AP Calculus AB '21-22
Integral Test v1

Name Solution Key

Score____________

NO CALCULATOR ALLOWED

1. Find $\int_{-3}^{3} x^2\, dx$ $\left.\frac{x^3}{3}\right]_{-3}^{3} = 9-(-9)$

a) -18 b) -9 c) 0 d) 9 e) 18

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2. $\frac{1}{4}\int_0^1 \frac{4x^3dx}{(x^4+1)^3} = \frac{1}{4}\int_1^2 u^{-3} = \frac{1}{4}\left[\frac{u^{-2}}{-2}\right]_1^2 = -\frac{1}{8u^2}\Big]_1^2 = -\frac{1}{32}-\left(-\frac{1}{8}\right) = \frac{3}{32}$

a) $\frac{3}{8}$ b) $\frac{3}{32}$ c) $\frac{1}{32}$ d) $\frac{1}{4}\ln 8$ e) 1

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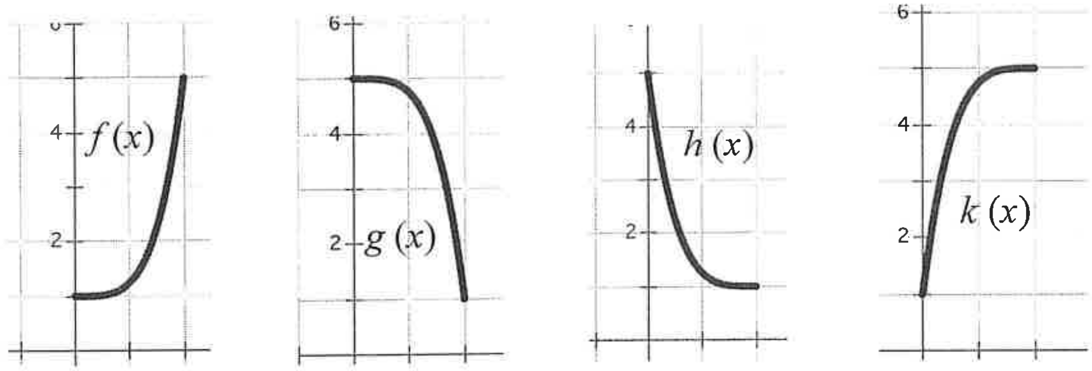
3. If $\int_0^6 f(x)\, dx = 9$, $\int_3^6 f(x)\, dx = 5$, and $\int_3^0 g(x)\, dx = -7$ then $\int_0^3 \left[\frac{1}{2} f(x) - 3g(x)\right] dx =$

a) -23 b) -19 c) $-\frac{17}{2}$ d) 19 e) 23

$\int_0^3 f(x)dx = \int_0^6 f(x)dx - \int_3^6 f(x)dx = 9-5 = 4$

$\frac{1}{2}\int_0^3 f(x)dx - 3\int_0^3 g(x)dx = \frac{1}{2}(4) - 3(7) = -19$

4. For which of the following functions would the left-hand Riemann sum be an underestimate?



a) $f(x)$ and $g(x)$ b) $h(x)$ and $k(x)$

LEFT UNDER = INCREASING

c) $f(x)$ and $h(x)$ d) $g(x)$ and $k(x)$

e) $f(x)$ and $k(x)$

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5. $\int_2^3 \frac{1}{9+x^2}\,dx = \frac{1}{3}\tan^{-1}\frac{x}{3}\Big]_2^3 = \frac{1}{3}\overset{=\pi/4}{\tan^{-1}1} - \frac{1}{3}\tan^{-1}\frac{2}{3}$

a) $\frac{\pi}{4}-\frac{1}{3}\tan^{-1}\left(\frac{2}{3}\right)$ b) $\frac{\pi}{12}-\frac{1}{3}\tan^{-1}\left(\frac{2}{3}\right)$

c) $\frac{1}{3}\tan^{-1}\left(\frac{2}{3}\right)$ d) $\frac{1}{3}\tan^{-1}\left(\frac{2}{3}\right)-\frac{\pi}{12}$

