AP Calculus AB '21-22
Spring Final Exam
Dr. Quattrin

Name Solution Key

Part I: Multiple choice – No Calculator.

1. If $g(x)=\cos^2(2x)$, then $g'(x)$ is $2\cos 2x(-\sin 2x)(2)$

(A) $-2\cos 2x\sin 2x$ (B) $-4\sin 2x\cos 2x$

(C) $-\sin^2(2x)$ (D) $-2\sin(2x)$ (E) $4\cos(2x)$

$u=1+x^2 \quad du=2x\,dx$

2. $\displaystyle\int \frac{4x}{1+x^2}\,dx = 2\int \frac{1}{u}\,du$

$2(2x)$

(A) $4\arctan x+C$ (B) $\dfrac{4}{x}\arctan x+C$ (D) $\dfrac{1}{2}\ln\left(1+x^2\right)+C$

(D) $2\ln\left(1+x^2\right)+C$ (E) $2x^2+4\ln|x|+C$

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3. Which of the following is the solution to the differential equation $\frac{dy}{dx} = y\sec^2 x$ with the initial condition $y(0) = -1$?

(A) $y = -e^{\tan x}$ (B) $y = -e^{(-1+\tan x)}$ (C) $y = -e^{\left(\sec^3 x - 2\sqrt{2}\right)/3}$

(D) $y = -\sqrt{2\tan x - 1}$ (E) $y = e^{(-1+\tan x)}$

$\frac{1}{y}dy = \sec^2 x\, dx$

$\ln|y| = \tan x + C$ $\rightarrow y = Ke^{\tan x} \rightarrow K = -1$

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4. A particle moves along the x-axis with acceleration at any time t given as $a(t) = 3t^2 + 4t + 6$. If the particle's velocity is 10 and its initial position is 2 when $t = 0$, what is the position function?

$V = t^3 + 2t^2 + 6t + C_1 \rightarrow C_1 = 10$

$x = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 10t + C_2$

(A) $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 12$

(B) $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 10t + 2$

(C) $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 2$

(D) $x(t) = 3t^4 + t^3 + t^2 + 10t + 2$

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5. $\frac{1}{4}\int_0^1 \frac{4x^3dx}{(x^4+1)^2} = \frac{1}{4}\int_1^2 u^{-2}\,du = -\frac{1}{4}u^{-1}\Big]_1^2 = -\frac{1}{4}\left[\frac{1}{2}-1\right] =$

(A) $\frac{1}{8}$ (B) $\frac{1}{32}$ (C) $\frac{1}{4}\ln 2$ (D) 1 (E) 16

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6. The basement of a house is flooding. The water pours in at a rate of $f(t)$ gallons per hour and is being pumped out at a rate of $r(t)$. When the pump is started, at time t = 0, there are 1200 gallons of water in the basement. Which of the following expresses the rate of change in the number of gallons of water in the basement at t hours?

