AB Calculus '21-22 Name Solution Key
Dx Apps I Test v1
No Calculator

1. Let g be the function defined by $\int_{-1}^{x} \frac{15-2t-t^2}{\sqrt{t^2+4}}\,dt$. On which intervals is g decreasing?

$g' = \frac{15-2x-x^2}{\sqrt{x^2+4}} \rightarrow$

g': $-$ 0 $+$ 0 $-$; -5, 3

a) $x \le -5$ or $3 \le x$
b) $-5 \le x \le 3$
c) $x \le -3$ or $5 \le x$
d) $-3 \le x \le 5$
e) none of these

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2. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that they have values given on the table below.

x	$f'(x)$	$f''(x)$	$g'(x)$	$g''(x)$
2	0	2	-8	0
4	8	0	0	3
8	0	-12	0	4

Then at $x = 2$, $f(x)$ has a:

a) Relative Maximum
b) Relative Minimum $f' = 0$; $f'' > 0$
c) Point of Inflection
d) Zero
e) None of these

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3. Suppose $f'(x)=\frac{(x+1)^3(x-4)^4}{(x^2+4)}$. Which of the following statements must be **true**?

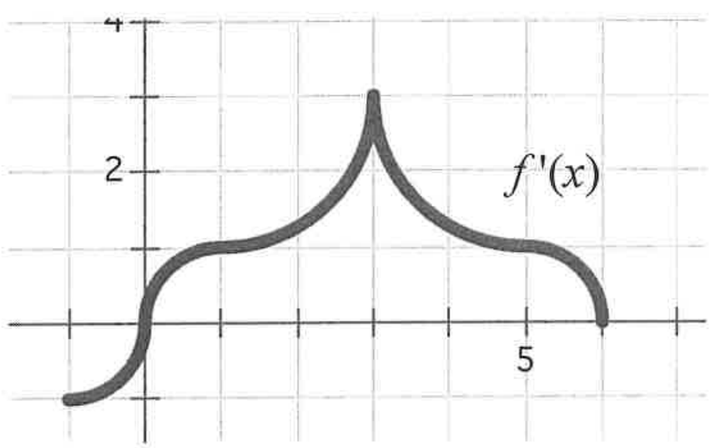
f' $-$ 0 $+$ 0 $+$
x -1 4

a) $f(x)$ has a point of inflection at $x=-1$ F

b) $f(x)$ is decreasing on $x\in(-\infty,-1)$ T

c) $f(x)$ has a relative minimum at $x=4$ F

d) $f(x)$ has a relative ~~minimum~~ max at $x=-1$ F



4. The figure above shows the graph of f', the derivative of f, for $-1\le x\le 6$. What is the value of x at which the absolute maximum of f occurs?

a) -1 b) 0 c) 1 d) 3 e) 6

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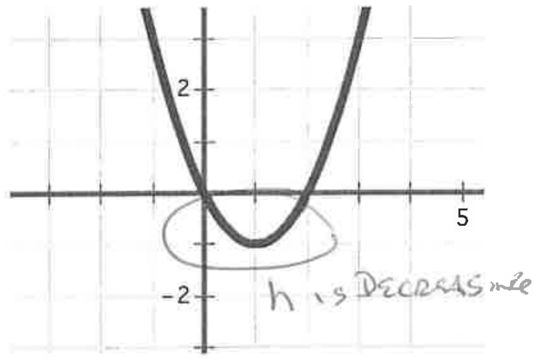
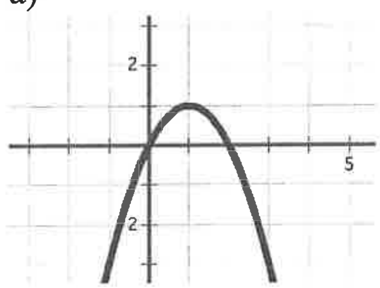
5. The function $h'(x)$ is graphed below.



Which of these functions represents $h(x)$?

a)



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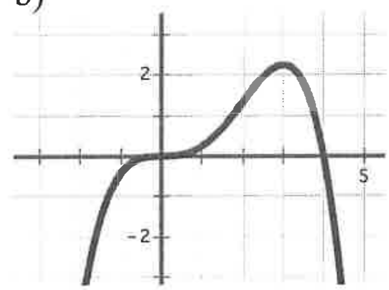
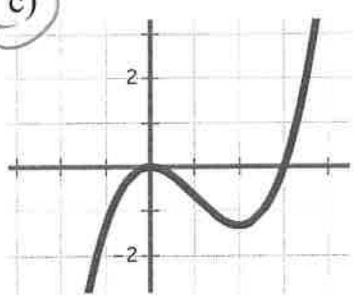
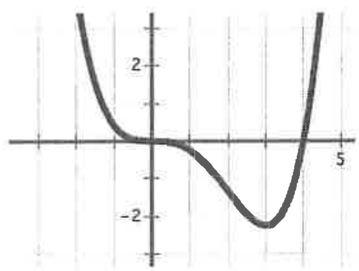