e) $\frac{1}{3}\tan^{-1}\left(\frac{2}{3}\right)-\frac{\pi}{4}$

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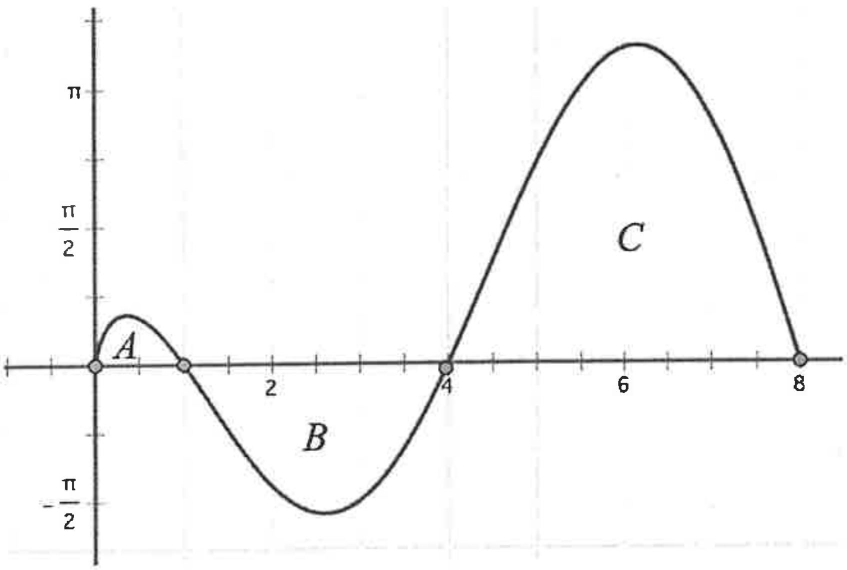
6. The basement of a house is flooding. The water pours in at a rate of $f(t)$ gallons per hour and is being pumped out at a rate of $r(t)$. When the pump is started, at time t = 0, there are 1200 gallons of water in the basement. Which of the following expresses the total number of gallons of water in the basement at time t hours?

a) $1200+\int_0^t [f(x)-r(x)]dx$ b) $\int_0^t [f(x)-r(x)]dx$

c) $f(t)-r(t)$ d) $\frac{1}{t}\int_0^t [f(x)-r(x)]dx$

e) $f'(t)-r'(t)$



In the figure above, A, B, and C are areas between the curve $f(x)$ and the x-axis.

If A=14, B=16, and C=50, what is the average value of $f(x)$ on $x\in[0,8]$

a) 6 b) 10 c) $\frac{38}{3}$ d) $\frac{80}{3}$ e) 80

$\frac{1}{8}\int_0^8 f(x)dx = \frac{1}{8}[14-16+50] = \frac{48}{8}$

$$\frac{1}{1/2-0}\int_0^{1/2}(e^{2x}-1)\,dx$$

$$= 2\int_0^{1/2}(e^{2x}-1)\,dx \qquad u=2x$$

$$= \cancel{2}\int_0^1 e^u-1\,du$$

$$= e^u - u\Big]_0^1$$

$$(e-1)-(e^0-0)$$

8. The average value of $y=e^{2x}+1$ on $x\in\left[0,\frac{1}{2}\right]$ is

a) $\frac{e}{2}$ b) $\frac{e}{4}$ c) $2e-1$ d) $\frac{e}{3}$ e) e

t (in minutes)	0	8	16	24	32	40	48
$V(t)$ (in m^3/min)	26	32	43	24	19	24	30

9. Waste flows through a sewage pipe. The table above shows the rate of flow at specific times. Using the Right Riemann rectangles, the approximate total volume of waste that flowed through the pipe over these 48 minutes is

a) $8[26+32+43+24+19+24+30]$

b) $16[32+24+24]$

c) $8[32+43+24+19+24+30]$

d) $8[26+32+43+24+19+24]$

e) $8[26+2(32)+2(43)+2(24)+2(19)+2(24)+30]$

AP Calculus AB '21-22

Chapter 3 v1 FRQ

Calculator Allowed

SOLUTIONS

Directions: Show all work.

1. Consider the function $f(x) = x^3 - 3x^2 - x + 3$.

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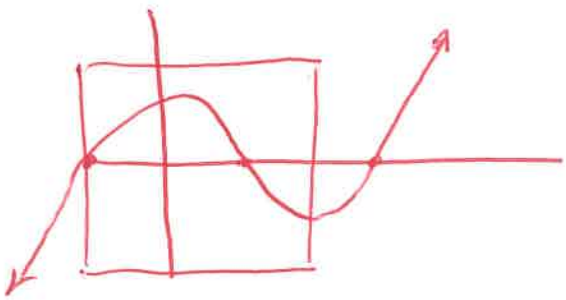
a) Draw the graph of $f(x)$ on $x \in [-1, 2]$.



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b) Prove that ~~the one~~ zero of $f(x)$ is at $x = 1$.

$f(x) = x^2(x-3) - 1(x-3)$

$(x^2-1)(x-3)$

$(x-1)(x+1)(x-3) = 0$

$x = \pm 1, 3$

$(-1, 0)\ (1, 0)$

$(3, 0)$

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c) Find the exact value of $\int_{-1}^{2} f(x)\,dx$. Show the antiderivative and boundary insertion steps.

