AB Calculus '19-20
Volume Practice Test
Calculator Allowed

Name Solution Key

Score: ______________

1. What is the area enclosed by $y = \ln(2x+1)$ and $y = 2\sin x$?

2.154

$$\int_0 \left[2\sin x - \ln(2x+1)\right] dx$$

a) 0.334

b) 0.661

c) 3.526

d) 0.825

e) 2.983

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2. A region is bounded by $y = \frac{2}{\sqrt{x}}$, the x-axis, the line $x = m$, and the line $x = 2m$, where $m > 0$. The area of this region:

a) is independent of m.
b) increases as m increases.
c) decreases as m increases.
d) increases until $m = \frac{1}{2}$, then decreases.
e) is none of the above

$$A = \int_m^{2m} \frac{2}{x^{1/2}} \, dx$$

$$= 4x^{1/2} \Big]_m^{2m}$$

$$= 4\sqrt{2m} - 4\sqrt{m}$$

$$= 4(\sqrt{2}-1)\sqrt{m}$$

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3. Let R be the region in the first quadrant bounded by $x = \sin^{-1} y$, the x-axis, and $x = \frac{\pi}{2}$. Which of the following integrals gives the volume of the solid generated when R is rotated about the line $x = \frac{\pi}{2}$?

$X = \sin^{-1} y$

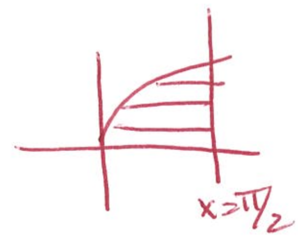
a) $\pi \int_0^{\pi/2} y^2 \, dy$

b) $\pi \int_0^1 \left(\frac{\pi}{2} - \sin^{-1} y\right)^2 dy$

c) $\pi \int_0^{\pi/2} \left(\sin^{-1} y\right)^2 dy$

d) $\pi \int_0^1 (\sin x)^2 \, dx$

e) $\pi \int_0^{\pi/2} \left(\frac{\pi}{2} - \sin x\right)^2 dx$



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4. Which of the following integrals gives the length of the graph $y = \sin 2x$ from $x=a$ to $x=b$?

$\frac{dy}{dx} = 2\cos 2x$

a) $\int_a^b \sqrt{1+4\sin^2 x \cos x} \, dx$

b) $\int_a^b \sqrt{1+\sin^2 2x} \, dx$

c) $\int_a^b \sqrt{1+4\cos^2 2x} \, dx$

d) $\int_a^b \sqrt{1+\cos^2 2x} \, dx$

e) $\int_a^b \sqrt{1+4\sin^2 x \cos^2 x} \, dx$

$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

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5. For $t \geq 0$ hours, H is a differentiable function of t that gives the change in temperature, in degrees Celsius per hour, at an Arctic weather station. Which of the following is the best interpretation of $\int_0^t H(x)\,dx$?

a) The total change in temperature during the first t hours.
b) The total change in temperature during the first day.
c) The average rate at which the temperature changed during the first t hours.
d) The rate at which the temperature is changing during the first day.
e) The rate at which the temperature is changing at the end of the 24th day.

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6. Let $\boldsymbol{R}$ represent the region in the first quadrant bounded by $y = -3x + 6$. Which expression gives the volume of the solid with base $\boldsymbol{R}$ whose cross-section perpendicular to the x-axis are semi-circles?

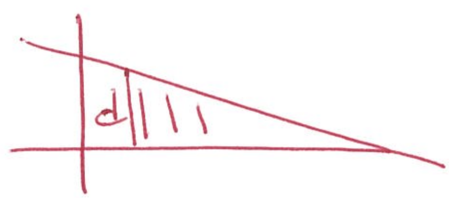
a) $\pi \int_0^2 (-3x+6)^2\,dx$

b) $\pi \int_0^6 \left(\frac{6-y}{3}\right)^2 dy$

c) $\frac{\pi}{4} \int_0^2 (-3x+6)^2\,dx$

d) $\frac{\pi}{8} \int_0^2 (-3x+6)^2\,dx$

e) $\frac{\pi}{2} \int_0^6 \left(\frac{6-y}{6}\right)^2 dy$



$d = \text{diameter} = 2r = -3x+6$

$\text{AREA}_{\triangle} = \frac{1}{2}\pi r^2$

$V = \int \frac{\pi}{2}\left(-\frac{3}{2}x+3\right)^2 dx$

$= \int \frac{\pi}{2}\left(\frac{-3x+6}{2}\right)^2 dx$

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7. What is the volume of the solid formed when the region bounded by $y = e^x - 1$, $y = 2$, and $x = 0$ is rotated around the y-axis?