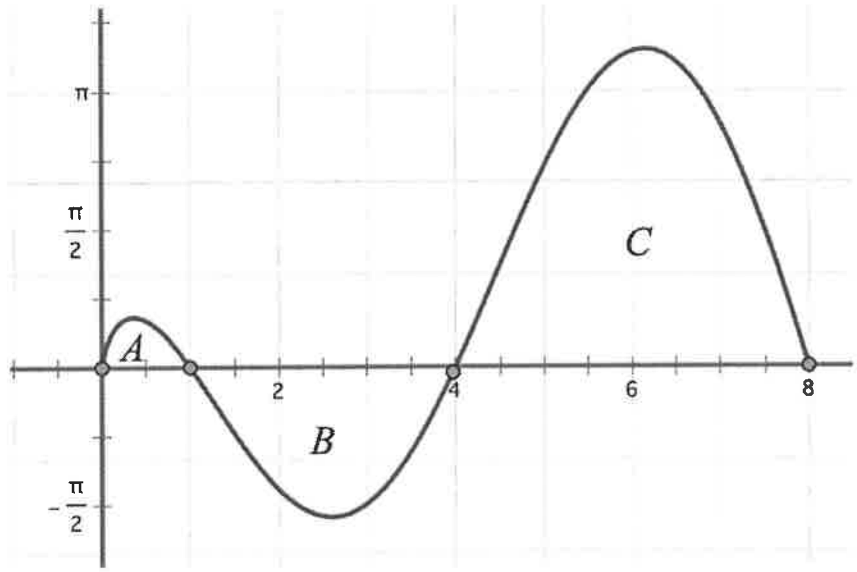
(A) $1200+\int_0^t[f(x)-r(x)]dx$

(B) $\int_0^t[f(x)-r(x)]dx$

(C) $f(t)-r(t)$

(D) $\frac{1}{t}\int_0^t[f(x)-r(x)]dx$

(E) $f'(t)-r'(t)$



In the figure above, A, B, and C are areas between the curve $f(x)$ and the x-axis.

7. If A=14, B=16, and C=50, what is the average value of $y = f(x)$ and the x-axis?

(A) 6 (B) 10 (C) 16 (D) $\frac{80}{3}$ (E) 48

$\frac{1}{8}\left[14 - 16 + 50\right] = \frac{48}{8}$

t in hours	0	12	24	36	48
$v(t)$ in km/hr	21	26.3	31.4	36.8	41.5

8. A Gravitational Slingshot Effect is sometimes used by space probes like Voyager 2 in order to increase its velocity without expending fuel. By flying close to the planet Saturn in a parabolic arc, the velocities on the table above were achieved by a probe. Which of the following is the setup for a right-hand Riemann sum which approximates $\int_0^{48} v(t)dt$

(A) $\cancel{(12)(21)}+(12)(26.3)+(12)(31.4)+(12)(36.8)+(12)(41.5)$

(B) $(12)(26.3)+(12)(31.4)+(12)(36.8)+(12)(41.5)$

(C) $(12)(21)+(12)(26.3)+(12)(31.4)+(12)(36.8)$

(D) $24(26.3)+24(36.8)$

(E) $12\left[21+2(26.3)+2(31.4)+2(36.8)+41.5\right]$

9. Let $f(x)$ be the function given by $f(x)=\sqrt{x+3}$. What is the y-intercept of the line tangent to $f(x)$ at $(1,2)$?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$ (E) $\frac{7}{4}$

$f'(x) = \frac{1}{2}(x+3)^{-1/2}$

$m = f'(1) = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x-1)$

$y = \frac{1}{4}x + \frac{7}{4}$

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10. $\int x\sqrt{3x}\,dx = \sqrt{3}\int x^{3/2}\,dx = \sqrt{3}\,\frac{x^{5/2}}{5/2} + c$

(A) $\frac{2\sqrt{3}}{5}x^{5/2}+C$ (B) $\frac{5\sqrt{3}}{2}x^{5/2}+C$ (C) $\frac{\sqrt{3}}{2}x^{1/2}+C$

(D) $2\sqrt{3x}+C$ (E) $\frac{5\sqrt{3}}{2}x^{3/2}+C$

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x	1	2	4	8
$f(x)$	-3	4	9	-1
$g(x)$	0	6	2	1
$f'(x)$	9	-4	3	2
$g'(x)$	10	1	3	5

11. Let $h(x) = g(x)f(x^3)$. What is the value of $h'(2)$?

(A) -6 (B) 2 (C) 11 (D) 24 (E) 143

$h'(2) = g(2) \cdot f'(8) \cdot 3(2)^2 + f(8) \cdot g'(2)$

$= 6 (2)(12) + (-1)(1)$

143

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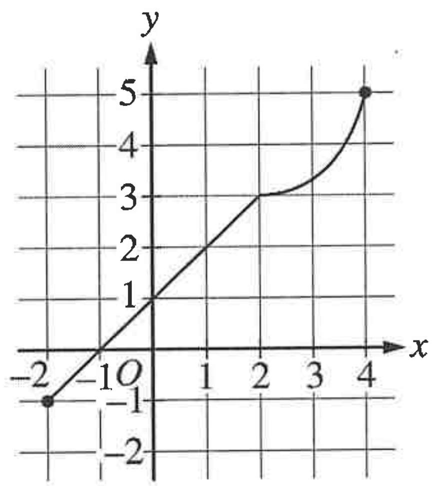
12. If $h(t) = \ln(t^2+1)$, then $h''(3) =$

