AP Calculus AB '21-22

Fall Final Part I

Calculator Allowed

Name:

Solution Key

1. Which of the following statements are **false**?

I. $\int \left(x\left(x^2+5\right)^3 \right) dx = \frac{1}{4}\left(x^2+5\right)^4 + c$ F

II. $\int \left(\frac{\cos\sqrt{x}}{\sqrt{x}} \right) dx = 2\sin\sqrt{x} + c$ T

III. $\int \tan x \, dx = \sec^2 x + c$ F

a) I only b) II only c) III only d) I and III e) II and III only

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2. If $f(x) = \sin^{-1}(\cos x)$, then $f'(x) =$ $\frac{1}{\sqrt{1-\cos^2 x}}(-\sin x) = \frac{1}{\sqrt{\sin^2 x}}(-\sin x)$

a) $\sin^{-2}(\cos x)$ b) $-\sin x \sin^{-2}(\cos x)$ c) $-\csc x$

d) -1 e) $\csc x$

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3. What is the average rate of change of the function $f(x) = x^4 - 5x$ on the closed interval [0, 3] ?

$f(0) = 0 \quad f(3) = 66$

(a) 8.5 (b) 8.7 (c) 22 (d) 33 (e) 66

$\frac{66 - 0}{3 - 0} = 22$

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4. Find the absolute maximum value of $y = 4x - x^2$ on $0 \le x \le 3$. $y' = 4 - 2x = 0$

$x = 2$

a) 4 b) –3 c) 0 d) 2 e) 3

x	y
0	0
2	4
3	5

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5. $\frac{1}{2}\int \frac{2x}{x^2 - 4}\,dx = \frac{1}{2}\ln|x^2 - 4| + c$

a) $\dfrac{-1}{4(x^2-4)^2} + C$ b) $\dfrac{1}{2(x^2-4)} + C$ c) $\dfrac{1}{2}\ln|x^2 - 4| + C$

d) $2\ln|x^2 - 4| + C$ e) $\dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + C$

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

6. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table above, find $h'(4)$, given that $h(x) = f(2x) \cdot g\left(\frac{1}{2}x\right)$.

$h'(4) = f(8) \cdot g'(2)(\frac{1}{2}) + g(2) f'(8)(2)$

$= 6(1)(1/2) + 8(-12)(2) =$

a) -189 b) -90 c) 0 d) 47 e) 64

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7. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

a) 0.4 b) 0.5 c) 2.6 d) 3.4 e) 5.5

$y - 2 = 5(x - 3)$

$-2 = 5(x - 3)$

$-.4 = x - 3$

$2.6 = x$

8. Let $p(x)$ be a differentiable function where $p'(x) = 4x^3 - 2$. If $p(2) = 8$, what is $p(1)$?

$p(x) = x^4 - 2x + C \rightarrow p(x) = x^4 - 2x - 4$

(A) -13 (B) -5 (C) 2 (C) 8 (D) 21

$p(1) = -5$

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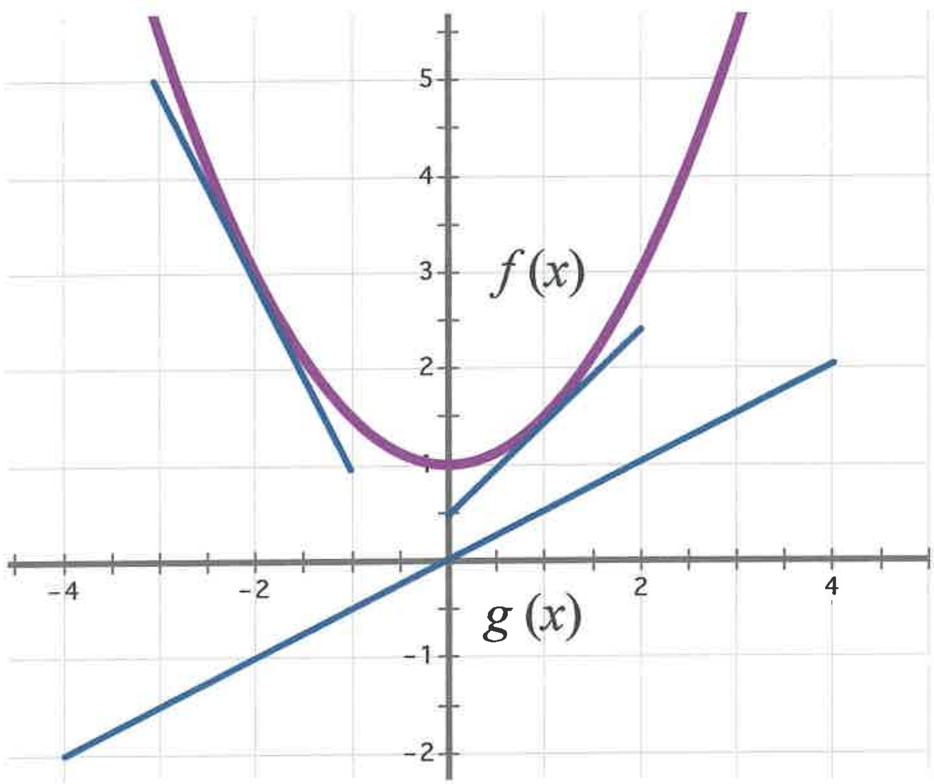
9. The figure below shows the graph of the functions *f* and *g*. The graphs of the lines tangent to the graph of *f* at $x = -2$ and $x = 1$ are also shown. If $B(x) = \frac{f(x)}{g(x)}$, what is $B'(-2)$?



