AP Calculus AB '20-21
Dr. Quattrin
Practice Fall Final Exam

Name Solution Key

Part I: Multiple choice – Circle correct answer.

1. Which of the following statements are **false**?

I. $\int (\csc u)\, dx = \ln|\csc u + \cot u| + c$ F

II. $\int a^u\, du = \dfrac{a^{u+1}}{u+1} + c,\ u \neq -1$ F

III. $\int \left(\dfrac{1}{\sqrt{9-x^2}}\right) dx = \sin^{-1}\dfrac{x}{3} + c$ T

(A) I only (B) I and II only (C) III only

(D) I and III (E) All these

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2. $\int (t-4)(t^2-8t)^5\, dt =$ $\frac{1}{2}\int (t^2-8t)^5 (2(t-4))dt = \frac{1}{2}\int u^5 du$

(A) $\dfrac{(t^2-8t)^6}{6}+C$ (B) $\dfrac{(t^2-8t)^6}{12}+C$ (C) $\dfrac{(t^2-8t)^6}{3}+C$

(D) $\dfrac{(t-4)^6}{6}+C$ (E) $\dfrac{(t-4)^6}{3}+C$

3.

h(t) (°C)	10.5	11.4	12.5	11.3
t (hours)	2	3	5	8

The continuous function h(t) gives the temperature, in °C, of a small town in Finland. Using a trapezoidal sum with subintervals indicated by the table, approximate the average temperature of the town over the interval 2 to 8 hours.

(A) 66.55
(B) 21.85
(C) 11.425
(D) 10.925
(E)4.50

$$\frac{10.5+11.4}{2}(1)+\frac{11.4+12.5}{2}(2)+\frac{12.5+11.3}{2}(3)$$

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4. The ABSOLUTE minimum value of $g(x)=-x^3+2x^2$ on $[-1,3]$ occurs when $x=$

(A) -1 (B) 0 (C) $4/3$ (D) 2 (E) 3

$g' = -3x^2+4x=0$

$= -x(3x-4)=0$

$x=0, 4/3$

x	g(x)	
-1	3	MAX
0	0	MIN
4/3		
3	-9	MIN

$y = (3x)^{1/4}$

6. Using the line tangent to $y = \sqrt[4]{3x}$ at $x = 27$, approximate $\sqrt[4]{90}$.

(A) 3.070

(B) 3.078

(C) 3.080

(D) 3.083

(E) 3.105

$\frac{dy}{dx} = \frac{1}{4}(3x)^{-3/4}(3) \quad m = \frac{3}{4}\left(\frac{1}{27}\right)^{3/4} = \frac{1}{36}$

$y - y_1 = m(x - x_1)$

$\sqrt[4]{3x} = \sqrt[4]{90} \rightarrow x = 30$

$y - 3 = \frac{1}{36}(x - 27)$

$y(30) = \frac{1}{12} + 3$

7.

x	1	2	4	8
$f(x)$	-3	4	9	-1
$g(x)$	0	6	2	1
$f'(x)$	9	-4	3	2
$g'(x)$	10	1	3	5

Let $h(x) = g(x) \cdot f(x^3)$. What is the value of $h'(2)$?

(A) -6

(B) 2

(C) 11

(D) 24

(E) 143

$h' = g(x) \cdot f'(x^3)(3x^2) + f(x^3) \cdot g'(x)$

$= g(2) \cdot f'(8)(12) + f(8) g'(2)$

8. If $j(x)$ is a continuous function where $\int_4^0 j(x)\,dx = 5$, then $\int_4^0 j(4-x)\,dx =$

(A) 5
(B) -5
(C) 11
(D) 21
(E) -4

$u = 4 - x$ $\quad u(0) = 4$

$du = -dx$ $\quad u(4) = 0$

$-\int_0^4 j(u)\,du = \int_4^0 j(u)\,du = 5$

9.] To which of these functions does the Mean Value Theorem apply on the interval $[-1,3]$?

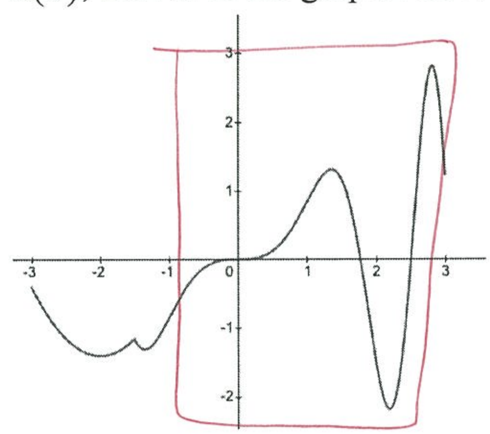
$f(x) = \begin{cases} 4x-1, & -1 \le x < 2 \\ x^2, & 2 \le x \le 3 \end{cases}$