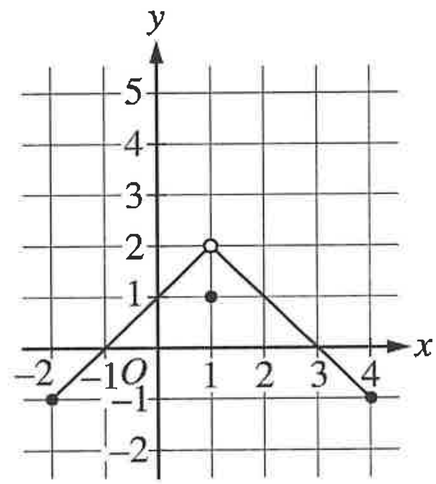
(A) $\ln 10$ (B) $\frac{1}{10}$ (C) $\frac{3}{5}$ (D) $\frac{3}{50}$ (E) $-\frac{4}{25}$

$h' = \frac{2t}{t^2+1}$ $h'' = \frac{(t^2+1)(2) - 2t(2t)}{(t^2+1)^2}$

$h''(3) = \frac{20-36}{100} = \frac{-16}{100}$



Graph of f



Graph of g

13. The graphs of the functions f and g are shown above. The value of $\lim_{x\to 1} f(g(x)) =$

a) 0 b) 1 c) 2 d) 3 e) dne

$= f\left[\lim_{x\to 1} g(x)\right] = f(2) = 3$

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14. On which of the following interval(s) is the function $y = -\frac{t^3}{3} + 3t^2 - 5t$ both increasing and concave up? $\rightarrow$ $y'+$ $y''+$

a) $(-\infty, 1)$ b) $(1,3)$ c) $(3,\infty)$ d) $(3,5)$ e) $(5,\infty)$

$y' = -t^2 + 6t - 5$

$y'' = -2t + 6$

y': − 0 + + 0 − (1, 5)

y'': + + − − (3)

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15. Suppose $f'(x)=\frac{(x+1)^3(4-x)^5}{(x^2+4)}$. Which of the following statements must be **true**?

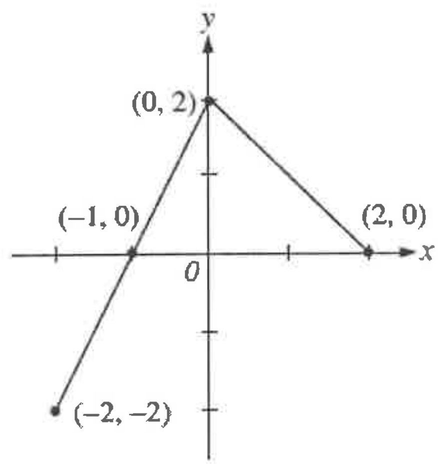
f' − 0 + 0 −
−1 4

a) $f(x)$ has a point of inflection at $x=-1$

b) $f(x)$ is increasing on $x\in(-\infty,-1)$ F

c) $f(x)$ has a relative minimum at $x=4$ F

d) $f(x)$ has a relative minimum at $x=-1$ T



Graph of *f*

16. The graph of the function *f* shown above consists of two line segments. If *g* is the function defined by $g(x)=\int_{-1}^{x} f(t)\,dt$, then the point of inflection on of $g(x)$ occurs at $x=$

a) -2 b) -1 c) 0 d) 1 e) 2

~~f' switches − to + so min~~

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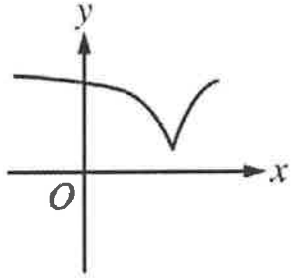
f' SWITCHES FROM INCREASING TO DECREASING

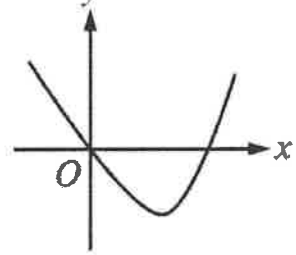
$f' > 0$

17. The function f is differentiable and increasing for all real numbers x, and the graph of f has three points of inflection. Of the following, which could be the graph of $f'(x)$, the derivative of f?

