AB Calculus '20-21
Dx Apps II Test v1
Calculator allowed

Name Solution Key

Score___________

1. A particle is moving along the $x$-axis and its position is given by $x(t)=te^{-2t}$. For what values of $t$ is the particle at rest?

(a) No values (b) 0 only (c) $\frac{1}{2}$ only (d) 1 only (e) 0 and $\frac{1}{2}$

$V = te^{-2t}(-2) + e^{-2t}(1)$

$= e^{-2t}(-2t+1) = 0$

$t = 1/2$

---

2. A particle moves along a straight line with its position at any time $t \geq 0$ given by $s(t)=\int_0^t \left(x^3-2x^2+x\right)dx$, where s is measured in meters and t is in seconds. The maximum velocity attained by the particle on $0 \leq t \leq 3$ is

a) $\frac{1}{3}$ m/s b) $\frac{4}{27}$ m/s c) $\frac{27}{4}$ m/s d) 12 m/s

---

$V = t^3 - 2t^2 + t$

$V' = 3t^2 - 4t + 1 = 0$

$(3t-1)(t-1) = 0$

$t = 1/3, 1$

$V'$: + 0 − 0 +

1/3 1 3

MAX MAX

$v(1/3) = \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \frac{1}{3} = \frac{4}{27}$

$v(1) = 1 - 2 + 1 = 0$

3. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 cm, what is the rate of change, in square centimeters per second, of the surface area of the sphere?

(a) $-108\pi$
(b) $-72\pi$
(c) $-48\pi$
(d) $-24\pi$
(e) $-16\pi$

$A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(3)(-2)$

---

4. The acceleration of a particle is given by $a(t) = 4e^{2t}$. When $t = 0$, the position of the particle is $x = 2$ and $v = -2$. Determine the position of the particle at $t = \frac{1}{2}$.

a) $e - 3$
b) $e - 2$
c) $e - 1$
d) $e$
e) $e + 1$

$v(t) = 2e^{2t} + c_1 \rightarrow (0, -2) \rightarrow -2 = 2e^{0} + c_1$

$\cancel{-2-2e} = c_1 - 4$

$x' = 2e^{2t} - 4$

$x(t) = e^{2t} - 4t + \cancel{2at} + c_2$

$= e^{2t} - 4t + 1$

---

$x(\tfrac{1}{2}) = e - 2 + 1$

5. A particle travels along a straight line with a velocity of $v(t) = 3e^{-t^2}\sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

a) 0.835 b) 1.625 c) 2.055 d) 2.261 e) 7.025

---

6. Gravel is being dumped from a conveyor belt at a rate of 35 ft$^3$/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 15ft high?

a) 0.27 ft/min b) 1.24 ft/min c) 0.14 ft/min

d) 0.2 ft/min e) 0.6 ft/min

---

7. The population of a herd of bison in Yellowstone National Park is modeled by the function $B$ that satisfies the logistic differential equation $\frac{dB}{dt} = 0.2B\left(1 - \frac{B}{900}\right)$, where $t$ is time in years, and $B(0) = 120$. What is the value of $B$ that maximizes the rate of change of $B$?

a) 120 b) 450 c) 900 d) 4500 e) 9000

---

8. A particle is moving along the $x$-axis and its position at time $t \geq 0$ is given by $s(t)=(t-2)^2(t-5)$

$v=(t-2)^2+(t-5)(2(t-2))$

$=(t-2)[t-2+2t-10]$

$=(t-2)(3t-12)$

$=3t^2-22t+24$

$a(t)=6t-22$

Which of the following is (are) true?

I. The particle changes direction at $x = 2$ and $x = 5$. F

II. The particle is slowing down on [0, 2]. T

III. The particle is speeding up on [2, 5]. F

a) I, II and III
b) II and III only
c) I and III only
d) II only
e) I only

---

9. If $\sin^{-1} x = \ln y$, then $\frac{dy}{dx} =$

$\frac{1}{\sqrt{1-x^2}} = \frac{1}{y}\frac{dy}{dx}$

(a) $\frac{y}{\sqrt{1-x^2}}$ (b) $\frac{xy}{\sqrt{1-x^2}}$ (c) $\frac{y}{1+x^2}$ (d) $e^{\sin^{-1} x}$ (e) $\frac{e^{\sin^{-1} x}}{1+x^2}$

---

AB Calculus '20-21 Name SOLUTION KEY
Dx Apps II Test v1
Calculator allowed Score__________
Directions: Show all work.

| $t$ in hours | 0 | 12 | 24 | 36 | 48 |
|---|---|---|---|---|---|
| $v(t)$ in km/sec | 21 | 26.3 | 31.4 | 36.8 | 41.5 |

1. A Gravitational Slingshot Effect is sometimes used by space probes like Voyager 2 in order to increase its velocity without expending fuel. By flying close to the planet Saturn in a parabolic arc, the velocities on the table above were achieved by a probe. (In the original *Star Trek* episode "Tomorrow is Yesterday," the Enterprise used this effect around a black hole to time-travel to 1967.)

---

a. Approximate the probe's acceleration at $t=30$.

(1) $a(t) \approx \frac{36.8-31.4}{36-24} = \frac{5.4}{12} = .45 \frac{\text{KM}}{\text{SEC}^2}$

---

b. Use a trapezoidal approximation for $\int_0^{48} v(t)\,dt$. Using the correct units, explain the meaning of this result.

