AP Calculus AB '20-21
Anti-Derivative MCQ Test v4

Name Solution Key

Score___________

1. $\int x\sqrt{3x}\,dx =$ $= \sqrt{3}\int x^{3/2}dx = \sqrt{3}\,\frac{x^{5/2}}{5/2} + C$

a) $\frac{2\sqrt{3}}{5}x^{5/2}+C$ b) $\frac{5\sqrt{3}}{2}x^{5/2}+C$ c) $\frac{\sqrt{3}}{2}x^{1/2}+C$

d) $2\sqrt{3x}+C$ e) $\frac{5\sqrt{3}}{2}x^{3/2}+C$

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2. Which of the following statements are true?

T I. $\int\left(x^5\sin x^6\right)dx = -\frac{1}{6}\cos x^6 + c$ II. $\int \tan x\,dx = \sec^2 x + c$ F

III. $\int\left(\left(x^3+x\right)\sqrt[4]{x^4+2x^2-5}\right)dx = \frac{1}{5}\left(x^4+2x^2-5\right)^{5/4}+c$ T

a) I only b) II only c) III only

d) I and III e) II and III only

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3. $\int \frac{x^2-1}{x}dx =$ $\int x - \frac{1}{x}\, dx = \frac{x^2}{2} - \ln|x| + c$

a) $\frac{1}{2}\ln|x^2-1|+c$ b) $\frac{1}{2}(x^2-1)^2+c$ c) $\frac{x^2}{2}-\ln|x|+c$

d) $x-\frac{1}{x}+c$ e) $1+x^{-2}+c$

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4. $\int \frac{4x}{1+x^2}\, dx =$ $2\int \frac{1}{u}\, du$ $u = 1+x^2$ $du = 2x\, dx$

$2\ln|u| + c$

a) $4\arctan x + C$ b) $\frac{4}{x}\arctan x + C$ c) $\frac{1}{2}\ln(1+x^2)+C$

d) $2\ln(1+x^2)+C$ e) $2x^2+4\ln|x|+C$

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5. Which of the following is the solution to the differential equation $\frac{dy}{dx} = y\sec^2 x$ with the initial condition $y\left(\frac{\pi}{4}\right) = -1$?

a) $y = -e^{\tan x}$ b) $y = -e^{(-1+\tan x)}$ c) $y = -e^{\left(\sec^3 x - 2\sqrt{2}\right)/3}$

d) $y = -\sqrt{2\tan x - 1}$ e) $y = e^{(-1+\tan x)}$

$\frac{1}{y}dy = \sec^2 x\,dx \Rightarrow \ln|y| = \tan x + C \rightarrow \cancel{y = Re^{\tan x}}$

$0 = 1 + C \Rightarrow C = -1 \quad y = -e^{-1+\tan x}$

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6. $\int \frac{e^{x^2} - 2x}{e^{x^2}}\,dx =$ $\int 1\,dx + \int e^{-x^2}(2x\,dx)$

$x + e^{x^2} + c$

a) $-e^{-x^2} + c$
b) $-e^{x^2} + c$
c) $x - e^{x^2} + c$
d) $x + e^{-x^2} + c$
e) $x - e^{-x^2} + c$

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7. $\int \left(2-\sin\frac{t}{5}\right)^2 \cos\frac{t}{5}\,dt =$

$u = 2 - \sin\frac{t}{5}$

$du = -\cos\frac{t}{5}\left(\frac{1}{5}dt\right)$

a) $-\frac{5}{3}\left(2-\sin\frac{t}{5}\right)^3 + c$

b) $\frac{5}{3}\left(2-\cos\frac{t}{5}\right)^3 + c$

c) $\frac{1}{3}\left(2-\sin\frac{t}{5}\right)^3 + c$

d) $5\left(2-\sin\frac{t}{5}\right)^3 + c$

e) $-\frac{5}{3}\left(2-\cos\frac{t}{5}\right)^3 + c$

$\int = -5\int u^2\,du$

$= \frac{-5u^3}{3} + c$

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8. For $\int \sec^2 x \tan^6 x\,dx$, the correct u-substitution is

a) $u = \sec x$

b) $u = \tan x$

c) either $u = \sec x$ or $u = \tan x$

d) neither $u = \sec x$ nor $u = \tan x$

e) to convert to sine and cosine

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9. A particle moves along the x-axis with acceleration at any time t given as $a(t) = 3t^2 + 4t + 6$. If the particle's velocity is 10 and its initial position is 2 when $t = 0$, what is the position function?

