AB Calculus '20-21
Limit Test v1
No Calculator

Name Solution Key

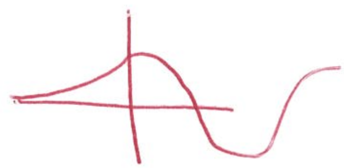
1. Let $f(x)=\begin{cases} e^x, \text{ if } x\le 0 \\ \cos x, \text{ if } 0<x \end{cases}$. Which of the following statements is **false** about f?

$f'(x)=\begin{cases} e^x \to f'(0)=1 \\ -\sin x \to f'(0)=0 \end{cases}$

(a) f is continuous at $x=0$. T

(b) f is differentiable at $x=0$. F

(c) f has a local maximum at $x=0$. T



d) f has a point of inflection at $x=0$. T

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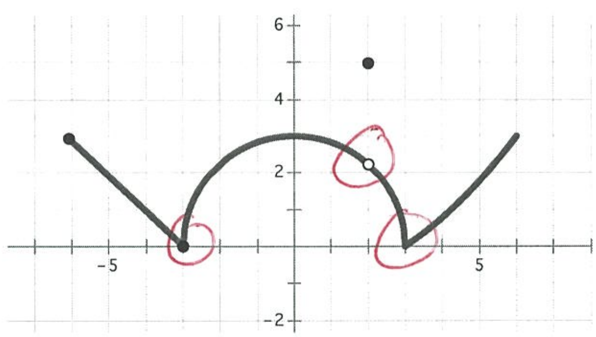
2. The function f is defined on the interval $x\in[-6, 6]$ and has the graph shown below.



For which of the following values is f not differentiable?

(a) -3 only (b) 3 only (c) 3 only

(d) -3 and 3 only (e) -3, 2, and 3

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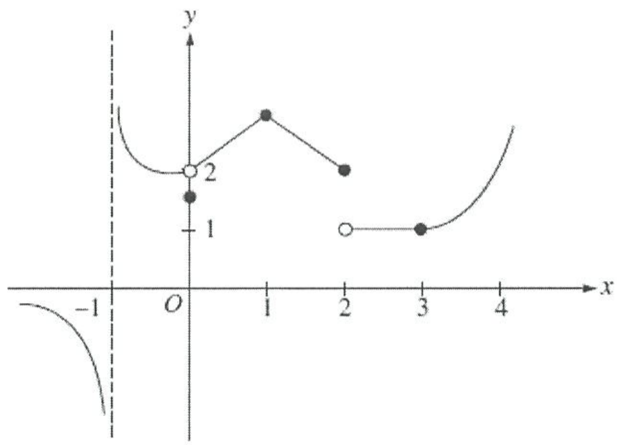
3. The function f is shown below. Which of the following statements about the function f, shown below, is false?



a) $\lim_{x \to 0} f(x)$ exists T

b) $\lim_{x \to 2} f(x)$ exists F

c) f is continuous at $x = 1$

d) $\lim_{x \to 3} \frac{f(x) - 5}{x - 3}$ does not exist

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4. $\lim_{x \to 0} \frac{3e^{3x} - 3}{\ln(1-x)^2} =$ L'H $\lim_{x \to 0} \frac{3e^{3x}(3)}{\frac{2}{1-x}} = \frac{9}{2}$

a) $-\frac{9}{2}$ b) $-\frac{3}{2}$ c) $\frac{3}{2}$ d) $\frac{9}{2}$ e) The limit does not exist

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5. $\lim_{x\to\infty} \frac{3x^5+3x^4+2x^3+x^2+1}{4x^5-9x^4+4x^3+15} =$

(a) 0 (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 3 (e) DNE

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x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
3	0	5	0	5

6. Given that $f(x)$ is a thrice differentiable, continuous function on the interval $(0, 4)$ with the table values given above. $\lim_{x\to 3} \frac{(x-3)^3}{f(x)} =$ L'H $\lim_{x \to 0} \frac{3(x-3)^2}{f'(x)} = \frac{0}{5}$

(a) 0 (b) $\frac{7}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{6}$ (e) dne

