# AP Calculus AB '19-20

# Spring Practice Final Part IA

# Calculator NOT Allowed

Name:

Solution Key

1. If $y = x^2 e^{2x}$, then $\frac{dy}{dx} =$ $x^2 e^{2x}(2) + 2xe^{2x} = 2xe^{2x}(x+1)$

a) $2xe^{2x}$

b) $4xe^{2x}$

c) $xe^{2x}(x+1)$

[✓ d)] $2xe^{2x}(x+1)$

e) $xe^{2x}(x+2)$

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2. $\int \sqrt[5]{3^x}\,dx =$ $\int 3^{x/5}\,dx = 5\int 3^{x/5}\frac{1}{5}\,dx = \frac{5}{\ln 3}\cdot 3^{x/5} + C$

a) $5\sqrt[5]{3^x} + C$

[✓ b)] $\frac{5}{\ln 3}\sqrt[5]{3^x} + C$

c) $\frac{\sqrt[5]{3^x}}{\ln 3} + C$

d) $\sqrt[5]{3^x} + C$

e) $\frac{\sqrt[5]{3^x}}{5\ln 3} + C$

3. $\lim_{x\to 5}\frac{x-5}{|5-x|}$ $\Rightarrow \lim_{x\to 5^-} = -1$ ; $\lim_{x\to 5^+} = +1$

a) -1 b) 1 c) 0 d) $\infty$ [✓ e)] DNE

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4. $\int\left(2-\sin\frac{t}{5}\right)^2\cos\frac{t}{5}\,dt$ $= -5\int u^2\,du$

$= -\frac{5}{3}u^3 + c$

$u = 2 - \sin\frac{t}{5}$

$du = -\cos\frac{t}{5}\left(\frac{1}{5}dt\right.$

[✓ a)] $-\frac{5}{3}\left(2-\sin\frac{t}{5}\right)^3+c$

b) $\frac{5}{3}\left(2-\cos\frac{t}{5}\right)^3+c$

c) $\frac{1}{3}\left(2-\sin\frac{t}{5}\right)^3+c$

d) $5\left(2-\sin\frac{t}{5}\right)^3+c$

e) $-\frac{5}{3}\left(2-\cos\frac{t}{5}\right)^3+c$

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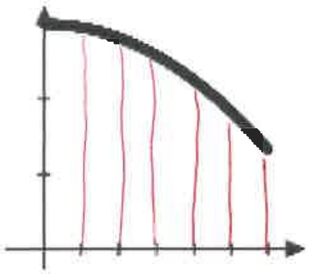
5. The graph of the function $f$ is shown below for $0 \le x \le 3$.



Of the following, which has the smallest value?

a) $\int_1^3 f(x)\,dx$

b) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 6 equal sub intervals. $> \int_1^3$

[✓ c)] Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 6 equal sub intervals. $< \int_1^3$

d) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 6 equal sub intervals.

e) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 6 equal sub intervals.

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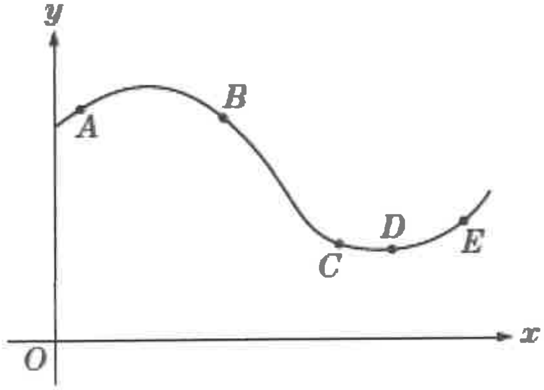
6. At which of the five points on the graph in the figure below are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

DECREASING &

CONCAVE DOWN



(a) A [✓ (b)] B (c) C (d) D (e) E

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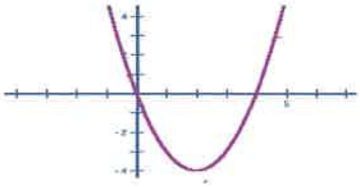
7. Let $f(x)=\begin{cases} \tan x, \text{ if } x\leq 0 \\ -\ln|x+1|, \text{ if } 0<x \end{cases}$. Which of the following statements is true about $f$?

$$f' = \begin{cases} \sec^2 x \\ \frac{-1}{x+1} \end{cases}$$

[✓ I.] $f$ is continuous at $x = 0$.
~~II.~~ $f$ is differentiable at $x = 0$.
[✓ III.] $f$ has a local maximum at $x = 0$.

a) I only b) II only c) III only

d) I and II only [✓ e)] I and III only



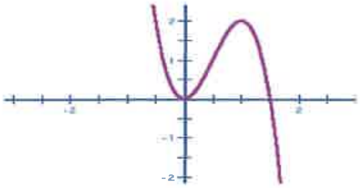
f' INC f' DEC f' INC

f'' + 0 − 0 +

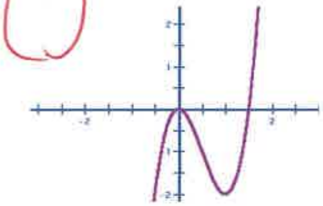
0 4

8. The graph of $f''$, the second derivative of $f$, is given above. Choose the answer that represents the graph of $f'$.

