AP Calculus AB '21-22

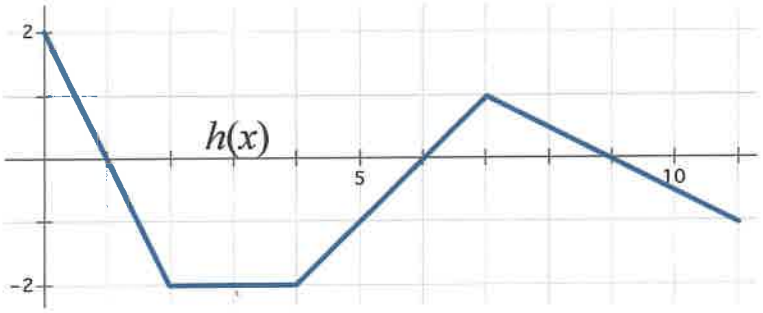
Fall Final Part II

Calculator Allowed

Name:

SOLUTION KEY

$$f(x)=4x-x^3$$



x	$g(x)$	$g'(x)$
0	-1	1
2	1	3
4	3	6
6	6	12
8	4	8

1. Let $f(x)$ be the function defined by the equation above, let $h(x)$ be the function whose graph is given above, and let $g(x)$ be a differentiable function with selected values for $g(x)$ and $g'(x)$ given on the table above.

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(2) (a) Find the equation of the line tangent to $g(x)$ at $x=4$.

$g(4)=3$
$g'(4)=6$

$y-3=6(x-4)$

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(2) (b) Let K be the function defined by $K(x)=h(f(x))$. Find $K'(1)$.

$K'(1)=h'(f(1))\cdot f'(1)$
$=h'(3)\cdot f'(1)$
$=0\cdot 1$
$=0$

$f'(x)=4-3x^2$
$f'(1)=1$

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2 (c) Let M be the function defined by $M(x)=g(x)\cdot f(x)$. Find $M'(6)$.

$M'(6) = g(6)\cdot f'(6) + f(6)\cdot g'(6)$

$= 6(-104) + (-192)(12)$

$= -2928$

$f'(6) = 4-3(6^2)$

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3 (d) Let J be the function defined by $J(x)=\dfrac{g(x)}{h(2x)}$. Find $J'(4)$.

$J'(4) = \dfrac{h(8)\cdot g'(4) - g(4)\,h'(8)\cdot 2}{[h(8)]^2}$

$= \dfrac{(\frac{1}{2})(6) - 3(-\frac{1}{2})(2)}{(\frac{1}{2})^2} = \dfrac{3+1.5}{1/4}$

$= 24$

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2. Consider the differential equation $\frac{dy}{dx}=\frac{y}{x^2+4}$. Let $y=f(x)$ be the particular solution to the differential equation with the initial condition $f(0)=4$. The function $y=f(x)$ is defined for all real numbers.

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a) Find the equation of the line tangent to $y=f(x)$ at $f(0)=4$

(2)

$\left.\frac{dy}{dx}\right|_{(0,4)} = \frac{4}{0+4} = 1$

$y-4=1(x-0)$

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(1) b) Use your answer in part a) to approximate $f(-0.1)$.

$f(-0.1) \approx y(-0.1) = 1(-.1)+4$

$= 3.9$

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c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 4$.

(6)

$$\int \frac{1}{y} = \int \frac{1}{x^2+4}$$

$$\ln|y| = \frac{1}{2}\tan^{-1}\frac{1}{2}x + C$$

$$e \qquad e$$

$$y = e^{\frac{1}{2}\tan^{-1}x/2 + C} = Ke^{\frac{1}{2}\tan^{-1}x/2}$$

$$(0,4) \rightarrow \quad 4 = Ke^{0} \rightarrow K = 4$$

$$y = 4e^{\frac{1}{2}\tan^{-1}\frac{x}{2}}$$

3. Dr. Quattrin decides to lease solar panels from Sunrun Solar. After a year, he reanalyzes his PG&E bill to track both his consumption of electricity ($C_e(t)$) and his production of electricity ($P_e(t)$) over the course of a year. The table below show the consumption of electricity, measured in kilowatts per month (kWs/mo).

t months	0	1	2	3	4	5	6
$C_e(t)$ in	326.5	660.0	667.1	538.4	420.5	412.1	347.8

t months	7	8	9	10	11	12
$C_e(t)$	287.5	303.1	322.4	342.5	390.3	384.2

$P_e(t) = 407 - 374.2\cos\left[\frac{\pi}{6}t\right]$ models the production in kW per month gathered by the solar panels and that PG&E buys back.

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a) How much power does PG&E buy back from the Quattrins over the course of **first six months** of the year? Indicate the units.

(2)

$$\int_0^6 P_e(t)\,dt = 2442 \text{ kW}$$

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b) Using the midpoint Riemann sum , approximate the amount of power the Quattrins consume over the course of the **first six months** of the year.

(3)

$$\int_0^6 C_e(t)\,dt \approx 2(660) + 2(538.4) + 2(412.1)$$

$$= 3221 \text{ kW}$$

$$M(t) = -0.518(x+1)^4 + 16.137(x+1)^3 - 166.554(x+1)^2 + 610.665(x+1) - 90.552$$

c) Assume the equation $M(t)$ above is an appropriate model for the $C_e(t)$ data. Find the time during the year when PG&E owes the maximum refund to Dr Q.

(4)

CRITICAL VALUES WHEN $P_e(t) = m(t)$ (1)

$t = 3.6363787, 9.3641142$ (1)

t	$\int_0^t P-m$
0	326.5 KW
(1) 3.636	1611.323 KW
9.364	−138.385 KW
12	486.537 KW

$t = 3.636$ MONTHS.

(1)

End of

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