AB Calculus '21-22 Name Solution Key

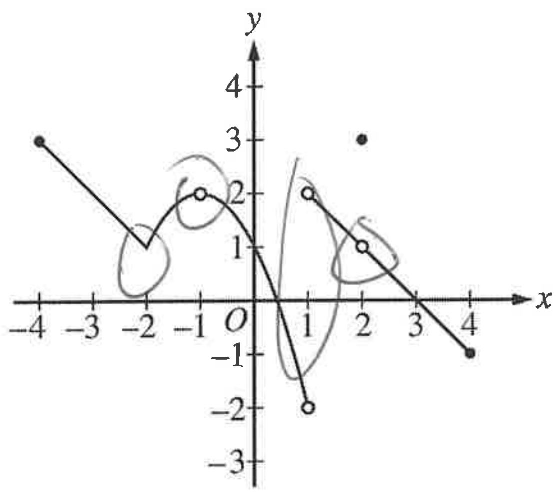
Limit Test v2

Dr. Quattrin score______

No Calculator

1. $\lim_{h\to 0}\dfrac{2\left(\frac{1}{3}+h\right)^3-2\left(\frac{1}{3}\right)^3}{h} \overset{L'H}{=} \lim_{h\to 0}\dfrac{6\left(\frac{1}{3}+h\right)^2}{1} =$

(a) 0 (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ (e) DNE



Graph of f

2. The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-4,4)$ is f not differentiable?

a) One b) Two c) Three d) Four e) Five

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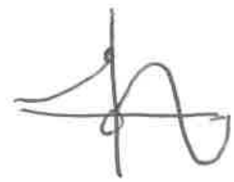
3. At $x=0$, the function given by $f(x)=\begin{cases} e^x, & \text{if } x \le 0 \\ \sin x, & \text{if } 0 < x \end{cases}$ is



(A) Undefined

(B) Continuous but not differentiable

(C) Differentiable but not continuous

(D) Neither continuous nor differentiable

(E) Both continuous and differentiable

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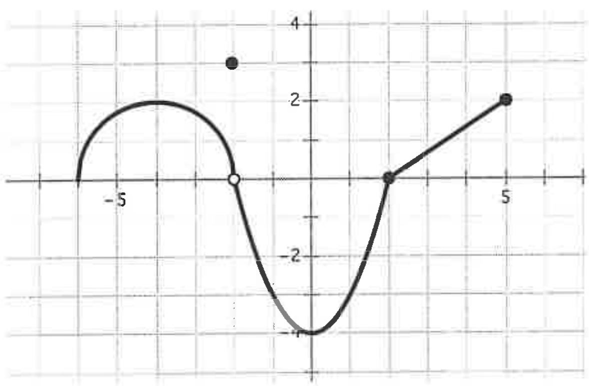
4. The function f is shown below. Which of the following statements about the function f, shown below, is **true**?



a) $\lim_{x \to 0} f(x)$ does not exist F

b) $\lim_{x \to 2} f(x)$ exists T

c) f is continuous at $x=-2$

d) $\lim_{h \to 0} \frac{f(1-h)+3}{h}$ does not exists

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5. $\lim_{x\to\infty} \frac{4x^5+3x^4+2x^3+x^2+1}{3x^5-9x^4+4x^3+15} =$

(a) 0 (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 3 (e) DNE

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6. Let f be defined by $f(x)=\begin{cases} 4x^2+10, & \text{if } x<1 \\ mx^3+8, & \text{if } 1<x \end{cases}$. Determine the value of m for which is continuous for all real x.

$x=1 \Rightarrow 14 = m+8 \Rightarrow m=6$

a) 6 b) −2 c) 8 d) 14 e) None of these

BUT $f(1)$ DNE

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7. $\lim_{x\to 0} \frac{\int_0^{x^3} \cos t^2 \, dt}{x^3} =$ L'H $\lim_{x\to 0} \frac{\cos x^6 (3\cancel{x^2})}{3\cancel{x^2}}$ $= \cos 0 = 1$

(a) 0 (b) 1 (c) $\frac{1}{3}$ (d) 3 (e) DNE

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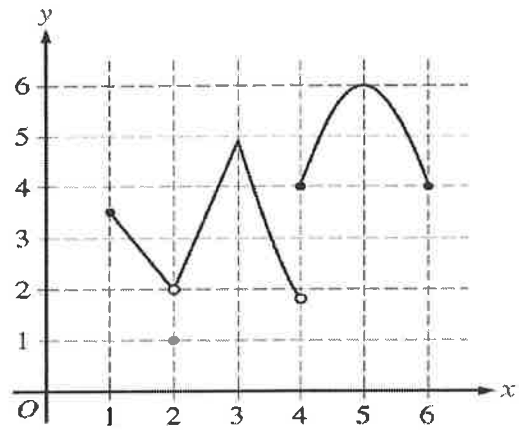
8. The function f is defined on the interval $x \in [1, 6]$ and has the graph shown below.