NOT CONTINUOUS

$g(x) = \begin{cases} 3x^4 - x, & x \le 1 \\ x+1, & x > 1 \end{cases}$

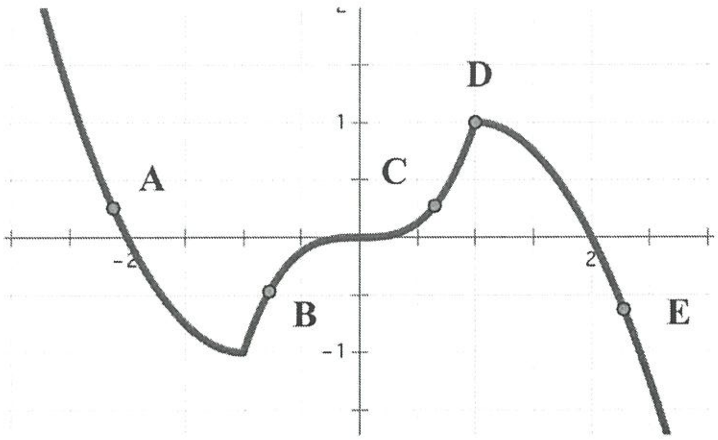
NOT DIFFERENTIABLE

$h(x)$, shown in the graph below



(A) $f(x)$ only (B) $g(x)$ only (C) $h(x)$ only

(D) $g(x)$ and $h(x)$ only (E) None of these



DECREASING AND CONCAVE UP

10. At what point on the above curve is $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$

(A) A (B) B (C) C (D) D (E) E

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11. For $t \geq 0$ hours, H is a differentiable function of t that gives the rate of change in temperature, in degrees Celsius per hour, at an Arctic weather station. In what units would $\frac{1}{t}\int_0^t H(x)dx$ be measured?

$\frac{1}{HR} \int_0^t \frac{°C}{HR}$ HRS $= \frac{°C}{HR}$

(A) degrees Celsius
(B) degrees Celsius per hour
(C) degrees Celsius per hour per hour.
(D) hours per degrees Celsius
(E) hours

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12. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t+1) - 2t + 1$. The total distance traveled by the particle from 0 to ~~3~~ 2 is

$\int_0^2 |v(t)|\,dt$

(A) 0.667 (B) 0.704 (C) 1.540 (D) 2.667 (E) 2.901

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13. Find the particular solution to $(x^2+1)\frac{dy}{dx} = y$, where $y(0) = 2$.

$\frac{1}{y}\,dy = \frac{1}{x^2+1}\,dx$

$\int dy = \tan^{-1}x + c$

$y = e^{\tan^{-1}x + c}$

$= ke^{\tan^{-1}x}$

(A) $y = 2e^{\tan^{-1}x}$ (B) $y = e^{\tan^{-1}x}$ (C) $y = e^{(\tan^{-1}x)\ln 2}$

~~(D)~~ $y = \sqrt{2\tan^{-1}x}$ ~~(E)~~ $y = \sqrt{2\tan^{-1}x + 4}$

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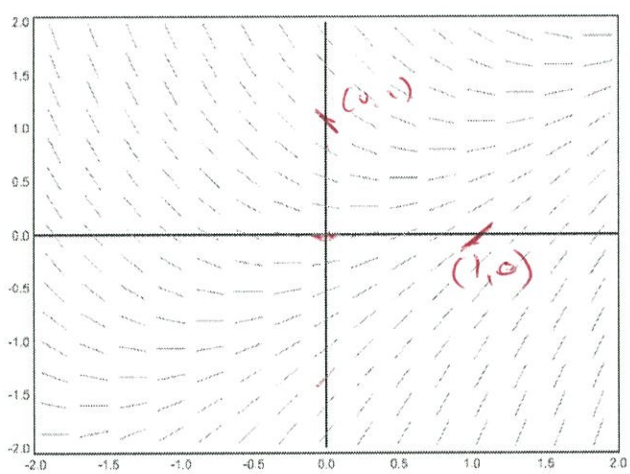
14. Which of the following differential equations corresponds to this slope field?



AT $(0,1)$ $\frac{dy}{dx} = -1$

~~(A)~~ $\frac{dy}{dx} = x$ ~~(B)~~ $\frac{dy}{dx} = xy$ ~~(C)~~ $\frac{dy}{dx} = y - x$

~~(D)~~ $\frac{dy}{dx} = x + y$ (E) $\frac{dy}{dx} = x - y$
