AB Calculus '21-22
Dx Apps II v1
Calculator allowed

Name Solution Key

Score___________

1. A particle moves along the $y$-axis so that at any time $t \geq 0$, it velocity is given $v(t) = \sin(2t)$. If the position of the particle at time $t = \frac{\pi}{2}$ is $y = 3.5$, the particle's position at time $t = 0$ is

(a) 2 (b) 2.5 (c) 3 (d) 3.5 (e) 4

$y(t) = \int \sin 2t \, dt = -\frac{1}{2} \cos 2t + C$

$3.5 = -\frac{1}{2} \cos \pi + C \rightarrow C = 3$

$y = -\frac{1}{2} \cos 2t + 3 \rightarrow y(0) = -\frac{1}{2} + 3$

2. A particle moves on the $x$-axis so that its position is given by $x(t) = t^2 - 6t + 5$. For what value of $t$ is the acceleration of the particle zero?

(a) 1 (b) 2 (c) 3 (d) 4 (e) No such value of $t$

$v(t) = 2t - 6$

$a(t) = 2$

3. A spherical balloon slowly deflates at a rate of 16 cubic inches per hour. At what rate is the radius of the balloon changing when the balloon has a diameter of 4 inches? $\Rightarrow r = 2$

a) $-\frac{1}{\pi}\ {in}/{hr}$  b) $-\frac{1}{4\pi}\ {in}/{hr}$  c) $\frac{1}{\pi}\ {in}/{hr}$

$V = \frac{4\pi}{3} r^3$

d) $\frac{1}{4\pi}\ {in}/{hr}$  e) None of these

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-16 = 4\pi (2)^2 \frac{dr}{dt}$

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4. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time $t$ seconds, $0 \le t$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = 2v - 32$ with $v(0) = 50$. Which of the following must be **false**?

✓ a) $v = 16 + e^{2t+c}$  ✓ b) $\frac{1}{2}\ln|2v-32| = t + c$

✓ c) $\ln|v-16| = 2t + c$  ✓ d) $v = 16 + ke^{2t}$

e) None are false.

$\frac{1}{2}\int \frac{1\,(2)}{2v-32}\,dv = \int dt$

$\frac{1}{2}\ln|2v-32| = t + c$

$\ln|2v-32| = 2t + c$

$\int \frac{1}{v-16}\,dv = 2\,dt$

$\ln|v-16| = 2t + c$

$v - 16 = e^{2t+c} = Ke^{2t}$

5. A particle travels along a straight line with a velocity of $v(t) = 3e^{-t^2} \sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \leq t \leq 2$ seconds?

a) 0.835 b) 1.625 c) 1.661 d) 2.261

$$\int_0^2 |v(t)| dt$$

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6. A cup has the shape of a right circular cone. The height of the cup is 9 inches, and the radius of the opening is 3 inches. Water is poured into the cup at a constant rate of 0.3 in$^3$ /sec. What is the rate at which the water level is rising when the depth of the water in the cup is 2 in? (The volume of a cone of height $h$ and radius $r$ is $V = \frac{\pi}{3} r^2 h$.

$$\frac{r}{h} = \frac{1}{3} \Rightarrow r = \frac{1}{3} h$$

(a) 0.095 in/sec
(b) 0.215 in/sec
(c) 0.419 in/sec
(d) 0.859 in/sec
(e) 3.142 in/sec

$$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3} \left(\frac{1}{3} h\right)^2 h = \frac{\pi}{27} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$$

$$.3 = \frac{\pi}{9} (2)^2 \frac{dh}{dt}$$

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7. The rate at which the population of a certain alien creature on another planet grows according to the differential equation $\frac{dA}{dt} = .125\left(40 - \frac{A}{150}\right)$, where $A$ in creatures at time $t$ days. The equation $y = A(t)$ is the particular solution to the differential equation wherein there are 10 creatures at time $t = 0$.
What is the maximum population for these creatures?

a) 150 b) 250 c) 600 d) 6000

$$\frac{dA}{dt} = .125\left(\frac{1}{150}\right)(6000 - A)$$

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8. Consider a particle moving such that its position is described by the function $x(t) = \frac{t^5}{5} - t^4$. When does the particle attain its minimum acceleration?

