AB Calculus '20-21
Volume Test v1
Calculator Allowed

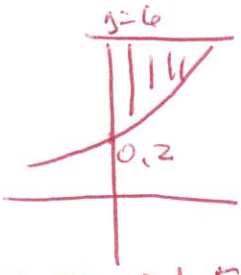
Name SOLUTION KEY

Score: 7

1. The area of the region bounded by the graphs of $y = e^x + 1$, $x = 0$ and the line $y = 6$ is

(A) $5\ln 5 - 4$ (B) $8\ln 2 - 3$ (C) $5\ln 5 + 5$

(D) $10\ln 2 - 4$ (E) $10\ln 2 + 4$



$A = \int_0^{\ln 5} (6 - (e^x + 1))\,dx = 5x - e^x \Big]_0^{\ln 5} = 5\ln 5 - 5 + 1$

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2. The region enclosed by the graphs of $y = 3e^x$ and $y = 3$ on $0 \le x \le 1$ is revolved about the y-axis. The volume of this solid is

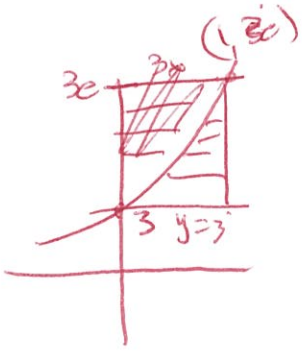
(A) $\pi \int_3^{3e} \left(3 - \ln\frac{y}{3}\right)^2 dy$

(B) $\pi \int_0^1 \left(3e^x - 3\right)^2 dx$

(C) $\pi \int_3^{3e} \left(1 - \ln^2\frac{y}{3}\right) dy$

(D) $\pi \int_0^3 \left(1 - \ln\frac{y}{3}\right)^2 dy$

(E) $\pi \int_3^{3e} \left(1 - \ln\frac{y}{3}\right)^2 dy$



$x = \ln\frac{y}{3}$

Disk

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3. Which of the following is the length of $y=\ln(\cos x)$ between $0 \le x \le \frac{\pi}{3}$?

(A) $\ln(2+\sqrt{2})$ (B) $\ln(1+\sqrt{3})$ (C) $\ln(2-\sqrt{3})$

(D) $\ln(2+\sqrt{3})$ (E) $\ln(\sqrt{3}-1)$

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4. A region is bounded by $y=\frac{1}{x}$, the x-axis, the line $x=m$, and the line $x=2m$, where $m>1$. If the area of the region equals 0.125, what is the value of m?

(A) 1.133 (B) 1.334 (C) 2.998

(D) 7.510 (E) There is no such value

$A = \int_m^{2m} \frac{1}{x}\,dx$

$= \ln x \Big]_m^{2m}$

$= \ln 2m - \ln m = \ln 2 \neq .125$

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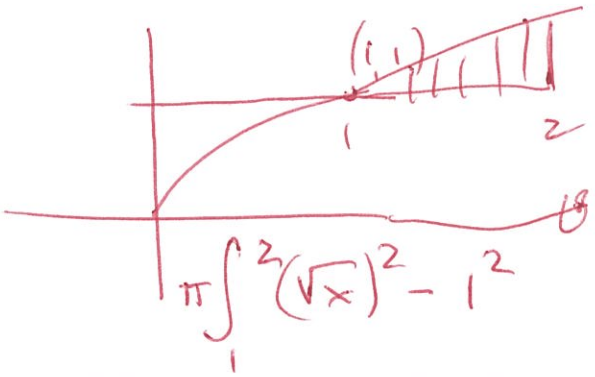
5. Let R be the region in the first quadrant bounded by $y=\sqrt{x}$ and the line $y=1$ on $1\le x\le 2$. Which of the following integrals gives the volume of the solid generated when R is rotated about the x-axis?

(a) $\pi\int_1^2 (x-1)\,dx$ (b) $\pi\int_1^2 (x-1)^2\,dx$

(c) $\pi\int_1^2 \left(\sqrt{x}-1\right)^2\,dx$ (d) $\pi\int_1^2 (2-x)^2\,dx$

(e) $\pi\int_1^2 \left(2-\sqrt{x}\right)^2\,dx$



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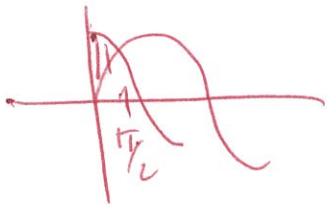
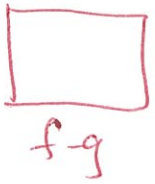
6. The base of a solid is the region bounded by $y=\sin x$ and $y=\cos x$ for $0\le x\le\frac{\pi}{4}$. If each cross-section of the solid perpendicular to the x-axis is a square, the volume of the solid is

(a) 0.306 (b) 0.256 (c) 0.315 (d) 0.286 (e) 0.257





$V=\int_0^{\pi/4} (\cos x - \sin x)^2\,dx$

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7. The region $\boldsymbol{R}$ is bounded by the lines $y = 2x - 4$, $x = 3$, and $y = 0$. Which of these expressions gives the volume of the solid formed by revolving $\boldsymbol{R}$ around the line $x = 5$?