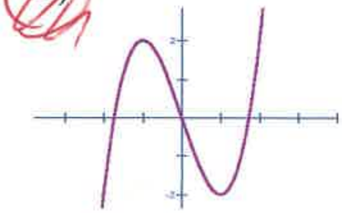
A)



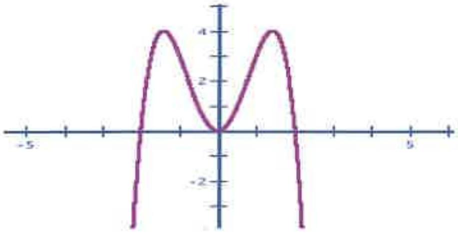
[✓ B)]



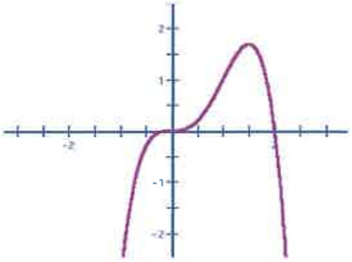
~~C)~~



~~D)~~



~~E)~~



9. Let *f* be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of *f* at $x = 3$ is used to find an approximation to a zero of *f*, that approximation is

a) 0.4 b) 0.5 [✓ c)] 2.6 d) 3.4 e) 5.5

$y - 2 = 5(x-3)$

$y = 5x - 13 = 0$

---

10. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(a) The function has no relative extremum.

(b) The graph of the function has one point of inflection and the function has two relative extrema.

[✓ (c)] The graph of the function has two points of inflection and the function has one relative extremum.

(d) The graph of the function has two points of inflection and the function has two relative extrema.

(e) The graph of the function has two points of inflection and the function has three relative extrema.

$f' = 4x^3 - 6x^2 = 2x^2(2x-3)$

$f'' = 12x^2 - 12x = 12x(x-1)$

11. If $y=5+\int_0^{2x} e^{-t^2}\,dt$, which of the following is true?

$\frac{dy}{dx} = 0 + e^{-(2x)^2}(2)$

(a) $\frac{dy}{dx}=e^{-x^2}$ and $y(0)=5$

[✓ (b)] $\frac{dy}{dx}=2e^{-x^2}$ and $y(0)=5$

~~(c)~~ $\frac{dy}{dx}=e^{-4x^2}$ and $y(2)=5$

(d) $\frac{dy}{dx}=2e^{-4x^2}$ and $y(0)=5$

~~(e)~~ $\frac{dy}{dx}=2e^{-4x^2}$ and $y(2)=5$

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12. Find $\frac{dy}{dx}$ if $x^2+9x+9xy-y^2=16$

$2x+9+2x\frac{dy}{dx}+9y-2y\frac{dy}{dx}=0$

a) $\frac{dy}{dx}=\frac{x+9+9y}{y-9x}$

b) $\frac{dy}{dx}=\frac{2x+9+9y}{2x-9y}$

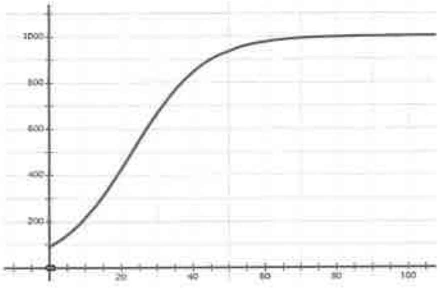
c) $\frac{dy}{dx}=\frac{2x-9+9y}{2y-9x}$

[✓ d)] $\frac{dy}{dx}=\frac{2x+9+9y}{2y-9x}$

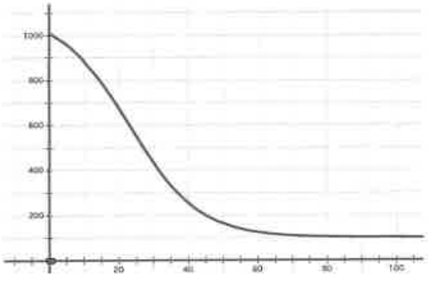
e) None of these

13. Which of the following graphs is of the solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = .1y(1000 - y)$? LOGISTIC

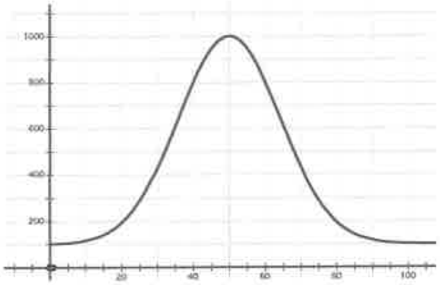
[✓ a)]



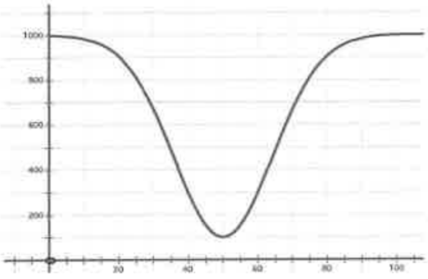
b)