Graph of f

Which of the following is true?

a) $\lim\limits_{x \to 2} f(x) = 1$ F

b) $\lim\limits_{x \to 3} \dfrac{f(x) - f(3)}{x - 3} = dne$ T

c) $\lim\limits_{x \to 3} f(x) = f(6)$ F

d) $\lim\limits_{x \to 4^-} f(x) = 4$ F

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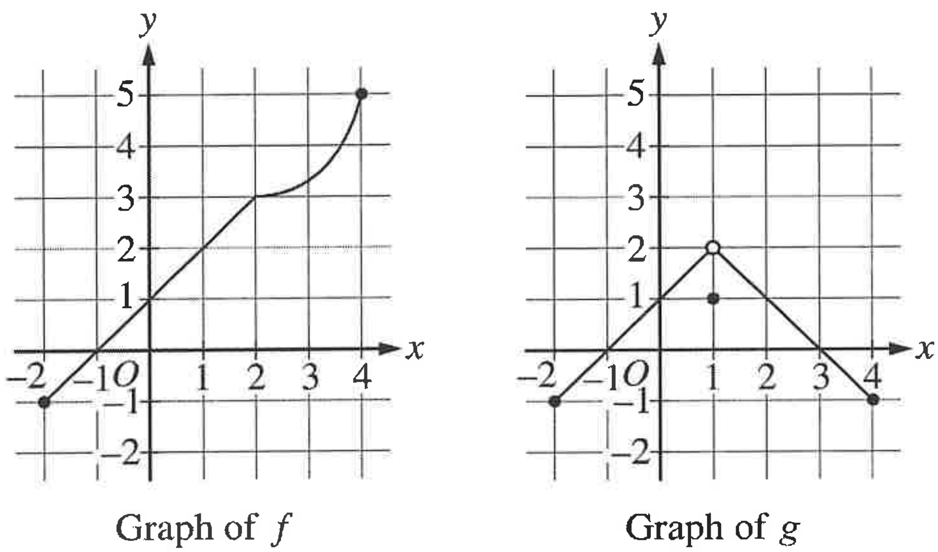
9. Which of the following functions is NOT differentiable at $x = \frac{\pi}{2}$?

(a) $f(x) = x^2$ (b) $f(x) = e^x$ (c) $f(x) = \ln(x+1)$

(d) $f(x) = \sec x$ (e) $f(x) = \cot x$

VA @ $x = \frac{\pi}{2}$



10. The graphs of the functions f and g are shown above. The value of $\lim_{x\to 1} f(g(x)) =$ $f(\lim_{x\to 1} g(x)) = f(2) = 3$

a) 0 b) 1 c) 2 d) 3 e) dne

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11. $\lim_{x\to\pi^+} \dfrac{x}{\tan x} = \dfrac{\pi}{0^+}$

a) ∞ b) $-\infty$ c) 1 d) -1

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12. Let $f(x)=\begin{cases} \ln(1-x), & \text{if } x \le 0 \\ \tan x, & \text{if } 0 < x \end{cases}$. Which of the following statements is **false** about f?

ln 1 = TAN 0 SO CONTINUOUS

(a) f is continuous at $x = 0$. T

$f'(x) = \begin{cases} \frac{-1}{1-x} \\ \sec^2 x \end{cases}$

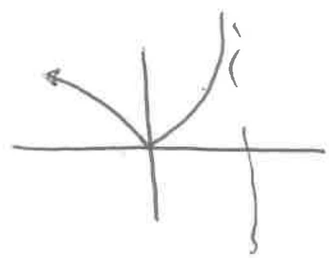
(b) f is not differentiable at $x = 0$. T