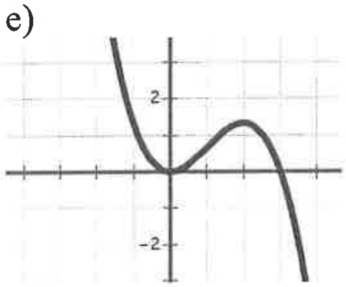
c)



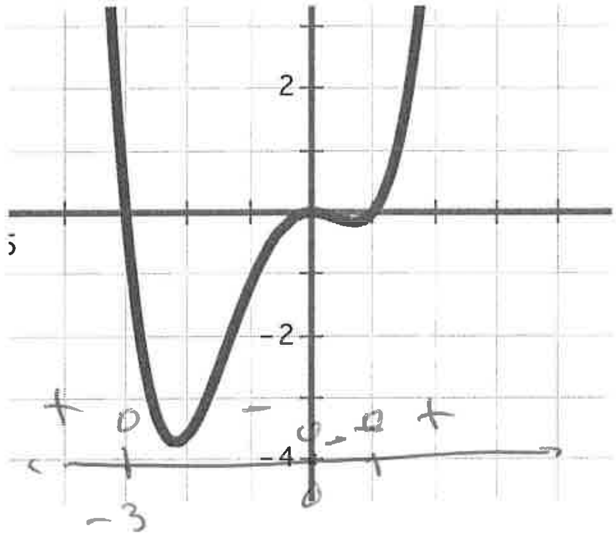
d)



e)



6. The graph below is of $g''(x)$, the **second** derivative of $g(x)$. Which of these statements is **false** about $g(x)$?

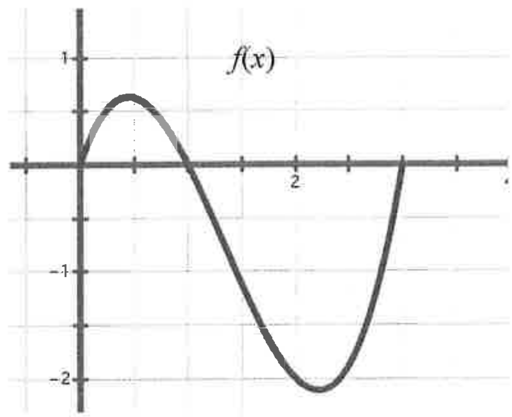


a) $g(x)$ is concave up on the interval $(2, 3)$ T

b) $g(x)$ has a point of inflection at $x = 1$ T

c) The derivative of $g(x)$ is increasing on $(2, 3)$ T

d) $g(x)$ has a point of inflection at $x = 4$ F



7. The graph of differentiable equation f is shown above. If $h(x)=\int_2^x f(t)\,dt$, which of the following is true?

$h(2) = 0$

a) $h(2)<h'(2)<h''(2)$

b) $h'(2)<h''(2)<h(2)$

$h'(2) = f(2) < -2$

c) $h'(2)<h(2)<h''(2)$

d) $h''(2)<h(2)<h'(2)$

$h''(2) \approx -1$

e) $h''(2)<h'(2)<h(2)$

8. Suppose $f'(x)=(1-x)^2(3-x)^5(x-5)^3$. Of the following, which best describes the graph of $f(x)$?

f': $-$ 0 $-$ 0 $+$ 0 $-$; 1, 3, 5

a) $f(x)$ has relative minimum at $x=1$, a relative maximum at $x=3$, and a points of inflection at $x=5$

b) $f(x)$ has relative minimum at $x=3$, a relative maximum at $x=1$, and a points of inflection at $x=5$

c) $f(x)$ has relative minimum at $x=5$, a relative maximum at $x=3$, and a points of inflection at $x=1$

d) $f(x)$ has relative minimum at $x=1$, a relative maximum at $x=5$, and a points of inflection at $x=3$

e) $f(x)$ has relative minimum at $x=3$, a relative maximum at $x=5$, and a points of inflection at $x=1$

