1$

ii) $\lim_{x\to 0^-} h(x) = 1$ $\lim_{x\to 0^+} h(x)=e^0=1$

$\lim_{x\to 0} h(x)=1$

iii) $\lim_{x\to 0} h(x) = h(0)$

$\therefore$ CONTINUOUS

b) For $x \ne 0$, express $h'(x)$ as a piecewise-defined function. Find the value of x for which $h'(x)=-3$

$h'(x) = \begin{cases} -2\cos x & \text{IF } x<0 \\ -4e^{-4x} & \text{IF } 0<x \end{cases}$

$h'=-3 \Rightarrow -2\cos x = -3 \rightarrow \cos x = 1.5$ NEVER

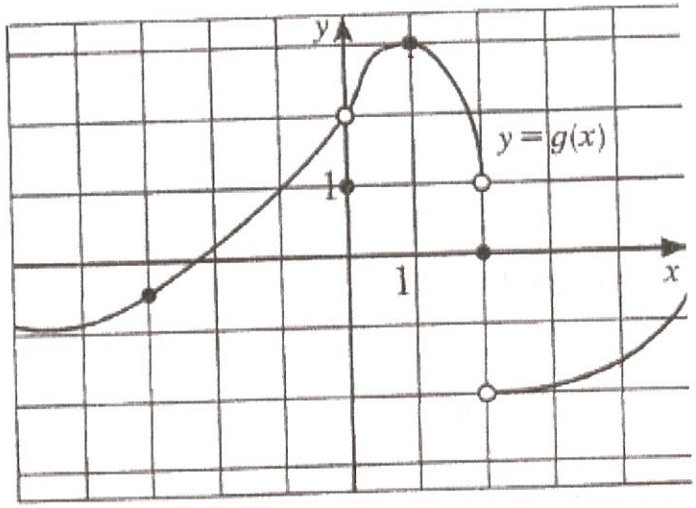
OR $-4e^{-4x} = -3 \Rightarrow e^{-4x} = 3/4 \Rightarrow x = \frac{-1}{4}\ln 3/4$

c) Find the average value of $h(x)$ on $x \in [-1,1]$

$\frac{1}{1-(-1)}\int_{-1}^{1} h(x) = \frac{1}{2}\int_{-1}^{0} 1-\sin 2x \, dx + \frac{1}{2}\int_{0}^{1} e^{-4x}\, dx$

$= \frac{1}{2}\left[x + \frac{1}{2}\cos 2x\right]_{-1}^{0} + \left[\frac{-1}{8}e^{-4x}\right]_0^1$

$= \frac{1}{2}\left(\frac{1}{2}+1+\frac{1}{2}\cos -2\right) + -\frac{1}{8}e^{-4} + \frac{1}{8}$



2. For this graph, find

(a) $\lim_{x\to 0^-} f(x) =$ 2

(b) $\lim_{x\to 2^-} f(x) =$ 1

(c) $\lim_{x\to 0^+} f(x) =$ 2

(d) $\lim_{x\to 1^-} f(x) =$ 3

(e) $\lim_{x\to 1^+} f(x) =$ 3

(f) $\lim_{x\to 1} f(x) =$ 3

(g) $f(0) = 1$

(h) $\lim_{x\to 2^+} f(x) = -2$

(i) $f(2) = 0$

(j) $\lim_{x\to 2} f(x) =$ DNE

(k) $f(1) = 3$

(l) $f(-3) = -1/2$

m) $f(4) = -1.6$