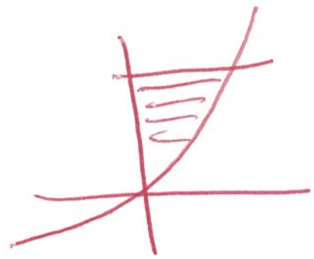
a) 1.296

b) 3.233

c) 5.930

d) 10.354

e) 25.199

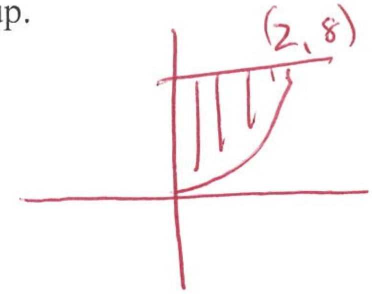


$x = \ln(y+1)$

$V = \pi \int_0^2 \ln^2(y+1)\, dy =$

8. Let S be the region bounded by $y = x^3$, $y = 8$, and $x = 0$.

a) Find the area of S. Show the setup.



$$A = \int_0^2 (8 - x^3)\,dx$$

$$= \left[8x - \frac{1}{4}x^4\right]_0^2$$

$$= 12$$

b) Find the volume of the solid generated by revolving S around the x-axis. Show the anti-differentiation steps. WASWSE

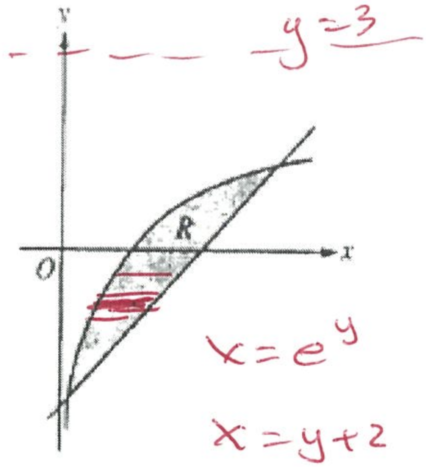
$$V = \pi \int_0^2 \left[8^2 - (x^3)^2\right] dx$$

$$= \pi \int_0^2 (64 - x^6)\,dx$$

$$= \pi \left[\cancel{8}64x - \frac{1}{7}x^7\right]_0^2$$

$$= 344.678$$

9. Let $\boldsymbol{R}$ be the region bounded by $y = \ln x$ and $y = x - 2$.



$x = e^y$

$x = y + 2$

a) Find the volume of the solid whose base is $\boldsymbol{R}$ and whose cross-sections perpendicular to the y-axis are isosceles right triangles with the one leg is in the base.

$$V = \int_{-1.841}^{1.146} \frac{1}{2}\left[y+2-e^y\right]^2 dy$$

$A = \frac{1}{2}s^2$

s

$= .773$

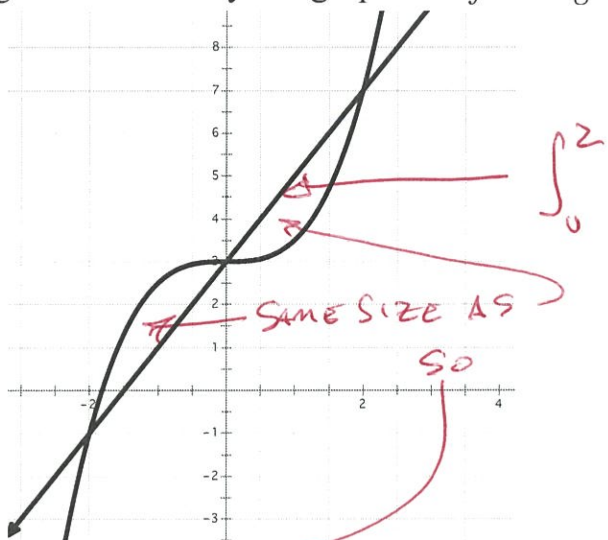
b) Find the volume of the solid formed by revolving $\boldsymbol{R}$ around the line $y = 3$.

WASHER

$$V = \pi \int_{.156}^{3.146} (3-(x-2))^2 - (3-\ln x)^2 \, dx$$

$= 39.280$

10. Let f and g be the functions given by $f(x)=\frac{1}{2}x^3+3$ and $g(x)=2x+3$. Let R be the two-part region enclosed by the graphs of f and g shown below.



(a) Find the area of region R. Do not use Math 9.

$$R = 2\int_0^2 (2x+3) - \left(\tfrac{1}{2}x^3+3\right)\,dx$$

$$= 2\left[x^2 + \cancel{3x} - \tfrac{1}{8}x^4 - \cancel{3x}\right]_0^2 = 2\left[4-2\right] = 4$$

(b) Let the base of the solid be the region R. Find the volume of the solid where the cross-sections perpendicular to the *x*-axis are semicircles. Show the set-up before using Math 9.

$A = \frac{\pi}{2}r^2$ $\quad r$

$2r = (2x+3) - \left(\tfrac{1}{2}x^3+3\right) = 2x - \tfrac{1}{2}x^3$

$r = x - \tfrac{1}{4}x^4$

$$V = 2\int_0^2 \frac{\pi}{2}\left(x - \tfrac{1}{4}x^4\right)^2 dx$$

$$= 1.257$$

(c) Find the volume of the solid generated when R is revolved about the y-axis.

$f(x) \to y = \frac{1}{2}x^3+2 \to [2(y-2)]^{1/3} = x$

$g(x) \to y = 2x+3 \to x = \frac{y-3}{2}$

$$V = \pi \int_{-1}^{7} \left[(2(y-2))^{1/3}\right]^2 - \left[\frac{y-3}{2}\right]^2 dy$$

$= 28.908$