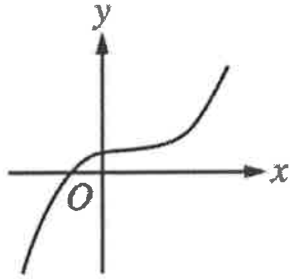
(A)



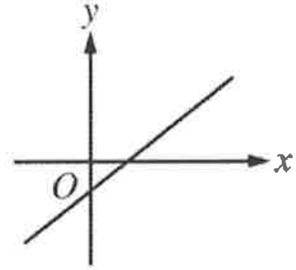
(B)



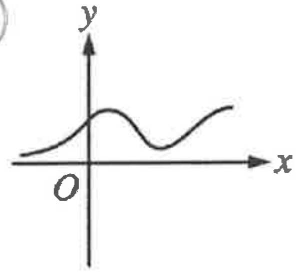
(C)



(D)



(E)



18. Suppose $f'(x)=(1-x)^2(3-x)^5(x-5)^3$. Of the following, which best describes the graph of $f(x)$?

f' − 0 − 0 + 0 −   1   3   5   POI @ $x=1$

a) $f(x)$ has relative minimum at $x=1$, a relative maximum at $x=3$, and a points of inflection at $x=5$

b) $f(x)$ has relative minimum at $x=3$, a relative maximum at $x=1$, and a points of inflection at $x=5$

c) $f(x)$ has relative minimum at $x=5$, a relative maximum at $x=3$, and a points of inflection at $x=1$

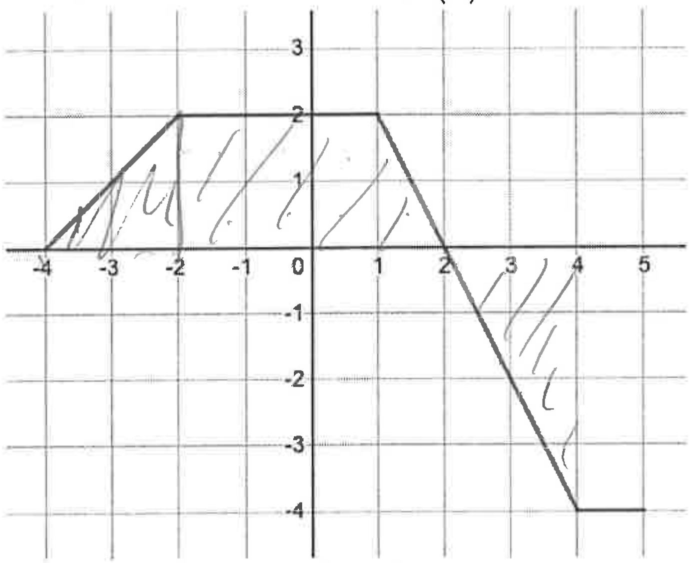
d) $f(x)$ has relative minimum at $x=1$, a relative maximum at $x=5$, and a points of inflection at $x=3$

e) $f(x)$ has relative minimum at $x=3$, a relative maximum at $x=5$, and a points of inflection at $x=1$

19. The graph below gives the graph of $f'(x)$, the derivative of $f(x)$. If it is known that $f(-4) = -2$, what is the value of $f(4)$?