(c) f has a local maximum at $x = 0$. F

$x \to 0^- \to -1$

d) f does not have a point of inflection at $x = 0$. F

$x \to 0^+ \to 1$

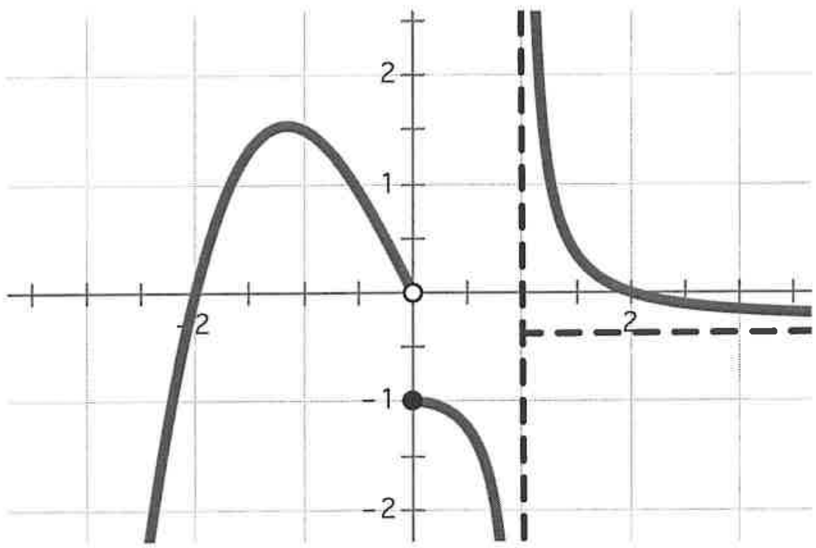


AB Calculus '21-22
Limit Test v2
Calculator allowed

Name Solution Key

Score 21

Directions: Show all work.



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1. For this graph, find

(a) $\lim_{x\to -1^-} f(x) = 1.5$ (b) $\lim_{x\to 0^-} f(x) = 0$ (c) $\lim_{x\to 1} f(x) =$ DNE

(d) $\lim_{x\to -1} f(x) = 1.5$ (e) $\lim_{x\to 0^+} f(x) = -1$ (f) $\lim_{x\to -1^+} f(x) = 1.5$

(g) $f(-1) = 1.5$ (h) $f(0) = -1$

(i) $f(1) =$ DNE (j) $f(2) = 0$

(k) $\lim_{x\to \infty} f(x) = -.4$ (l) $\lim_{x\to -\infty} f(x) = -\infty$

2. Let $f(x) = \begin{cases} x-1, \text{ if } x<1 \\ \sin[\pi(x-1)], \text{ if } 1 \le x \end{cases}$.

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3 (a) Is $f(x)$ continuous at $x=1$? Justify your answer.

i) $f(1)$ EXISTS AND $f(1)=0$

ii) $\lim_{x \to 1^-} f(x) = 1-1=0 = \sin 0 = \lim_{x \to 1^+} f(x)$ $\therefore$ $\lim_{x \to 1} f(x)$ EXISTS

iii) $\lim_{x \to 1} f(x) = f(1)$

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2 (b) Find $f'(-1)$ and $f'(2)$.

$f' = \begin{cases} 1 & \text{IF } x<1 \\ \pi \cos \pi(x-1) & \text{IF } x>1 \end{cases}$

~~$f(-1) = -1-1 = -2$~~

$f'(-1) = 1$

$f'(2) = \pi \cos \pi = -\pi$

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2 (c) Express $f'(x)$ as a piecewise-defined function. Explain why $f'(1)$ does not exist.

$$f'(x) = \begin{cases} 1 & \text{IF } x < 1 \\ \pi \cos[\pi(x-1)] & \text{IF } x \geq 1 \end{cases}$$

$$\lim_{x \to 1^-} f'(x) = 1 \neq \lim_{x \to 1^+} f'(x) = \pi$$

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2 (d) Find $\lim\limits_{x \to 1^-} \dfrac{f(x)}{x^2 - x}$. Justify your answer.

$$\lim_{x \to 1^-} f(x) = 0 = \lim_{x \to 1} x^2 - x$$

$$\therefore \lim_{x \to 1^-} \frac{f(x)}{x^2 - x} \overset{L'H}{=} \lim_{x \to 1^-} \frac{f'(x)}{2x - 1} = \frac{1}{1} = 1$$

(c) Express $f'(x)$ as a piecewise-defined function. Explain why $f'(0)$ does not exist.

(d) Find $\lim\limits_{x \to \pi} \dfrac{f'(x)}{\pi - x}$. Justify your answer.