a) $t=0$ b) $t=2$ c) $t=3$

d) $t=4$ e) $t=5$ f) There is NO minimum

$$v(t) = t^4 - 4t^3$$

$$a(t) = 4t^3 - 12t^2$$

$$a' = 12t^2 - 24t$$

$$12t(t-2)$$

$a'$: + 0 − 0 +

0 2

min

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9. Given that $\frac{dy}{dx} = \frac{\tan^2 y}{x}$, find $\frac{d^2 y}{dx^2}$ $= \dfrac{x\left[2\tan y \sec^2 y \frac{dy}{dx}\right] - \tan^2 y}{x^2}$

$= \dfrac{2x\tan y \sec^2 y \left(\frac{\tan^2 y}{x}\right) - \tan^2 y}{x^2}$

a) $\dfrac{2x \tan y \sec^2 y - \tan^2 y}{x^2}$

b) $\dfrac{x \sec^2 y - \tan^2 y}{x^2}$

c) $\dfrac{\tan^2 y \sec^4 y - \tan^2 y}{x^2}$

d) $\dfrac{2x \tan^2 y \sec^2 y - \tan^2 y}{x^2}$

e) $\dfrac{2 \tan^3 y \sec^2 y - \tan^2 y}{x^2}$

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Directions: Show all work.

Name Solution Key

Score__________

1. On the first offensive play of the 2020 49ers vs Jets game, running back Raheem Mostert set a record as the ballcarrier to hit the highest velocity in a game. According to Pro Football Focus (PPF), he achieved a velocity of 23.09 mph on his 80-yard touchdown run. (He actually went 87 yards from his starting position in the backfield.) As a football fan and stats advocate, Dr. Quattrin used replay and a stopwatch to build the table of approximate velocities (below) during the run.

| Time $t$ in seconds | 0 | 1.1 | 2 | 3.74 | 4.82 | 6.18 | 7.51 | 9.19 | 11.95 |
|---|---|---|---|---|---|---|---|---|---|
| $v(t)$ in yards/sec | 0 | 4.38 | 6.24 | 9.75 | 11.28 | 10.52 | 8.35 | 7.02 | 4.62 |

2

a) Approximate Mostert's acceleration at $t = 3$ seconds. Indicate the units.

$$a(3) \approx \frac{9.75 - 6.24}{3.74 - 2} = 2.006 \ \frac{\text{yds}}{\text{sec}^2}$$

1

b) Use a Right-hand Riemann Sum to approximate the total number of yards run. [EC. Why is this more than 87 yards?]

$$\int_0^{11.95} v(t)\,dt = 1.1(4.38) + .9(6.24) + 1.74(9.75) + 1.08(11.28) + 1.36(10.58)$$
$$+ 1.33(8.35) + 1.68(7.02) + 2.76(4.62) = 89.288 \text{ yds}$$

[Because of ~~pace~~ estimates]

2

c) One model of the data on the table is $M(t)=6.2te^{-0.21t}$. According to this model, find Mostert's approximate acceleration at $t=3$ seconds. Indicate the units.

$M'(3) = 1.222 \frac{\text{YDS}}{\text{SEC}^2}$

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4

d) According to the model $M(t)=6.2\cancel{t}e^{-0.21\cancel{t}}$, find Mostert's maximum velocity. Indicate the units.