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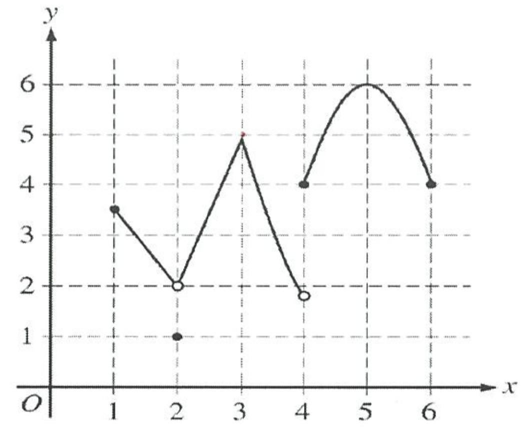
7. The function f is defined on the interval $x \in [-5, 5]$ and has the graph shown below.



Graph of f

Which of the following is true?

a) $\lim_{x\to 2} f(x) = 1$ F

b) $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = dne$ $= f'(3)$ T

c) $\lim_{x\to 3} f(x) = f(6)$ F

d) $\lim_{x\to 4^-} f(x) = 4$

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8. Let m and b be real numbers and let the function f be defined by

$$f(x) = \begin{cases} 3x^2 - mx + 5 & \text{for } x \le 1 \\ mx + b & \text{for } x > 1 \end{cases}$$

If f is both continuous and differentiable at $x = 1$, then

(a) $m = 3,\ b = 2$
(b) $m = 3,\ b = -2$
(c) $m = -3,\ b = 2$
(d) $m = -3,\ b = -2$
(e) None of these

$f' = \begin{cases} 6x - m \\ m \end{cases}$

$x = 1 \to 6 - m = m$
$6 = 2m$
$3 = m$

$3 - 3 + 5 = 3 + b$
$2 = b$

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9. Which of the following functions is NOT differentiable at ~~$x=1$~~ $x=0$?

(a) $f(x)=x^2$ (b) $f(x)=e^x$ (c) $f(x)=\ln(x+1)$

(d) $f(x)=\begin{cases} \dfrac{1}{x+1} & \text{for } x \neq -1 \\ 0 & \text{for } x=-1 \end{cases}$ (e) $f(x)=\cot x$

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10. $\lim_{x\to 0} \dfrac{\int_0^{3x} e^t\,dt}{\tan x} =$ L'H $\lim_{x\to 0} \dfrac{e^{3x}(3)}{\sec^2 x} = 3$

(a) 0 (b) 1 (c) $\frac{1}{3}$ (d) 3 (e) DNE

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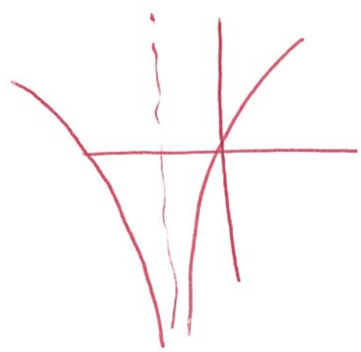
11. A function $f(x)$ has a vertical asymptote at $x=-2$. The derivative of $f(x)$ is negative for all $x<-2$ and positive for all $-2<x$. Which of the following statements are **true**?



a) $\lim_{x\to -2^-} f(x) = -\infty$ and $\lim_{x\to -2^+} f(x) = -\infty$

b) $\lim_{x\to -2^-} f(x) = -\infty$ and $\lim_{x\to -2^+} f(x) = +\infty$

c) $\lim_{x\to -2^-} f(x) = +\infty$ and $\lim_{x\to -2^+} f(x) = +\infty$

d) $\lim_{x\to -2^-} f(x) = +\infty$ and $\lim_{x\to -2^+} f(x) = -\infty$

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12. At $x=-3$, the function given by $f(x)=\begin{cases} 10-x^2, & \text{if } x<-3 \\ e^{x+3}, & \text{if } -3\le x \end{cases}$ is

$x=-3 \to f(-3^{\mp})=\begin{cases}1\\1\end{cases}$ CONT

(A) Undefined

(B) Continuous but not differentiable

$f'(x)=\begin{cases}-2x\\e^{x+3}\end{cases}$ $f'(-3^{\pm})=\begin{cases}6\\1\end{cases}$

