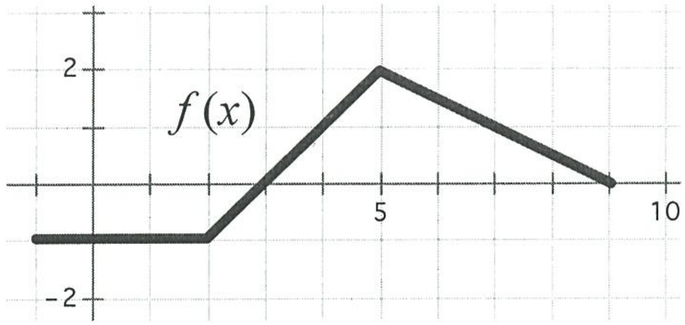
AP Calculus AB '20-21
Dr. Quattrin
Fall Final Exam v1

Name Solution Key

Part II: Free Response – Show all work.



x	$g(x)$	$g'(x)$
0	-1	1
2	1	3
4	3	6
6	6	12
8	4	8

1. Let $f(x)$ be the function whose graph is given above and let $g(x)$ be a differentiable function with selected values for $g(x)$ and $g'(x)$ given on the table above. Furthermore, let h be the function defined by $h(x) = \ln(x^2 + 4)$.

(a) Find the equation of the line tangent to $f(x)$ at $x = 4$.

2

$f(4) = 1$

$f'(4) = 1$ 1

$y - 1 = 1(x - 4)$

1

(b) Let K be the function defined by $K(x) = h(f(x))$. Find $K'(3)$.

2

$K' = h'(f(x)) \cdot f'(x)$ 1

$K'(3) = h'(0) \cdot f'(3) = 0 \cdot 1 = 0$

1

(c) Let M be the function defined by $M(x) = g(x) \cdot f(x)$. Find $M'(6)$.

2

$M'(6) = g(6) f'(6) + f(6) g'(6)$ 1

$= 6\left(-\frac{1}{2}\right) + 12\left(\frac{3}{2}\right)$

$= \cancel{2}\;15$

1

(d) Let J be the function defined by $J(x) = \dfrac{g(x)}{h\left(\frac{1}{2}x\right)}$. Find $J'(8)$.

3

$J' = \dfrac{h(\frac{1}{2}x)\, g'(x) - g(x)\, h'(\frac{1}{2}x)(\frac{1}{2})}{h(\frac{1}{2}x)}$ 1

$J(8) = \dfrac{h(4)\, g'(8) - g(8)\, h'(4)\,(\frac{1}{2})}{(h(4))^2}$ 1

$= \dfrac{\ln 20\,(8) - 4\left(\frac{16}{20}\right)\left(\frac{1}{2}\right)}{(\ln 20)^2} = \dfrac{8 \ln 20 - \frac{8}{5}}{(\ln 20)^2}$ 1

$= 2,492$

2. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function $y = f(x)$ is defined for all real numbers.

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a) Find the equation of the line tangent to $y = f(x)$ at $f(1) = 2$

2

$m = \frac{1}{1}$ $y - 2 = 1(x-1)$

1   1

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b) Use your answer in part a) to approximate $f(0.9)$.

1

$f(0.9) \approx y = 2 + (.9 - 1) = 1.9$

1

6

c) Find $y=f(x)$, the particular solution to the differential equation with the initial condition $f(1)=2$.

$$\frac{dy}{dx} = \frac{y-1}{x^2}$$

1 $$\int \frac{1}{y-1}\,dy = \int x^{-2}\,dx$$

2 $$\ln|y-1| = \frac{x^{-1}}{-1} + c \quad 1$$

$$y-1 = e^{\frac{-1}{x}+c} = ke^{-1/x}$$

$$f(1)=2 \rightarrow 1 = ke^{-1/x}$$

$$k = e^{1}$$

1

$$y-1 = e\cdot e^{-1/x} = e^{1-1/x}$$

1 $$y = 1 + e^{1-\frac{1}{x}}$$

t days	0	1	2	3	4
$H(t)$ in mm per day	0	0.9	1.4	1.7	2.1

3. A small plant is purchased from a nursery and the change in height of the plant is measured at the end of each day for four days. The data, where $H(t)$ is measured in millimeters per day and t is measured in days, are listed above.

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a. Estimate $H'(3)$. Show the work that leads to your answer. Indicate the units.

2

$H'(3) \approx \frac{2.1-1.4}{4-2} = .35 \ \frac{mm}{DAY^2}$

1 1

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b. USING THE CORRECT UNITS Explain the difference between $H(3)$ and $H'(3)$ in terms of the plant's growth.

2

H (3) IS HOW FAST THE PLANT IS GROWING IN MM/DAY 1

H'(3) IS THE RATE OF CHANGE OF THE RATE OF GROWTH IN MM/DAY² 1

3

c. Use right-hand rectangles with subintervals indicated by the table to approximate $\int_0^4 H(t)\,dt$. Using correct units, explain the meaning of this value in the context of the problem.

1 1

$\int_0^4 H(t)dt = 1(0.9) + 1(1.4) + 1(1.7) + 1(2.1) = 6.1$ MM

2

THE PLANT HAS GROWN 6.1 MM IS THESE FOUR DAYS

1

2

d. Using correct units, explain the meaning of $\frac{1}{4}\int_0^4 H(t)\,dt$ in the context of the problem.

1

$\frac{1}{4}\int_0^4 H(t)dt =$ THE AVERAGE RATE OF GROWTH IN MM/DAY OVER THESE FOUR DAYS

1