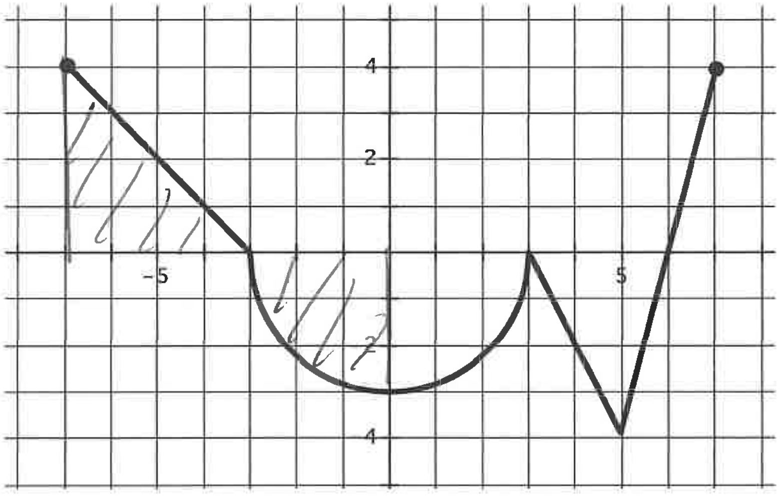
c)



d)



e) None of these



14. Let $g(x)=\int_0^x f(t)\,dt$ for $-7\le t\le 7$, where the graph of the differentiable function *f* is shown above. $g(-7)=$

a) $8-\frac{9\pi}{4}$ [✓ b)] $\frac{9\pi}{4}-8$ c) $8-\frac{9\pi}{2}$

d) $\frac{9\pi}{2}-8$ e) $2-\frac{9\pi}{2}$

$$\int_0^{-7} = \int_0^{-3} + \int_{-3}^{-7} = \frac{9\pi}{4} + (-8)$$

15. On which of the following interval is the graph of $y = 2x^3 - 3x^2 - 12x + 15$ both decreasing AND concave up?

$\frac{dy}{dx} = 6x^2 - 6x - 12 = 6(x-2)(x+1)$

a) $(-\infty, -1)$ b) $\left(-1, \frac{1}{2}\right)$ c) $(-1, 2)$

$\frac{d^2y}{dx^2} = 12x - 6$

[✓ d)] $\left(\frac{1}{2}, 2\right)$ (e) $(2, \infty)$

$y'$ + 0 − 0 +

−1 2

$y''$ − 0 +

1/2

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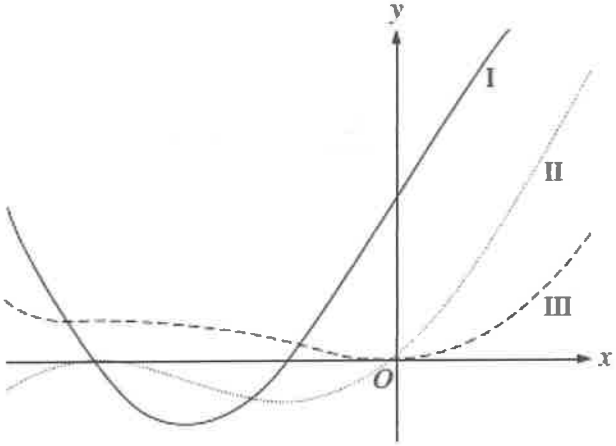

16. Three graphs labeled I, II, and III are shown above. One is the graph of $f(x)$, one is the graph of $f'(x)$, and one is the graph of $f''(x)$. Which of the following correctly identifies each of the three graphs?

| | $f(x)$ | $f'(x)$ | $f''(x)$ |
|---|---|---|---|
| (a) | I | II | III |
| (b) | I | III | II |
| (c) | II | I | III |
| (d) | II | III | I |
| [✓ (e)] | III | II | I |

I is II' BECAUSE THE ZEROS OF I MATCH THE EXTREME POINTS OF II

∴ C OR E IS CORRECT

ZEROS OF III IS NOT THE EXT OF I

SO C IS WRONG

17. The Mean Value Theorem does not apply to $f(x)=|x-3|$ on $x\in[1,4]$ because

a) $f(x)$ is not continuous on $x\in[1,4]$

[✓ b)] $f(x)$ is not differentiable on $x\in[1,4]$

c) $f(1)=f(4)$

d) $f(1)>f(4)$

e) None of these

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18. $\lim_{x\to 0}\dfrac{\int_0^{x^3}\cos t^2\,dt}{x^3}=$ L'H $\lim_{x\to 0}\dfrac{\cos x^6(\cancel{3x^2})}{\cancel{3x^2}}=\cos 0$

a) 0 [✓ b)] 1 c) $\frac{1}{3}$ d) 3 e) DNE

19. The region ***R*** is bounded by the lines $y=2x-4$, $x=3$, and $y=0$. Which of these expressions gives the volume of the solid formed by revolving ***R*** around the line $x=5$?

