AP Calculus AB ’19 – ‘20
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Derivative Test

Name Solution Key

Multiple choice – Circle correct answer.

1. A normal line to the graph of a function f at the point $(x, f(x))$ is defined to be the line perpendicular to the tangent line at that point. An equation of the **normal** line to the curve $y = \sqrt[3]{x^2 - 1}$ at the point where $x = 3$ is

$y(3) = 2$

$y'(x) = \frac{1}{3}(x^2-1)^{-2/3}(2x)$

$m = y'(3) = \cancel{\frac{1}{3}(4)^{-2/3}}(6) =$

$= \frac{1}{3}\left(\frac{1}{4}\right)6 = 1/2$

$y - 2 = -2(x - 3)$

a) $y + 12x = 38$
b) $y - 4x = 10$
c) $y + 2x = 4$
d) $y + 2x = 8$
e) $y - 2x = -4$

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2. If $f(x) = \sin^3(3 - x)$, then $f'(x) =$ $3\sin^2(3-x)\cos(3-x)(-1)$

a) $3\sin^2(3 - x)$

b) $-3\sin^2(3 - x)$

c) $-3\cos^2(3 - x)$

d) $3\sin^2(3 - x)\cos(3 - x)$

e) $-3\sin^2(3 - x)\cos(3 - x)$

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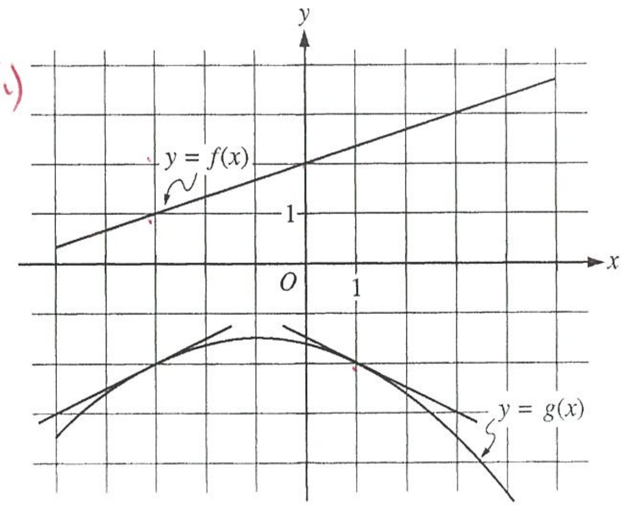
3. The figure below shows the graph of the functions f and g. The graphs of the lines tangent to the graph of g at $x=-3$ and $x=1$ are also shown. If $B(x)=f(g(x))$, what is $B'(1)$?



$B' = f'(g(1))\cdot g'(1)$
$= f'(-2)\, g'(1)$
$= \left(\frac{1}{3}\right)\left(-\frac{1}{2}\right)$

a) $-\frac{1}{2}$

b) $-\frac{1}{6}$

c) $\frac{1}{6}$

d) $\frac{1}{3}$

e) $\frac{1}{2}$

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4. Which of the following statements must be true?

I. $\frac{d}{dx}\left(x\sec^{-1}x\right)=\sec^{-1}x+\frac{1}{\sqrt{x^2-1}}$

II. $\frac{d}{dx}\left(\frac{3-2x}{3x+2}\right)=\frac{13}{(3x+2)^2}$

III. $\frac{d}{dx}\ln(1-x)=\frac{1}{x-1}$

a) I only   b) II only   c) II and III only

d) I and III only   e) I, II, and III

I $x\left(\frac{1}{|x|\sqrt{x^2-1}}\right) + \sec^{-1}x\,(1)$ TRUE

II $\frac{(3x+2)(-2)-(3-2x)(3)}{(3x+2)^2} = \frac{-13}{(3x+2)^2}$ FALSE

III $= \frac{1}{\ }(-1) = \frac{1}{\ }$ TRUE

CHAIN RULE, NOT PRODUCT RULE

5. If $f(x)=\sin\left[g(x)\right]$, then $\frac{d}{dx}\left[f(x)\right]$ is $= \cos(g(x)) \cdot g'(x)$

a) $\sin x \cdot g'(x) + g(x) \cdot \cos x$

b) $\sin x \cdot g'(x) - g(x) \cdot \cos x$

c) $\cos x \cdot g'(x)$

d) $\cos\left[g(x)\right] \cdot g'(x)$

e) None of these

6.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	3	4	-1
2	4	-1	7	8
4	1	2	2	1

Selected values of *f*, *g*, and their derivatives are indicated in the table above. Let $h(x) = g\left(f\left(x^2\right)\right)$. What is the value of $h'(2)$?

a) -8 b) -4 c) 1 d) 2 e) 16

$h' = g'(f(x^2)) \cdot f'(x^2)(2x)$

$h'(1) = g'(f(2)) \cdot f'(2)(4)$

$= g'(4)\, f'(2)\, (4)$

$= \cancel{8 \cdot 2 \cdot 2}$

$= 1 \cdot (-1)(4) = -4$

7. Find the approximate value of $\sqrt[3]{7}$ using the tangent approximation for $\sqrt[3]{x}$ at $x = 8$.

a) $\frac{25}{12}$ b) $\frac{23}{12}$ c) $\frac{15}{8}$ d) $\frac{7}{4}$ e) $\frac{13}{8}$