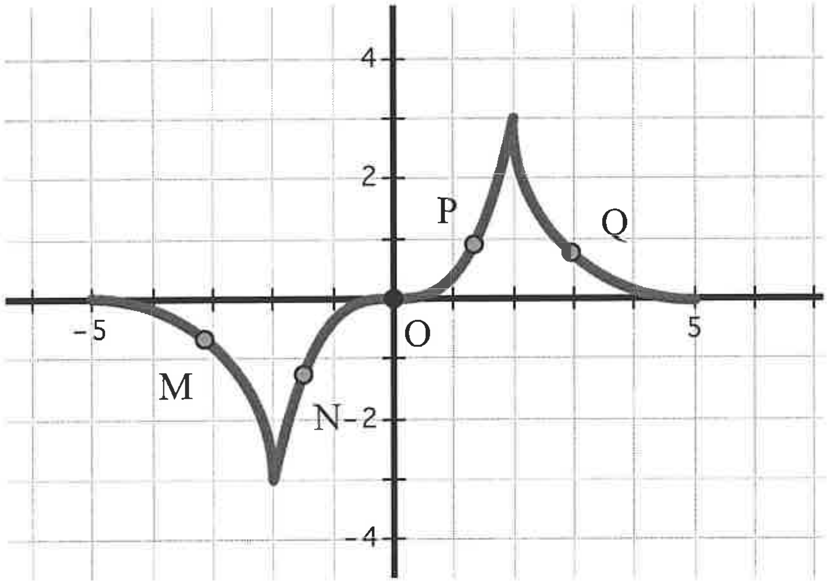
INITIAL $+\int_{-4}^{4}$

$= -2 + 9 - 4$



graph of f'(x)

a) -1   b) 0   c) 1   d) 3   e) 17



20. At what point on the above curve is $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$

INC CON DOWN

a) M b) N c) P d) Q

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21. A particle moves along the x-axis and its position for time $t \geq 0$ is $x(t) = e^{t^2}$. When $t = 1$, the acceleration of the particle is

(a) $3e$ (b) $4e$ (c) $5e$ (d) $6e$ (e) none of these

$v = e^{t^2}(2t)$

$a = e^{t^2}(2) + 2t\,e^{t^2}(2t)$

$= 2e^{t^2}(1 + 2t^2)$

$a(1) = 2e^1(3)$

22. Choose the integral expression that would result in the total distance traveled on the interval [0, 3] if the velocity is given by $v(t)=e^t-6$. $=0$

$-\int_0^{\ln 6} + \int_{\ln 6}^{3}$ $\qquad e^t=6$, $t=\ln 6$

(a) $\int_0^{\ln 6}\left(e^t-6\right)dt-\int_{\ln 6}^{3}\left(e^t-6\right)dt$ (b) $\int_3^{\ln 6}\left(e^t-6\right)dt-\int_{\ln 6}^{0}\left(e^t-6\right)dt$

(c) $\int_0^{\ln 6}\left(e^t-6\right)dt+\int_{\ln 6}^{3}\left(e^t-6\right)dt$ (d) $\int_{\ln 6}^{3}\left(e^t-6\right)dt-\int_0^{\ln 6}\left(e^t-6\right)dt$

(d) $\int_0^{3}\left(e^t-6\right)dt$

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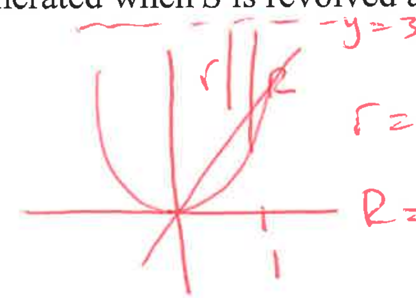
23. Let S be the region enclosed by the graphs of $y=2x$ and $y=2x^2$ for $0\le x\le 1$. What is the volume of the solid generated when S is revolved about the line $y=3$?



$r=3-2x$

$R=3-2x^2$

a) $\pi\int_0^1\left(\left(3-2x^2\right)^2-\left(3-2x\right)^2\right)dx$

b) $\pi\int_0^1\left(\left(3-2x\right)^2-\left(3-2x^2\right)^2\right)dx$

c) $\pi\int_0^1\left(4x^4-4x^2\right)dx$

$\pi\int_0^1 (3-2x^2)^2-(3-2x)^2$

d) $\pi\int_0^2\left(\left(3-\frac{y}{2}\right)^2-\left(3-\sqrt{\frac{y}{2}}\right)^2\right)dx$

e) $\pi\int_0^2\left(\left(3-\sqrt{\frac{y}{2}}\right)^2-\left(3-\frac{y}{2}\right)^2\right)dx$

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24. Consider a particle moving such that its position is described by the function $x(t) = t^4 - \frac{t^5}{5}$. When does the particle attain its ~~maximum~~ MIN velocity?

$V = 4t^3 - t^4 \rightarrow V' = 12t^2 - 4t^3 = 4t^2(t-3)$

V': $-$ 0 $-$ 0 $+$; 0, 3 MIN

a) $t=0$ b) $t=2$ c) $t=3$

d) $t=4$ e) $t=5$ f) There is no minimum

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25. Consider the closed curve in the x-y plane given by $2y - x + xy = 8$. Which of the following is correct?