HORIZONTAL RECTANGLES

(a) $\int_2^3 \left((2x-4)^2-3^2\right)dx$

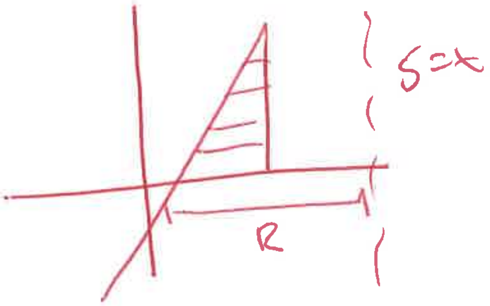
(b) $\int_2^3 \left((2x-9)^2-2^2\right)dx$

~~TOP~~

(c) $\int_0^2 \left(\left(\frac{y+4}{2}\right)^2-3^2\right)dy$

[✓ (d)] $\int_0^2 \left(\left(\frac{y-6}{2}\right)^2-2^2\right)dy$

(e) $\int_0^6 \left(\left(\frac{y+4}{2}\right)^2-3^2\right)dy$



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20. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below, find $h'(4)$, given that $h(x) = f(2x) \cdot g\left(\frac{1}{2}x\right)$.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 2 | 4 | -2 | 8 | 1 |
| 4 | 10 | 8 | 4 | 3 |
| 8 | 6 | -12 | 2 | 4 |

[✓ a)] -18~~9~~7 b) -90 c) 0 d) 47 e) 64

$$h' = f(2x) \cdot g'\left(\tfrac{1}{2}x\right) \cdot \tfrac{1}{2} + g\left(\tfrac{1}{2}x\right) \cdot f'(2x)(2)$$

$$h'(4) = f(8) \cdot g'(2) \cdot \tfrac{1}{2} + g(2) \cdot f'(8)\,2$$

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$$= \frac{10 \cdot 8}{2} + 8(-12)(2)$$

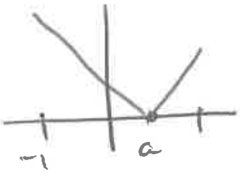
[illegible] $5 - 192$

$= -187$

21. If the average value of the function $f(x)=|x-a|$ on $[-1, 1]$ is $\frac{5}{8}$, what is/are the values of $a$?

(a) $\pm 1$ [✓ (b)] $\pm\frac{1}{2}$ (c) $\pm\frac{1}{4}$ (d) 0 (e) None of these

$$\frac{1}{1-(-1)}\int_{-1}^{1}|x-a|\,dx=\frac{5}{8}$$



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MISSING TABLE

22. A small plant is purchased from a nursery and the change in height of the plant is measured at the end of each day for four days. The data, where $H(t)$ is measured in inches per day and $t$ is measured in days, are listed above. Using the trapezoidal rule, which of the following represents an estimate of the average rate of growth of the plant over the four-day period?

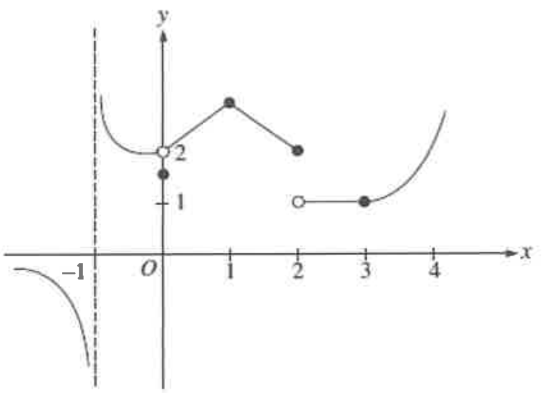
(a) $\frac{1}{4}(0+1.3+1.5+2.1+2.6)$

(b) $\frac{1}{4}\left[\frac{1}{2}(0+1.3+1.5+2.1+2.6)\right]$

[✓ (c)] $\frac{1}{4}\left[\frac{1}{2}(0+2(1.3)+2(1.5)+2(2.1)+2.6)\right]$

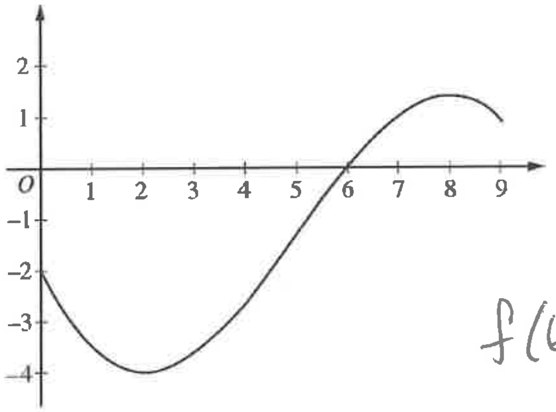
(d) $\frac{1}{4}\left[\frac{1}{2}(0+2(1.3)+2(1.5)+2(2.1)+2(2.6))\right]$

(e) $\frac{1}{4}\left[\frac{1}{4}(0+2(1.3)+2(1.5)+2(2.1)+2.6)\right]$



23. The graph of a function *f* is shown above. If $\lim_{x \to b} f(x)$ exists and $f(x)$ is not continuous at $b$, then $b =$