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$$\int_{-1}^{2} (x^3 - 3x^2 - x + 3)\,dx$$

$$= \frac{x^4}{4} - x^3 - \frac{1}{2}x^2 + 3x \Big]_{-1}^{2}$$

$$= (4 - 8 - 2 + 6) - \left(\frac{1}{4} + 1 - \frac{1}{2} - 3\right) = \frac{9}{4}$$

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d) Find the exact area between the x-axis and $f(x)$ on $x \in [-1, 2]$. Show the antiderivative and boundary insertion steps.

$$A = \int_{-1}^{1} f(x)\,dx - \int_{1}^{2} f(x)\,dx$$

$$= \left[\frac{x^4}{4} - x^3 - \frac{1}{2}x^2 + 3x\right]_{-1}^{1} - \left[\frac{x^4}{4} - x^3 - \frac{1}{2}x^2 + 3x\right]_{1}^{2}$$

$$= \left[\left(\cancel{\tfrac{1}{4}} - 1 - \cancel{\tfrac{1}{2}} + 3\right) - \left(\cancel{\tfrac{1}{4}} + 1 - \cancel{\tfrac{1}{2}} - 3\right)\right] - \left\{(4 - 8 - 2 + 6) - \left(\frac{1}{4} - 1 - \frac{1}{2} + 3\right)\right\}$$

$$4 - \left(-\frac{7}{4}\right)$$

$$= 5.75$$

The Carbon Sequestration Problem

2. One innovative approach to global warming is to capture carbon dioxide that is created as a byproduct of producing concrete and storing the CO_2 in the sandstone in depleted natural gas fields. At a particular site on a particular day, the CO_2 is injected into the sandstone at a rate of $I(t)=121\sin\left[\frac{\pi}{65}t^2\right]$. The CO_2 stabilizes the ground and forces remaining natural gas upward where it can be extracted at a rate of $E(t)=50-50\cos\left[\frac{\pi}{8}t\right]$. $I(t)$ and $E(t)$ are measured in metric tons per hour and t is measured in hours where $0\le t\le 8$.

(a) How many metric tons of CO_2 is injected into the field over $0\le t\le 8$?

TOTAL INJECTED $= \int_0^8 I(t)\,dt$

$= 492.320$ METRIC TONS

(b) Find the value of $I(4)$ and $I'(4)$. Using the correct units, explain the meaning of both. ANSWERS

$I(4) = 84.520$

$I'(4) = 33.480$

~~E(4)~~) 84.520 TONS/HR IS HOW FAST CO_2 IS BEING INJECTED INTO THE GAS FIELD AT $t=4$ HOURS

33.480 TONS/HR^2 IS HOW FAST THE RATE OF INJECTION IS INCREASING AT $t=4$ HOURS

OTHER THAN $t=0$

(c) Find the time, ~~if any~~, when the rate of injection of CO_2 is equal the rate of extraction of natural gas.

$t \approx 6.813$ HOURS $I(t) = E(t)$

AND $t=0$

(d) Find the total change of gasses in the sandstone during this 8-hour day. Using the correct units, explain the result.

92

$\int_0^8 (I(t) - E(t))\,dt = 92.320$ TONS

THE AMOUNT OF GAS IN THE FIELD HAS INCREASED BY 92.320 TONS ~~BE~~ DURING THE TIME INTERVAL FROM $t=0$ TO $t=8$.

The PHS Dog Problem

t in Month	1	2	4	6	7	9	12
$R(t)$ in Dogs received per month	208	195	201	240	236	220	247
$A(t)$ in Dogs Adopted per month	63	79	56	73	65	67	71

3. The Peninsula Humane Society consistently received lost, stray and abandon dogs over the course of the year and tries their best to find homes for them. The table above shows a sample of the numbers of dogs received and adopted out per month in 2019.

(a) Approximate $R'(5)$. Using the correct units, explain $R'(5)$ in context of the problem.

$$R'(5) = \frac{240-201}{6-4} = \frac{39}{2} = 19.5 \ \frac{\text{DOGS}}{\text{MO}^2}$$

THE RATE AT WHICH DOGS ARE RECEIVED AT PHS IS INCREASING BY 19.5 DOGS PER MONTH AT $t=5$ MONTHS

(b) Using a Midpoint Reimann Sum, find $\int_1^{12} A(t)dt$. Using the correct unots, explain $\int_1^{12} A(t)dt$.

$$\int_1^{12} A(t)dt \approx 3(79) + 3(73) + 5(67) = 791 \text{ DOGS}$$

APPROXIMATELY 791 DOGS HAVE BEEN ADOPTED FROM PHS DURING THESE 12 MONTHS

(c) Set up a left-hand Reimann Sum to approximate $\int_1^6 R(t)dt$. Using the correct units, explain the meaning of $\frac{1}{6-1}\int_1^6 R(t)dt$.

$\int_1^6 R(t) \approx \cancel{2}\,1(208) + 2(195) + 2(201) = 600$

$\frac{1}{6-1}\int_1^6 R(t)\,dt$ IS THE APPROXIMATE AVERAGE NUMBER OF DOGS PER MONTH RECEIVED AT PHS ~~DURING~~ BETWEEN $t = 1$ AND $t = 6$ MONTHS

(d) Assume that $R(t)$ and $A(t)$ are continuous and differentiable functions. If there were 105 dogs in the PHS kennels at the start of the year, set up an integral expression that would determine the number of dogs in the kennels at time t months.

~~TOTAL~~ DOGS $= 105 + \int_{1}^{t} R(x) - A(x)\,dx$