$V = t^3 + 2t^2 + 6t + C_1$

$10 = 0 + 0 + 0 + C_1 \rightarrow C_1 = 10$

a) $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 12$

b) $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 10t + 2$

$X = \int (t^3 + 2t^2 + 6t + 10)\,dt$

$\frac{1}{4}t^4$ - - - -

c) $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 2$

d) $x(t) = 3t^4 + t^3 + t^2 + 10t + 2$

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1. $\int\left(7x^5+7^x-\frac{1}{\sqrt[3]{x^7}}+\frac{1}{7x^3}\right)dx$ $=\int\left(7x^5+7^x-x^{-7/3}+\frac{1}{7}x^{-3}\right)dx$

$\frac{7x^6}{6}+\frac{7^x}{\ln 7}+\frac{3}{4}x^{-4/3}-\frac{1}{14}x^{-2}+C$

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2. $\int\frac{\csc^2(\ln x)}{x}dx$

$u=\ln x$
$du=\frac{1}{x}dx$

$=\int\csc^2 u\,du$

$=-\cot u+C$

$=-\cot(\ln x)+C$

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3. The acceleration of a particle is described by $a(t) = e^t - \sin 2t$. Find the distance equation for $x(t)$ if $v(0) = 0$ and $x(0) = 3$.

$v(t) = \int (e^t - \sin 2t) = e^t + \frac{\cos 2t}{2} + c_1$

$(0,0) \rightarrow 0 = e^0 + \frac{\cos 0}{2} + c_1$

$= 1 + \frac{1}{2} + c_1$

$-\frac{3}{2} = c_1$

$x(t) = \int e^t + \frac{1}{2}\int \cos 2t\,dt + \int -\frac{3}{2}\,dt$

$= e^t + \frac{1}{4}\sin 2t - \frac{3}{2}t + c_2$

$(0,3) \rightarrow 3 = e^0 + \sin 0 - 0 + c_2 \rightarrow c_2 = 2$

$x(t) = e^t + \frac{1}{4}\sin 2t - \frac{3}{2}t + 2$

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4. $\int \left(3\sqrt{x^3} - \sec(2x) + \frac{x}{e^{4x^2}}\right)dx$

$u_1 = 2x$
$du_1 = 2dx$

$u_2 = -4x^2$
$du_2 = -8x\,dx$

$3\int x^{3/2}dx - \frac{1}{2}\int \sec u_1\,du_1 + -\frac{1}{8}\int e^{u_2}du_2$

$3\frac{x^{5/2}}{5/2} - \frac{1}{2}\ln|\sec 2x + \tan 2x| - \frac{1}{8}e^{-4x^2} + C$

$\frac{6}{5}x^{5/2} - \frac{1}{2}\ln|\sec 2x + \tan 2x| - \frac{1}{8}e^{-4x^2} + C$

5. Find the particular solution $w = f(t)$ that passes through $(1, 0)$ if

$$\frac{dw}{dt} = \frac{(t^3)\sqrt{1-4w^2}}{w}$$

$$\frac{w}{(1-4w^2)^{1/2}} = t^3\,dt$$

$u = 1-4w^2$

$du = -8w$

$$-\frac{1}{8}\int u^{-1/2}\,du = \frac{t^4}{4} + c$$

$$-\frac{1}{8}\,\frac{u^{1/2}}{1/2} = \frac{t^4}{4} + c$$

$$-\frac{1}{4}(1-4w^2)^{1/2} = \frac{1}{4}t^4 + c$$

$$(1,0) \to -\frac{1}{4}(1)^{1/2} = \frac{1}{4} + c$$

$$-\frac{1}{2} = c$$

$$-\frac{1}{4}(1-4w^2)^{1/2} = \frac{1}{4}t^4 - \frac{1}{2}$$

$$(1-4w^2)^{1/2} = -t^4 + 2$$

$$1-4w^2 = (2-t^4)^2$$

$$-4w^2 = (2-t^4)^2 - 1$$

$$w^2 = -\frac{1}{4}(2-t^4)^2 + \frac{1}{4}$$

$$w = \pm\sqrt{-\frac{1}{4}(2-t^4)^2 + \frac{1}{4}}$$