(a) −1 [✓ (b) 0] (c) 1 (d) 2 (e) 3



Graph of $f$

$f(6) = h'(6) = 0$

$h(6)$ = NEGATIVE AREA

$h''(6) = f'(6) = m > 0$

24. The graph of differentiable equation $f$ is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

a) $h(6) < h'(6) < h''(6)$
b) $h(6) < h''(6) < h'(6)$
c) $h'(6) < h(6) < h''(6)$
[✓ d)] $h''(6) < h(6) < h'(6)$
e) $h''(6) < h'(6) < h(6)$

25. The population of bears grows according to the logistic equation

$$\frac{dB}{dt} = 2B - 0.02B^2 \quad = 2B\left(1 - \frac{1}{100}B\right)$$

where $B$ is the number of bears and $t$ is measured in years. Which of the following statements is false?

[✓ I.] The growth rate of bears is greatest at $B = 50$

[✓ II.] If $B > 100$, the population is decreasing.

~~III.~~ $\lim_{t \to \infty} B(t) = 50 \quad = 100$

a) I only  b) II only  [✓ c)] I and II only

d) I and III only  e) I, II, and III

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26. Let the function $f$ be differentiable on the interval $[0,\ 2.5]$ and define $g$ by $g(x) = f(f(x))$. Use the table to find $g'(1)$.  $f'(f(1)) \cdot f'(1)$

| $x$ | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
|---|---|---|---|---|---|---|
| $f(x)$ | 1.7 | 1.8 | 2.0 | 2.4 | 3.1 | 4.4 |
| $f'(x)$ | 0.1 | 0.3 | 0.6 | 1.1 | 2.0 | 2.2 |

a) 0.8  [✓ b)] 1.2  c) 1.6  d) 2.0  e) 2.4

28. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time $t$ seconds, $0 \leq t$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = 2v - 32$ with $v(0) = 50$. Which of the following must be true?

$$\frac{1}{2}\int \frac{1}{2v-32}(2dv) = \int 2\, dt$$

$$\frac{1}{2}\ln|2v-32| = 2t + c$$

a) $v(t) = v^2 - 32t + c$

b) $\ln|2v - 32| = t + c$

c) $2\ln|2v - 32| = t + c$

[✓ d)] $\frac{1}{2}\ln|2v - 32| = t + c$

e) $\ln|v - 16| = 2t + c$

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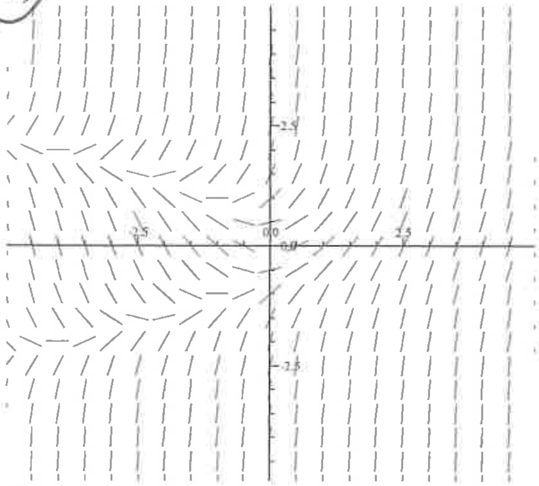
29. If $a$ and $b$ are positive constants, then $\lim_{x\to\infty} \frac{\ln(bx+1)}{\ln(ax^2+3)} = \lim_{x\to\infty} \frac{\frac{b}{bx+1}}{\frac{2ax}{ax^2+3}}$

a) 0 [✓ b)] $\frac{1}{2}$ c) $\frac{ab}{2}$ d) 2 e) $\infty$

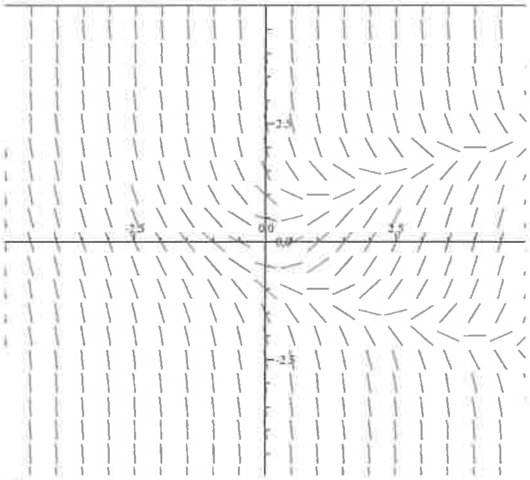
$$= \lim_{x\to\infty} \frac{abx^2+3b}{2abx^2+2ax}$$

27. Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = x + y^2$?