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9. Given $g(t)=t\sqrt{t+6}$ on $x\in[-2,\ 0]$ is both continuous and differentiable, the Mean Value Theorem guarantees that $g'(t)=$

a) 0 b) $\sqrt{2}$ c) $-\sqrt{2}$

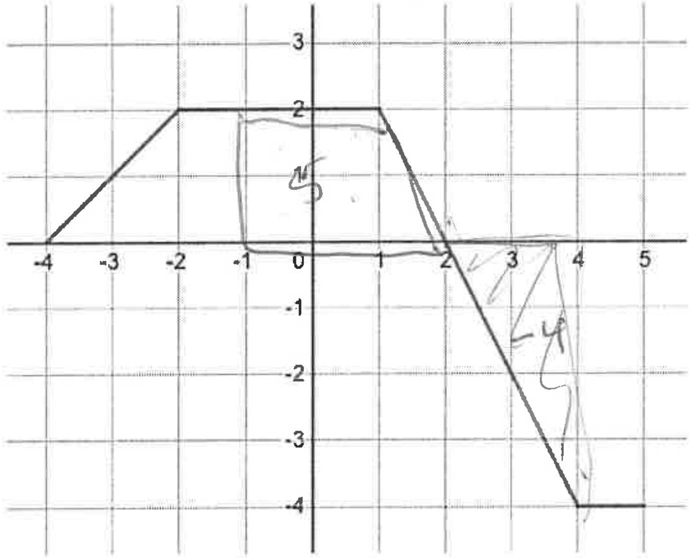
d) -2 e) 2

$$g'(c)=\frac{g(0)-g(-2)}{0-(-2)}=\frac{0-(-4)}{0+2}=\cancel{+2}\qquad 4/2=2$$

10. The graph below gives the graph of $f'(x)$, the derivative of $f(x)$. If it is known that $f(-1)=3$, what is the value of $f(4)$?

$f(4) = 3 + \int_{-1}^{4} f'(t)\,dt$

$= 3+3$



graph of f'(x)

a) 3 b) 4 ~~c)~~ 6 d) 7 e) 9

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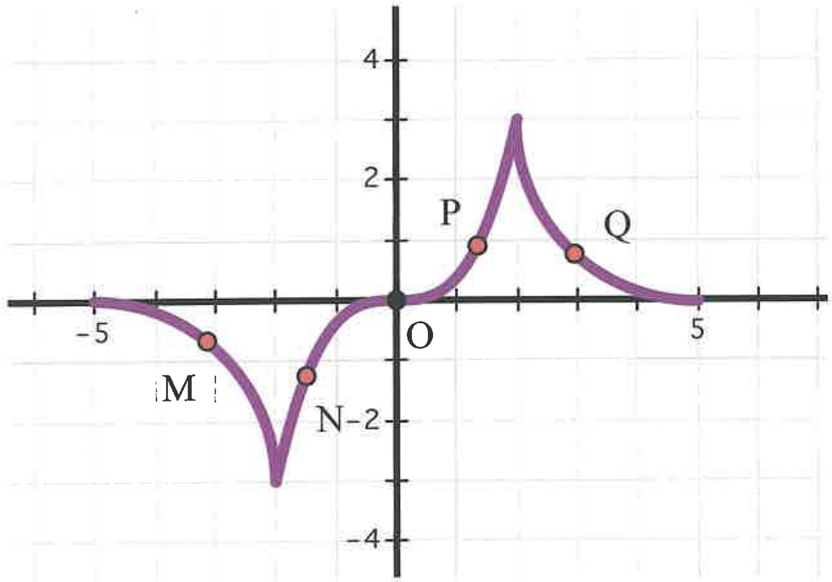
11. Find the absolute maximum value of $y = 4x - x^2$ on $0 \le x \le 3$.

a) 4 b) -3 c) 0 d) 2 e) 3

CV	y
0	0
2	4
3	3



12. At what point on the above curve is $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$

a) M b) N c) P d) Q

INCREASING
CONCAVE UP

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Name Solution Key

Calculator allowed

Score 27

Directions: Show all work.

x	$-3 \le x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x \le 3$
$f'(x)$	Negative	0	Negative	0	Positive
$f''(x)$	Positive	dne	Negative	dne	Negative

1. A function f is continuous on the interval $x \in [-3, 3]$ such that $f(-3) = 6$ and $f(3) = 1$. The functions f' and f'' have the properties given above.

a) Find all the values of x for which f has a maximum or a minimum on $x \in [-3, 3]$. Justify your answer.