a) $-\frac{1}{18}$ b) $-\frac{1}{2}$ c) 0 d) $\frac{1}{2}$ e) $\frac{1}{18}$

$$\frac{g(-2) \cdot f'(-2) - f(-2)\, g'(-2)}{[g(-2)]^2} = \frac{(-1)(-2) - (3)(1/2)}{1}$$

x	2	5	10	14	16
$f(x)$	12	28	34	30	28

10. Let f be a differentiable function on the closed interval [2, 14] and which has values as shown on the table above. Using the sub-intervals defined by the table values and using left-hand Riemann sums, $\int_5^{16} f(x)\,dx =$

a) 336

$5(28)+4(34)+2(30)$

b) 346

c) 372

d) 376

e) 430

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11. A particle moves along the x-axis so that at any time $t \geq 0$, its acceleration is given $a(t) = -4\sin(2t)$. If $v(0) = 7$ and $x(0) = 0$, then the particle's position equation is

$\int a(t) = +2\cos 2t + C_1$

$7 = 2\cos 0 + C \Rightarrow C = 5$

a) $x(t) = \sin(2t) + 5t$

$\int v = \sin 2t + 5t + C_2$

b) $x(t) = \sin(2t) + 7t$

$0 = 0 + 0 + C_2 \Rightarrow C_2 = 0$

c) $x(t) = \sin(2t) + 9t$

d) $x(t) = 16\sin(2t) + 7t$

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12. The minimum value of $f(x)=\frac{4}{\sqrt{x}}+3\sqrt{x}$ is

$f' = -2x^{-3/2} + \frac{3}{2}x^{-1/2} = 0$

a) $\frac{3}{4}$ b) $\frac{4}{3}$ c) 0 d) $4\sqrt{3}$ e) $\frac{19\sqrt{3}}{2}$

$\cancel{+}2 = \frac{3}{2}x$

$\frac{-4}{3} = x$

$y(4/3) =$

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13. Find $\frac{d^2y}{dx^2}$ if $y=\frac{x+2}{x-3}$.

a) $\frac{-2}{(x-3)^2}$ b) 0 c) $\frac{10}{(x-3)^3}$ d) $\frac{2}{(x-3)^2}$

$\frac{dy}{dx} = \frac{(x-3)(1) - (x+2)(1)}{(x-3)^2} = -5(x-3)^{-2}$

$\frac{d^2y}{dx} = 10(x-3)^{-3}$

14. For $t \geq 0$ hours, H is a differentiable function of t that gives the change in temperature, in degrees Celsius per hour, at an Arctic weather station. Which of the following is the best interpretation of $\int_0^t H(x)dx$? = TOTAL CHANGE

a) The change in temperature during the first t hours.
b) The change in temperature during the first day.
c) The average rate at which the temperature changed during the first t hours.
d) The rate at which the temperature is changing during the first day.
e) The rate at which the temperature is changing at the end of the first day.

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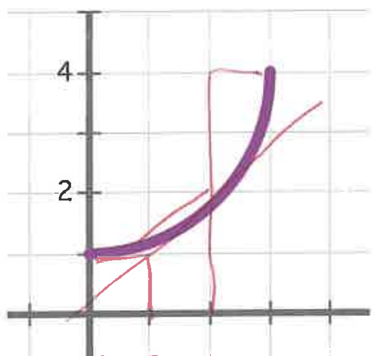
15. The graph of the function f is shown below for $0 \leq x \leq 3$.



TAN LINE UNDER

RIGHT OVER

TRAPEZ OVER

LEFT UNDER

Of the following statements is true?

a) The Right Riemann sum will give an under-approximation of $\int_0^3 f(x)dx$. F

b) The Left Riemann sum will give an over-approximation of $\int_0^3 f(x)dx$. F

c) The Trapezoidal Sum will give an under-approximation of $\int_0^3 f(x)dx$. F

d) The tangent line will give an under-approximation of $f(x)$

e) None of these are true.

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16. Identify is the first mistake (if any) in this process to solve:

$$\frac{dy}{dx} = xy + x$$

Step 1: $\frac{1}{y+1}dy = xdx$

Step 2: $\ln|y+1| = x^2 + c$ $\quad \frac{x^2}{2} + c$

Step 3: $|y+1| = e^{x^2} + c$

Step 4: $y = e^{x^2} + c$

a) Step 1 b) Step 2 c) Step 3

d) Step 4 e) There is no mistake.

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17. Insects destroy a crop at a rate of $R(t) = \frac{100e^{-0.1t}}{2 - e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons are destroyed during the time interval $7 \le t \le 14$?

a) 125 b) 100 c) 88 d) 50 e) 12

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18. Given that $\int_2^3 P(t)\,dt = 7$ and $\int_2^7 P(t)\,dt = -2$, what is $\int_7^3 P(t)\,dt =$

a) −9 b) −5 c) 5 d) 9

e) not enough information

$$\int_7^3 = -\int_3^7 = -\left[\int_2^3 - \int_2^7\right]$$

$$= -\left[7 - (-2)\right] =$$