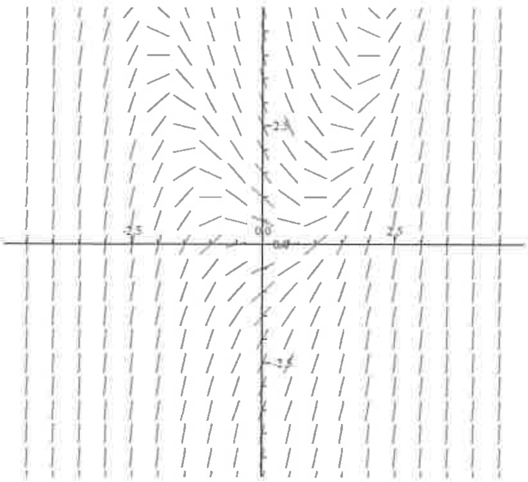
[✓ a)]



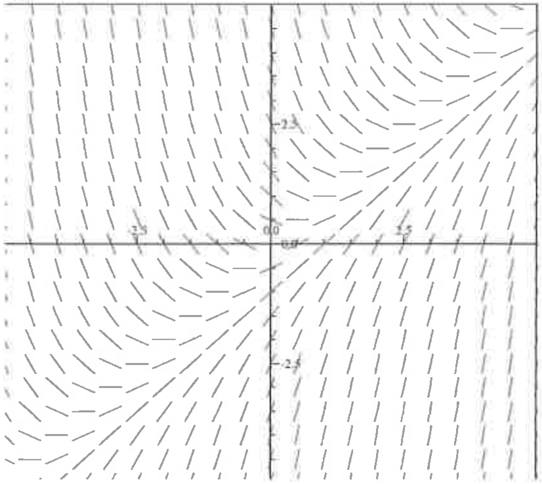
b)



c)



d)



30. Identify is the first mistake (if any) in this process:

$$\frac{dy}{dx} = xy + x$$

Step 1: $\frac{1}{y+1}dy = xdx$

Step 2: $\ln|y+1| = \frac{x^2}{2} + c$

Step 3: $|y+1| = e^{\frac{x^2}{2}} \cancel{+c} = Ke^{x^2}$

Step 4: $y = e^{x^2} + c$

a) Step 1 [✓ b) Step 2] ~~c) Step 3~~

d) Step 4 e) There is no mistake.

End of

AP Calculus AB '19-20

Spring Practice Final
Part IA

# AP Calculus AB ’19-20

# Spring Practice Final Part IB

## Calculator Allowed

Name:

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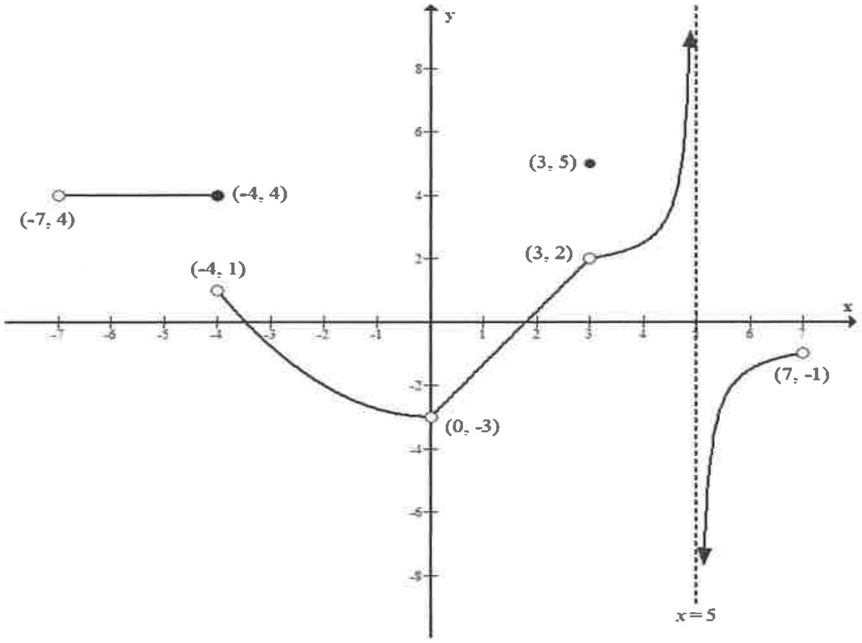
76. The function $f$ is defined on the interval $x \in [-1, 5]$ and has the graph shown below.



Which of the following is [✓ (are)] true?

[✓ I.] $\lim_{x\to 3} f(x) = 2$

~~II.~~ $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = 5 \to \lim_{x\to 3} f'(x) = 5$

[✓ III.] $\lim_{x\to -4^-} [1+f(x)] = f(3)$

a) I only  b) II only  c) III only

d) I and II only  [✓ e)] I and III only

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77. In the classic 2002 Amusement Park problem, equations $E(t)$ and $L(t)$ were given, representing the rate at which people were entering and leaving the park respectively, for time $9 \le t \le 23$, the hours during which the park was open, with t=9 corresponding to 9 am. Let us assume that $F(t) = E(t) - L(t)$. Which of the following is the best interpretation of $F(16)$?

a) The number of people in the park at 4 pm.
b) The number of people entering and leaving the park before 4 pm
c) The average number of people in the park between 9 am and 4 pm.
[✓ d)] The rate at which the number of people in the park is changing at 4 pm.
e) The rate of change of how quickly the number of people in the park is changing at 4 pm.