$f(x) = x^{1/3}$

$f(8) = 2$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(8) = \frac{1}{12}$

$y - 2 = \frac{1}{12}(x-8)$

$\sqrt[3]{7} = f(7) \approx y(7) = \frac{1}{12}(7-8) + 2 =$

$= \frac{23}{12}$

8a. $\frac{d}{dx}\left(\sqrt[3]{16+x^3}\right)$ $= \frac{1}{3}(16+x^3)^{-2/3}(3x^2$

$= \dfrac{x^2}{(16+x^3)^{2/3}}$

b. $\frac{d}{dx}\left(5e^{\cos(7x)}\right)$ $= 5e^{\cos 7x}(-\sin 7x)(7)$

$= -35 \sin 7x \cancel{\cos} e^{\cos 7x}$

c. $\frac{d}{dx}\left[x^4 - 14\sqrt[7]{x^9} + 8^x - \frac{1}{\sqrt[3]{x^7}} + \frac{1}{8x}\right]$ $= \frac{d}{dx}\left[x^4 - 14x^{9/7} + 8^x - x^{-7/3} + \frac{1}{8}x^{-1}\right]$

$= 4x^3 - 18x^{2/7} + 8^x \ln 8 + \frac{7}{3}x^{-10/3} - \frac{1}{8}x^{-2}$

d. $\frac{d}{dx}\left(\sec^{-1}(2x^2)\right)$ $= \dfrac{1}{2x^2\sqrt{(2x^2)^2-1}}(4x)$

$= \dfrac{2}{x\sqrt{4x^4-1}}$

9. $\frac{d}{dx}\left(\sqrt{\cos\left(1-x^2\right)}\right)$ $= \frac{d}{dx}\left(\cos\left(1-x^2\right)\right)^{1/2}$

$= \frac{1}{\cancel{2}}\left(\cos\left(1-x^2\right)\right)^{-1/2}\left(-\sin\left(1-x^2\right)\right)\left(-\cancel{2}x\right)$

$= \dfrac{x\sin\left(1-x^2\right)}{\cos^{1/2}\left(1-x^2\right)}$

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10. If $f(x)=e^{\cot 3x}$, find $f''(x)$.

$f' = e^{\cot 3x}\left(-\csc^2 3x\right)$

$f'' = e^{\cot 3x}\left(-2\csc 3x\left(-\csc 3x \cot 3x\right)(3) + \right.$

$\left.\left(-\csc^2 3x\right)\left(e^{\cot 3x}\left(-\csc^2 3x\right)(3)\right)\right)$

$= 6e^{\cot 3x}\csc^2 3x \cot 3x + 3e^{\cot 3x}\csc^4 3x$

$= 3\csc^2 x\, e^{\cot 3x}\left[2\cot 3x + \csc^2 3x\right]$

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11. If $f(x)=\tan^2(3x)$, find $f'(x)$ and $f''(x)$. Write answers in simplified, factored form.

$$f'(x) = 2\tan 3x \sec^2 3x\,(3)$$

$$= 6\tan 3x \sec^2 3x$$

$u = 6\tan 3x$ $\quad$ $v = \sec^2 3x$

$Du = 6\sec^2 3x\,(3)$ $\quad$ $Dv = 2\sec 3x \sec 3x \tan 3x\,(3)$

$= 18\sec^2 3x$ $\quad$ $= 6\sec^2 3x \tan 3x$

$$f''(x) = 6\tan 3x\left(6\sec^2 3x \tan 3x\right) + \sec^2 3x\left(18\sec^2 3x\right)$$

$$= 36\sec^2 3x \tan^2 3x + 18\sec^4 3x$$

$$= 18\sec^2 3x\left(2\tan^2 3x + \sec^2 3x\right)$$

12. $y=(4x^5-3)^7(7x^2+1)^5$. Find $\frac{dy}{dx}$ in factored form.

$u=(4x^5-3)^7$

$Du = 7(4x^5-3)^6(20x^4)$

$= 140x^4(4x^5-3)^6$

$V = (7x^2+1)^5$

$Dv = 5(7x^2+1)^4(14x)$

$= 70x(7x^2+1)^4$

$$\frac{dy}{dx} = \overset{u}{(4x^5-3)^7}\overset{Dv}{\left(\underset{70x(7x^2+1)^4}{\cancel{140x^4(4x^5-3)^6}}\right)} \overset{+}{+} \overset{v}{(7x^2+1)^5}\overset{Du}{(140x^4)(4x^5-3)^6}$$

$= 70x(4x^5-3)^6(7x^2+1)^4\left[(4x^5-3)+(7x^2+1)(2x^3)\right]$

$= 70x(4x^5-3)^6(7x^2+1)^4\left[4x^5-3+14x^5+2x^3\right]$

$= 70x(4x^5-3)^6(7x^2+1)^4(\cancel{22}\,18x^5\,\cancel{x}+2x^3-3)$