AP Calculus AB '20-21 Name_____
Integral Test v1

Score_____

CALCULATOR ALLOWED
Directions: Show all work.

1. Letters arrive at a post office at a rate of $P(t) = 8 + t\sin\frac{t^3}{80}$ hundred letters per hour over the course of a workday. The day begins at 9am ($t = 0$) and ends at 5pm ($t = 8$). There are 3 hundred letters in the office at 9am. Workers send letters out of the office at a constant rate of 5 hundred letters per hour.

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a) Find $P'(2)$. Using correct units, interpret the meaning of $P'(2)$ in the context of this problem.

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b) Find the total number of letters that arrive at the office between 9am and noon ($t = 3$). Round to the nearest whole number of letters.

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c) Write an expression for $L(t)$, the total number of letters in the post office at time *t*.

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d) What is the maximum number of letters in the office over the course of the workday ($0 \le t \le 8$)?

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2. A diabetic patient tests his blood glucose level every morning. After being put on insulin, the data below show the glucose levels in milligrams per deciliter (mg/dL) over one week.

t days	1	2	3	4	5	6	7
$G(t)$ (mg/dL)	233	198	185	168	147	130	147

Let $G(t)$ represent the glucose level where t is measured in days.

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a) Estimate $G'(3.7)$. Using the correct units, explain the meaning of the result.

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b) Use midpoint Riemann rectangles to approximate $\int_1^7 G(t)dt$. Using the correct units, explain the meaning of $\frac{1}{7}\int_1^7 G(t)dt$ in terms of the patient's glucose levels.

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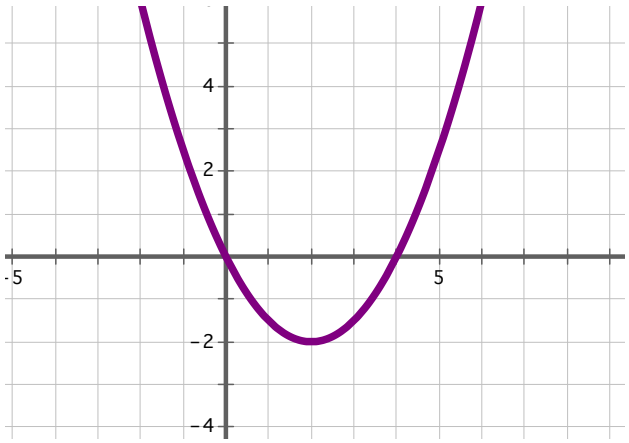
c) Ignoring the last data point, $M(t) = 237.6e^{-0.082t}$ is a model of $G(t)$. Find $M'(3.7)$. Is $M(t)$ decreasing at an increasing rate? Show the work that leads to your conclusion.

3. Consider the function $g(x)=\frac{1}{2}x^2-2x$ which has the graph below.



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a) Find the zeros of $g(x)$. Show the work that leads to your answer.

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b) Find the exact value of $\int_{-1}^{6} g(x)\,dx$. Show the antiderivative and boundary insertion steps.

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c) Find the exact area between the x-axis and $g(x)$ on $x \in [-1, 6]$. Show the antiderivative and boundary insertion steps.