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78. The concentration of a certain drug during the first 20 hours after it has been administered can be approximated by

$$p(t) = \frac{300t}{6t^2 + 5}, \quad 0 \le t \le 20$$

where $t$ is the time in hours after the medication is taken and $p(t)$ is the concentration in percent. Determine the average concentration during the first ten hours after the medication was taken.

$$\frac{1}{10}\int_0^b p(t)\,dt$$

a) 0.101

b) 0.480

c) 3.502

d) 7.719

[✓ e)] 11.989

| $x$ | 1.1 | 1.2 | 1.3 | 1.4 |
|---|---|---|---|---|
| $f(x)$ | 4.18 | 4.38 | 4.56 | 4.73 |

79. Let *f* be a function such that $f''(x) < 0$ for all *x* in the closed interval [1, 2]. Selected values of *f* are shown in the table above. Which of the following must be true about $f'(1.2)$?

(a) $f'(1.2) < 0$
(b) $0 < f'(1.2) < 1.6$
(c) $1.6 < f'(1.2) < 1.8$
(d) $1.8 < f'(1.2) < 2$
(e) $2 < f'(1.2)$

---

80. The first derivative of the function $g(x)$ is defined by

$$g'(x) = \frac{20x\sin(1-2x)}{x^2+4} \text{ for } 0 \le x \le 3.$$

On which of the following intervals is $g(x)$ concave down?

a) $x \in (0, 0.5) \cup (2.071, 3)$

b) $x \in (0.5, 2.071)$

c) $x \in (0, 0.256) \cup (1.354, 2.827)$

[✓ d)] $x \in (0.256, 1.354) \cup (2.827, 3)$

e) None of these

$g$ IS CONCAVE DOWN WHERE $g'(x)$ IS DECREASING.

SEE GRAPHER

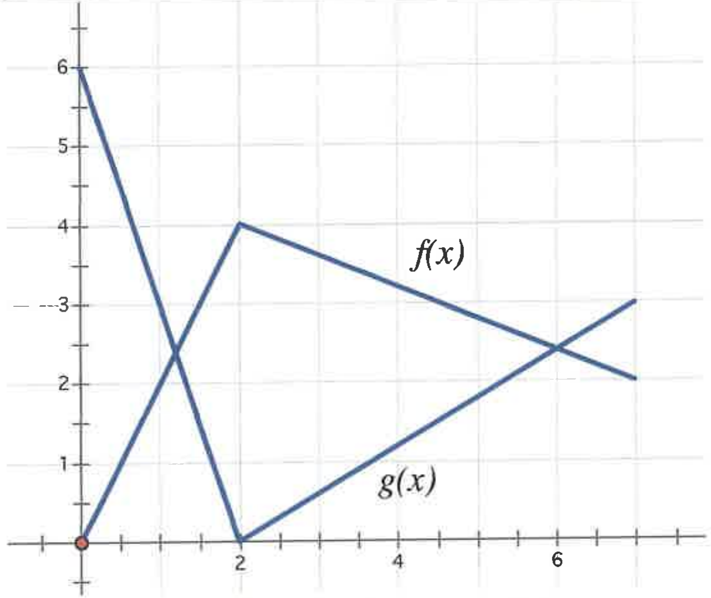
---

83. Oil is leaking from a tanker at the rate of $R(t) = 2000e^{-0.2t}$ gallons per hour, where $t$ is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

$\int_0^{10} e(t)\,dt =$

(a) 54 gallons
(b) 271 gallons
(c) 865 gallons
[✓ (d)] 8,647 gallons
(e) 14,778 gallons

---



84. The graphs of $f(x)$ and $g(x)$ are shown above. If $h(x) = g(f(x))$, then $h'(1) =$

$h' = \cancel{f'(g(x))\cdot g'}($

a) $-6$ b) $\frac{6}{5}$ c) $\frac{2}{15}$ d) $\frac{1}{5}$ [✓ e)] does not exist

$h' = g'(f(x))\cdot f'(x)$

---

$= g'(2)\cdot f'(1)$

81. A particle is moving along the $x$-axis such that it position is given by $x(t)=-4t^3+9t^2-6t-2$ for $t\geq 0$. When does the particle stop in order to switch from moving right to moving left?

a) $t=\frac{1}{2}$ only  [✓ b)] $t=1$ only  c) $t=\frac{3}{4}$ only

d) $t=\frac{1}{2}$ and $t=1$  e) never

$v$ − 0 + 0 −

$t$ ← $\frac{1}{2}$ $1$ →

$V = -12t^2 + 18t + 6 = -6(2t^2 - 3t + 1)$

---

82. A rocket rises vertically from a point on the ground 75,000 feet from a radar station. If the rocket is rising at a rate of 40000 ft/min at the instant when it is 100,000 feet high, what is the rate of change, in radians/min, of the rocket's angle of elevation from the radar station at that instant?

a) $\frac{18}{25}$  b) $\frac{8}{15}$  [✓ c)] $\frac{24}{125}$  d) $\frac{18}{125}$  e) $\frac{8}{25}$

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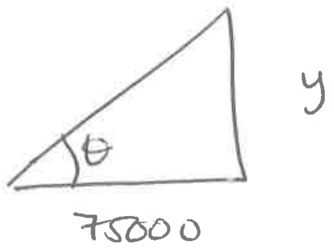