(C) Differentiable but not continuous

(D) Neither continuous nor differentiable

(E) Both continuous and differentiable

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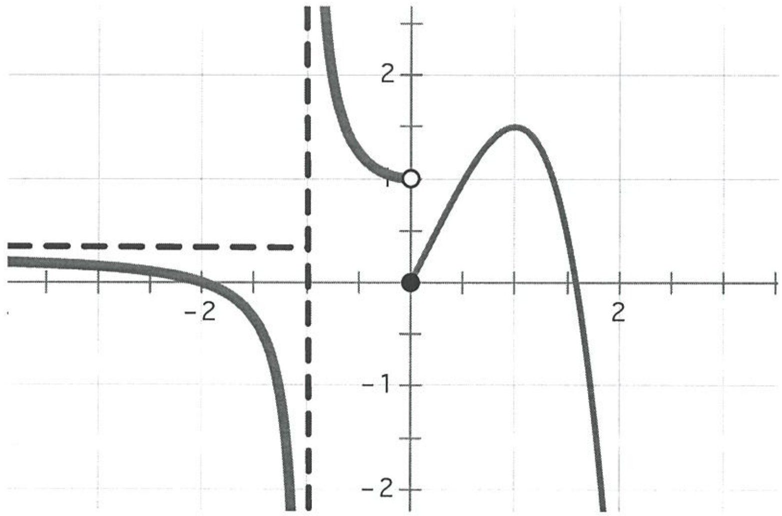
Score___________

Directions: Show all work.



1. For this graph, find

(a) $\lim_{x\to -1^-} f(x) = -\infty$

(b) $\lim_{x\to 0^-} f(x) = 1$

(c) $\lim_{x\to 1} f(x) =$ ~~DNE~~ 1.5

(d) $\lim_{x\to -1} f(x) =$ DNE

(e) $\lim_{x\to 0^+} f(x) = 0$

(f) $\lim_{x\to -1^+} f(x) = +\infty$

(g) $f(-1) =$ DNE

(h) $f(0) = 0$

(i) $f(1) = 1.5$

(j) $\cancel{f(3) =}$?

2. $f(x)=\begin{cases} 3x-2, \text{ if } x<1 \\ \ln(3x-2), \text{ if } 1 \le x \end{cases}$

a) Is $f(x)$ continuous at $x=1$? Why/Why not?

i) $f(1)$ EXISTS

ii) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1} 3x-2 = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} \ln(3x-2) = 0$

$\lim_{x \to 1} f(x)$ DNE

$\therefore$ NOT CONTINUOUS

b) Is $f(x)$ differentiable at $x=1$? Why/Why not?

NOT CONTINUOUS $\therefore$ NOT DIFFERENTIABLE

3. $$h(x)=\begin{cases} \dfrac{x}{4+x^2}, & \text{if } -2\le x<2 \\ \frac{1}{4}, & \text{if } x=2 \\ \dfrac{1}{4}-\sqrt{x^2-4}, & \text{if } 2<x\le 4 \end{cases}$$

a) Is $h(x)$ continuous at $x=2$? Why/Why not?

i) $h(2)$ EXISTS

ii) $\lim_{x\to 2^-} h(x) = \lim_{x\to 2} \frac{x}{4+x^2} = \frac{\cancel{4}\,2}{8} = \frac{1}{4}$

$\lim_{x\to 2^+} h(x) = \lim_{x\to 2} \frac{1}{4}-\sqrt{x^2-4} = 1/4$

$\therefore \lim_{x\to 2} h(x)$ EXISTS

iii) $\lim_{x\to 2} h(x) = h(2)$ $\therefore$ $h(x)$ IS CONTINUOUS

b) For $x\neq 2$, express $h'(x)$ as a piecewise-defined function. Is $h(x)$ differentiable at $x=2$? Why/Why not?

i) $h(x)$ IS CONTINUOUS

ii) $$h'(x) = \begin{cases} \dfrac{(4+x^2)(1)-x(2x)}{(4+x^2)^2} \\ -\frac{1}{2}(x^2-4)^{-1/2}(2x) \end{cases} = \begin{cases} \dfrac{4-x^2}{(4+x^2)^2} & \text{IF } x<2 \\ \dfrac{-x}{(x^2-4)^{1/2}} & \text{IF } 2<x \end{cases}$$

$\lim_{x\to 2^-} h' \neq \lim_{x\to 2^+} h'$ $\therefore$ $h(x)$ IS NOT DIFFERENTIABLE