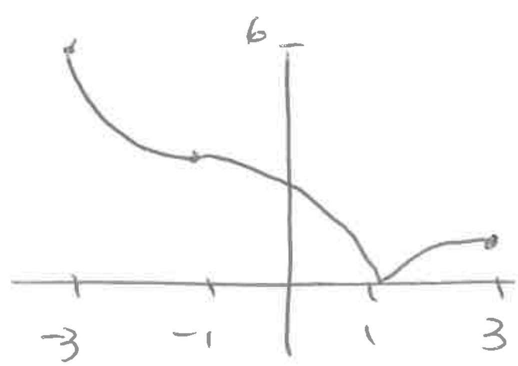
MAX @ $x = \pm 3$ BECAUSE OF THE END POINT CORALLARY

MIN @ $x = +1$ BECAUSE f' SWITCHES FROM $-$ TO $+$

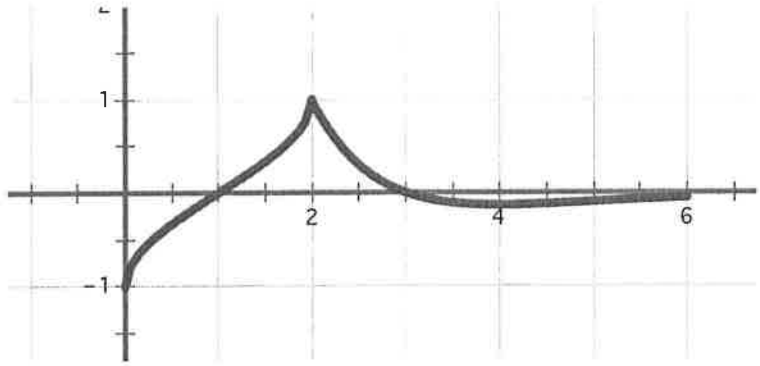
b) Find all the values of x for which f has a point of inflection on $x \in [-3, 3]$. Justify your answer.

POI @ $x = -1$ BECAUSE f'' CHANGES SIGN

c) Sketch a graph of $f(x)$.



2. Let $g(x)$ be a continuous function on $x \in [0, 6]$ where the graph of $g'(x)$ is the function shown below.



a) Identify the x-value(s) of the relative maximums of $y = g(x)$ on the interval $x \in [0, 6]$. Justify your answer.

X=3

X= 0 LEFT END POINT FOLLOWED BY −

b) Identify the x-value(s) of the relative minimums $y = g(x)$ on the interval $x \in [0, 6]$. Justify your answer.

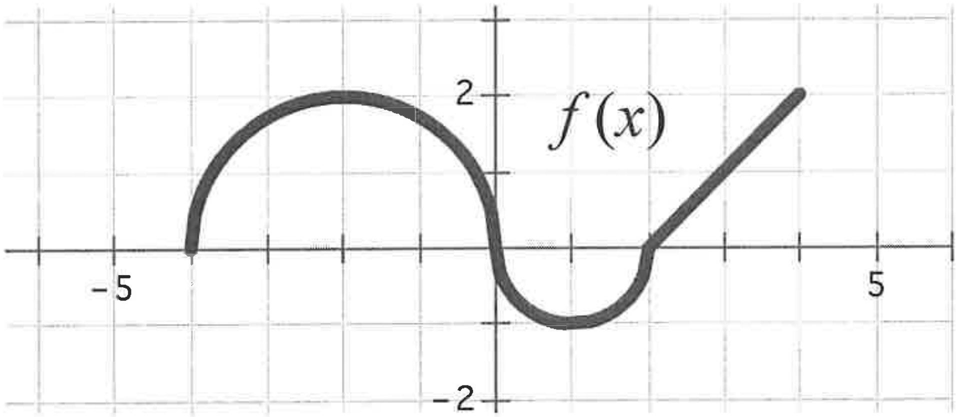
X=6 RIGHT END PRECEEDED BY −

~~X=3~~ X=1 g' SWITCHES − TO +

c) Where are the points of inflection on $y = g(x)$? Justify your answer.

X=2 & X=4 BECAUSE g' SWITCHES INC TO DEC OR VICE VERSA

3. Let $h(x)=\int_0^t f(t)dt$ on $x\in[-4,4]$. Let the graph of be comprised of two semicircles and a line segment as shown below.



Also, let $h(0)=\pi$.

(a) Find $h(2)$, $h'(2)$, and $h''(2)$.

$h(2)=\int_0^2 f(t)dt = -\pi/2$

$h'(2)=f(2)=0$

$h''(2)=f'(2)=$ DNE

(b) Find the average rate of change of $h(x)$ on $x\in[0,2]$.

$$\frac{h(2)-h(0)}{2-0} = \frac{-\pi/2 - 0}{2-0} = -\pi/4$$

(c) At what x-values is $h(x)$ decreasing and concave up? Justify your answer.

$f = h'$ IS NEGATIVE & INC

$\therefore x \in (1, 2)$

(d) What is the absolute maximum value of $h(x)$ on the interval $x \in [-4, 4]$?

MAXES @ $x = 0$ & $x = 4$

$h(0) = 0$

$h(4) = -\frac{\pi}{2} + 2 > 0 \therefore$ $2 - \frac{\pi}{2}$ IS ABS MAX