$\tan\theta = \frac{y}{75000}$

At 100,000 →

$\theta = \tan^{-1}\frac{4}{3}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{75000}\frac{dy}{dt}$

$\sec^2\left(\tan^{-1}\frac{4}{3}\right)\frac{d\theta}{dt} = \frac{40,000}{75000}$

85. If $\int_{-3}^{-1} g(x)\,dx = -19$ and $\int_{5}^{-1} g(x)\,dx = -14$, then $\int_{-3}^{5} g(x)\,dx =$

a) −33 [✓ b)] −5 c) 0 d) 5 e) 33

$$\int_{-3}^{5} = \int_{-3}^{-1} - \int_{5}^{-1} = -19 - (-14)$$

---

86. The base of a solid is the region bounded by $y = \sin x$ and $y = \cos x$ for $0 \le x \le \frac{\pi}{4}$. If each cross-section of the solid perpendicular to the $x$-axis is a square, the volume of the solid is

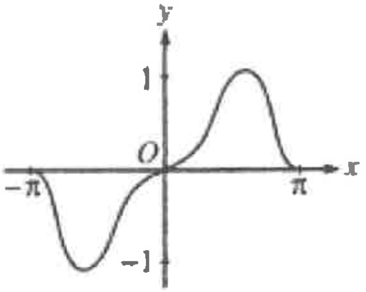
(a) 0.306 (b) 0.256 (c) 0.315 [✓ (d)] 0.286 (e) 0.257

$$V = \int_{0}^{\pi/4} [\cos x - \sin x]^2\,dx$$

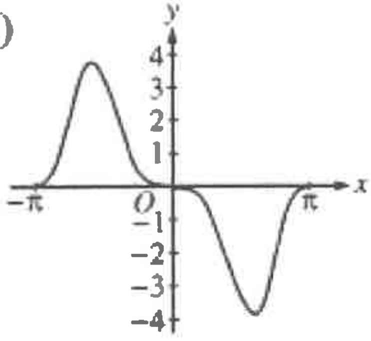
---

87. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $x \in [-\pi, \pi]$?

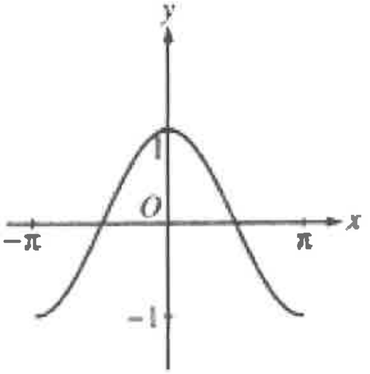
(A)



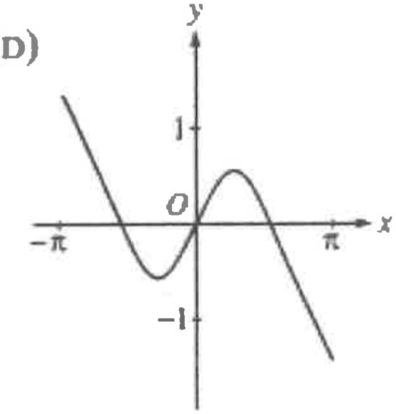
(B)



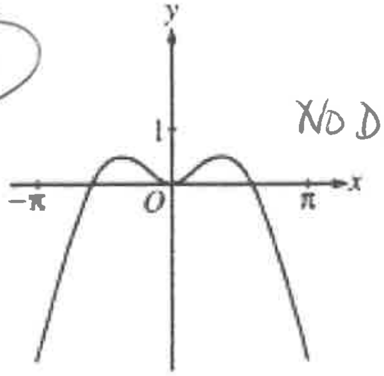
(C)



(D)



[✓ (E)]



NO DIAGONAL SYMMETRY

88. Let $f$ be the function given by $f(x)=2e^{4x^2}$. For what value of $x$ is the slope of the line tangent to the graph of $f$ at $(x, f(x))$ equal to 3?

[✓ (a) 0.168] (b) 0.274 (c) 0.318 (d) 0.342 (e) 0.551

$f' = 2e^{4x^2}(8x) = 3$ GRAPH

89. The rate at which ice is melting in a pond is given by $\frac{dV}{dt}=\sqrt{1+2^t}$, where V is the volume of the ice in cubic feet and t is the time in minutes. The amount of ice which has melted in the first five minutes is

a) 14.49 ft$^3$ b) 14.51 ft$^3$ [✓ c) 14.53 ft$^3$]

d) 14.55 ft$^3$ e) 14.57 ft$^3$

$V = \int_0^5 \sqrt{1+2^t}\,dt$

90. The average value of $y = e^x \cos x$ on $x \in \left[0, \frac{\pi}{2}\right]$ is

a. 0 [✓ b.] 1.213 c. 1.905 d. 2.425 e. 3.810

$$\text{Ave} = \frac{1}{\frac{\pi}{2} - 0} \int_0^{\pi/2} e^x \cos x \, dx =$$

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End of

AP Calculus AB '19-20

Spring Practice Final
Part IIb