$M'(t) = 6.2t\, e^{-.21t}(-.21) + e^{-.21t}(6.2)$

$= 6.2e^{-.21t}[-.21t + 1] = 0$

$t = 4.762$ SEC

$M(4.762) = 10.861 \frac{\text{YDS}}{\text{SEC}}$

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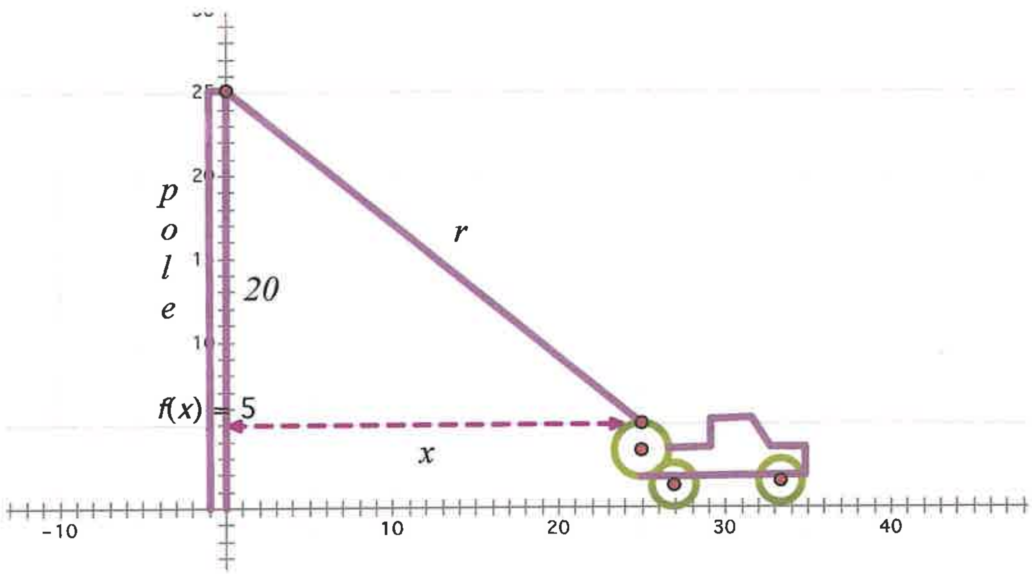
2. A telephone crew is replacing a phone line from one telephone pole to the next. The line is on a spool on the back of a truck, and one end is attached to the top of a 25' pole. The vertical distance from the top of the pole to the level of the spool is 20'.




The truck moves down the street at $20\ ^{ft}/_{sec}$.

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a) Find the length of line that has been rolled out when $t = 15$ sec.

At $t = 15$, $x = 20(15) = 300$ FEET

$r = \sqrt{20^2 + 300^2} = 300.666$ FT

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b) Find the rate at which the telephone line is coming off the spool when the truck is 150 feet from the pole.

$$r^2 = 20^2 + x^2$$

$$2r\frac{dr}{dt} = 2x\frac{dx}{dt} \rightarrow 2\sqrt{20^2+150^2}\,\frac{dr}{dt} = 2(150)(20)$$

$$\frac{dr}{dt} = 19.825\,\frac{FT}{SEC}$$

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c) What is the relationship between the angle $\theta$ and the truck's distance from the pole? Find $\theta$, in radians, when the truck is 140 feet from the pole.

$$\text{TAN}\,\theta = \frac{20}{x}$$

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d) Find the rate, in radians per second, at which the angle the line forms with horizontal is changing when the truck is 140 feet from the pole.

$$\text{SEC}^2\theta\frac{d\theta}{dt} = -20x^{-2}\frac{dx}{dt}$$

$$x = 140 \rightarrow \theta = \text{TAN}^{-1}\left(\frac{1}{7}\right)$$

$$= .142$$

$$\frac{d\theta}{dt} = (\cos^2 .142)\left(\frac{-20}{140^2}\right)(15) = -.015\,\frac{RAD}{SEC}$$

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3. As you know, when a course ends, students start to forget the material they have learned. One model (called the Ebbinghaus model) assumes that the rate at which a student forgets material is proportional to the difference between the amount $y$ (material which is currently forgotten) and the total amount of material learned. Based on this, the rate of loss would be determined by

$$\frac{dy}{dt} = k(100 - y),$$

where $y$ is the percentage of material forgotten and t is measured in weeks since the end of class. At the end of the class $(t = 0)$, the students have not forgotten anything $(y = 0)$.

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6 (a) Find the general solution to the differential equation.

$$\frac{1}{100-y}\,dy = k\,dt$$

$$-\ln|100-y| = kt + c$$

$$|100-y| = e^{-kt+c}$$

$$100 - y = Ce^{-kt} \qquad (0,0) \Rightarrow C = 100$$

$$y = 100 - 100e^{-kt}$$

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1 b) Find the particular solution to the differential equation if the students have forgotten half the material $(y = 50)$ after four weeks. FIND K

$$50 = 100 - 100e^{-k(4)}$$

$$\frac{1}{2} = e^{-4k}$$

$$K = .1732867$$

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2 c) How much material do they **still remember** after the Summer Break, 10 weeks later?

$$y = 100 - 100e^{-1.7328} = 82.323$$

THEY REMEMBER 17.677 % OF THE MATERIAL

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