(1) $\int_0^{48} v(t)\,dt \approx 12\left(\frac{21+26.3}{2}\right) + 12\left(\frac{26.3+31.4}{2}\right) + 12\left(\frac{31.4+36.8}{2}\right) + 12\left(\frac{36.8+41.5}{2}\right)$

$= 1509$ KM

(1) THE PROBE TRAVELED APPROXIMATELY 1509 KM IN THESE 48 HRS

---

c. Using your answer in b), approximate the average velocity of the probe between $t = 0$ and $t = 48$? Indicate the correct units.

1

$$\text{AveVel} = \frac{1}{48}\int_0^{48} v(t)\,dt$$

$$= 31.438 \ \frac{\text{km}}{\text{hr}}$$

1 1

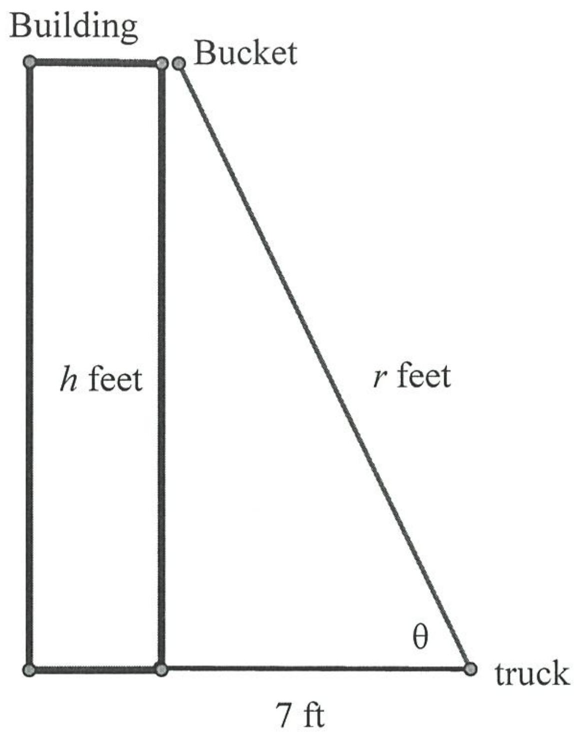
---

d) The data on the table can be approximated by the equation $v(t) = 0.000027x^2 + 0.4396x + 21$. Based on this equation, find the total distance traveled by the probe between $t = 0$ and $t = 48$ hours. Indicate the units.

3 $\int_0^{48} v(t) = 1515{,}415 \text{ km}$

---

2. A fire truck is parked 7 feet away from the base of a building and its ladder is extended to the top of the building. The ladder retracts at a rate of 0.5 feet per second, while the angle of the ladder changes such that the bucket at the end of the ladder comes down vertically.




a) How far is the ladder extended when the bucket is 10 feet above the ground?

1

$10^2 + 7^2 = r^2$

$r = 12.207$ FT

b) Find the rate at which the bucket is dropping vertically when the bucket is 10 feet above the ground.

1 $h^2 + 7^2 = r^2$

1 $2h\frac{dh}{dt} = 2r\frac{dr}{dt}$

$2(10)\frac{dh}{dt} = 2(12.207)(.5)$

1 $\frac{dh}{dt} = 0.610$

---

c) What is the relationship between the angle $\theta$ and the height of the bucket? Find $\theta$, in radians, when the bucket is 10 feet above the ground.

1 $\tan\theta = \frac{h}{7}$ 1 $\theta = \tan^{-1}\left(\frac{10}{7}\right) = 0.960$ RADIANS

---

d) Find the rate, in radians per second, at which the angle the ladder forms with the ground is changing when the bucket is 10 feet above the ground.

$\frac{d}{dt}\left(\tan\theta = \frac{1}{7}h\right)$

1 $\sec^2\theta\,\frac{d\theta}{dt} = \frac{1}{7}\frac{dh}{dt}$

$\frac{d\theta}{dt} = 0.029\ \frac{KM}{SEC}$

1 1

---

3. A research team is studying a group of monkeys living on Monkey Island in Cambodia. When they begin observing the monkeys $(t=0)$, there are 20 monkeys on the island. The researchers determine that the population $P$ grows logistically at a rate of $\frac{dP}{dt} = 3P - \frac{P^2}{40}$ monkeys per year.

---

a) According to this logistic model, what is the maximum population of monkeys on the island?

$$\frac{dP}{dt} = 3P\left(1 - \frac{P}{120}\right) \qquad A = 120$$

---

b) Further research shows the growth to be a bounded exponential function rather than a logistic growth function. Assuming a new model of $\frac{dM}{dt} = \frac{1}{40}(130 - M)$, find the particular solution to the differential equation, givsn that $M(0) = 20$.

$$\frac{1}{130-m}\,dm = \frac{1}{40}\,dt$$

$$-\ln|130-m| = \frac{1}{40}t + C$$

$$\ln|130-m| = -\frac{1}{40}t + C$$

$$130 - m = Ke^{-\frac{1}{40}t} \quad \rightarrow (0, 20) \rightarrow \quad 110 = K$$

$$M = 130 - 110e^{-\frac{1}{40}t}$$

---

c) Find $\lim_{t \to \infty} M$. Explain the meaning of this result, using the correct units.

$$\lim_{t \to \infty} M(t) = 130$$

THE MAXIMUM NUMBER OF MONKEYS ON THE ISLAND IS 130