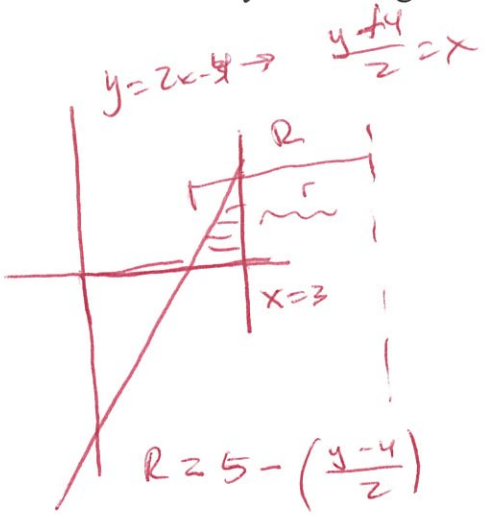
(a) $\int_2^3 \left((2x-4)^2 - 3^2 \right) dx$

(b) $\int_2^3 \left((2x-9)^2 - 2^2 \right) dx$

(c) $\int_0^2 \left(\left(\frac{y+4}{2} \right)^2 - 3^2 \right) dy$

(d) $\int_0^2 \left(\left(\frac{y+4}{2} \right)^2 - 2^2 \right) dy$

(e) $\int_0^6 \left(\left(\frac{y+4}{2} \right)^2 - 3^2 \right) dy$



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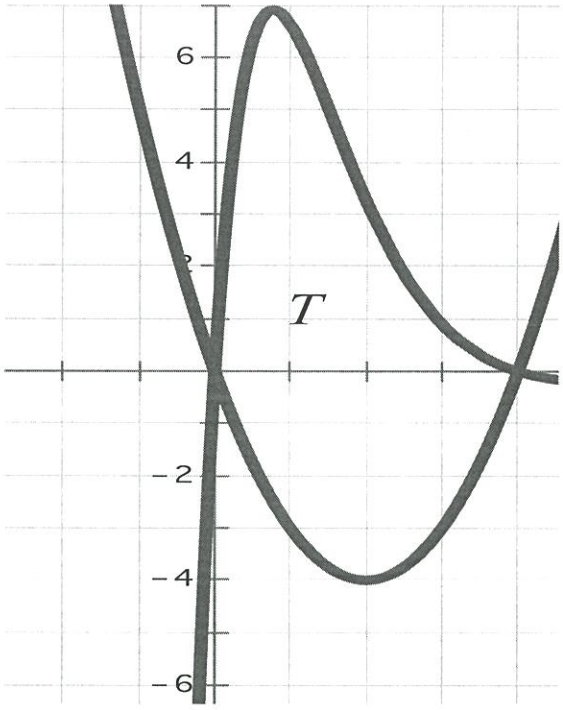
Score: 18

1. Region T is bounded by the curves $f(x)=-6(x^2-4x)e^{-x}$ and $g(x)=x^2-4x$



(a) Find the area of region T. Show the set-up.

(2) $A=\int_0^4 \left[-6(x^2-4x)e^{-x}-(x^2-4x)\right]dx$ (1)

$=23.326$ (1)

(b) Find the volume of the solid generated when R is revolved about the line $y=-4$.

(4) $V=\pi\int_0^4\left[\left[-6(x^2-4x)e^{-x}+4\right]^2-\left[x^2-4x+4\right]^2\right]dx$ (1) (1) (1)

$=676.640$ (1)

(c) Let the base of the solid be the region R. Find the volume of the solid where the cross-sections perpendicular to the x-axis are squares.

(3) $V=\int_0^4 (y_1-y_2)^2\,dx = 170.182$ (1) (1) (1)

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2. Mrs Quattrin bought a handmade cermaic pitcher that was created on a potter's wheel (think Patrick Swayze and Demi Moore in *Ghost*…or Leslie Nielsen and Priscilla Presley in *Naked Gun*). Wh\anting to know the volume, she measures the radius of the pitcher at one inch increments and records the data on the table below.

H inches	0	1	2	3	4	5	6	7
$r(h)$ inches	2	3.25	3.4	3.3	3	2.56	2	2.66

h is the height from the bottom of the pitcher and $r(h)$ is the radius at each height.

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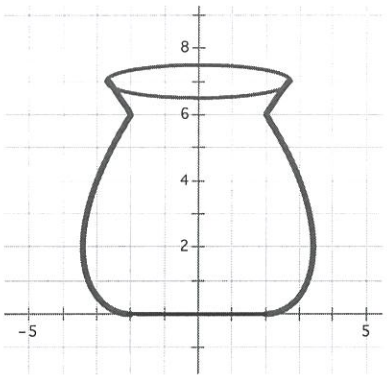
a) a) Using a right-hand Riemann sum, approximate $\pi\int_0^7\left[r(h)\right]^2 dh$, the volume of the pitcher.

(2) $\int \approx$ (1) π [$(3.25)^2 + (3.4)^2 + (3.3)^2 + 3^2 + (2.56)^2 + 2^2 + (2.66)^2$] (1)

$= 187.370$



(3)

b) Using a calculator to do a quartic regression on the data, Mrs Quattrin finds that the data is modeled by $R(h)=2+\frac{1}{4}(h-6)\sqrt{h}$. Find the volume of the pitcher if this model is rotated about the h-axis.

(1) (1)

$$V=\pi\int_0^7\left(2+\tfrac{1}{4}(h-6)\sqrt{h}\right)^2 dh$$

$$=12.666\pi$$

(1)

$$=39.792$$

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(4)

c) Using the kitchen faucet, Mrs Quattrin fills the pitcher with water. The faucet delivers water at $1.13\ {}^{in^3}/_{\text{sec}}$. The model indicates that the volume equation would be $V(h)=\pi\int_0^h\left(2+\frac{1}{4}(x-6)\sqrt{x}\right)^2 dx$. How fast is h changing when the height is 4.2 inches?

(1) (1)

$$\frac{dV}{dt}=\pi\left(2+\tfrac{1}{4}(x-6)\sqrt{x}\right)^2\frac{dh}{dt}$$

$$1.13=\pi\left(2+\tfrac{1}{4}(-1.8)\sqrt{4.2}\right)^2\frac{dh}{dt}$$

(1)

$$\frac{dh}{dt}=.309$$

(1)