$2\frac{dy}{dx} - 1 + x\frac{dy}{dx} + y(1) = 0$

$(2+x)\frac{dy}{dx} = 1 - y$

a) $\frac{dy}{dx} = \frac{1-y}{2+x}$ b) $\frac{dy}{dx} = \frac{y-1}{2+x}$

c) $\frac{dy}{dx} = \frac{1+y}{2+x}$ d) $\frac{dy}{dx} = -\frac{1+y}{2+x}$

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26. At $x=0$, the function given by $f(x)=\begin{cases} 2-e^x, \text{ if } x\le 0 \\ x^2+2x+1, \text{ if } 0<x \end{cases}$ is

$f' = \begin{matrix} -e^x \\ 2x+2 \end{matrix}$

$x=0 \to 2-e^0 = 1 = 0+0+1$ CONT

$\to -e^0 = -1 \neq 2(0)+2$

a) Undefined

b) Continuous but not differentiable

c) Differentiable but not continuous

d) Neither continuous nor differentiable

e) Both continuous and differentiable

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27. The function f defined on all the Reals such that $f(x)=\begin{cases} 10+bx-x^2, \text{ if } x\le -1 \\ 3+ke^{x+1}, \text{ if } x>-1 \end{cases}$.

$f'(x) = \begin{cases} b-2x \\ ke^{x+1} \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

CONTINUOUS

$10 - b - 1 = 3+k$

$9-b = 3+k$

DIFFERENTIABLE

$b+2 = +k$

$9-b = 3+b+2$

$4 = 2b$

$2 = b$

$k=4$

a) $b=-2,\ k=-4$

b) $b=2,\ k=4$

c) $b=4,\ k=2$

d) $b=-4,\ k=-2$

$$\frac{d}{dx}\int_9^{x^2}(t+1)\,dt = (x^2+1)(2x$$

28. If $h(x)=\int_9^{x^2}(t+1)dt$, then $\lim\limits_{x\to 3}\dfrac{h(x)}{3x-9}=$ $\dfrac{h'}{3} = \lim\limits_{x\to 3}\dfrac{(x^2+1)(2x)}{3}$

a) $-\infty$ b) ∞ c) 20 d) $10/3$ e) DNE

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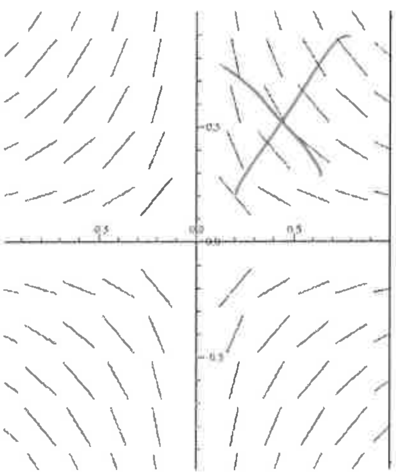
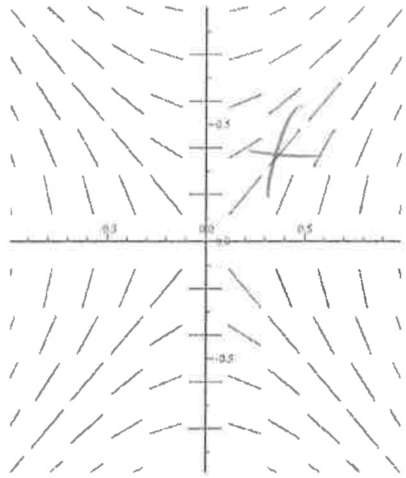
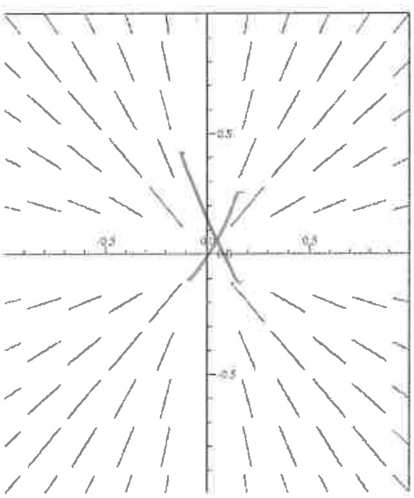
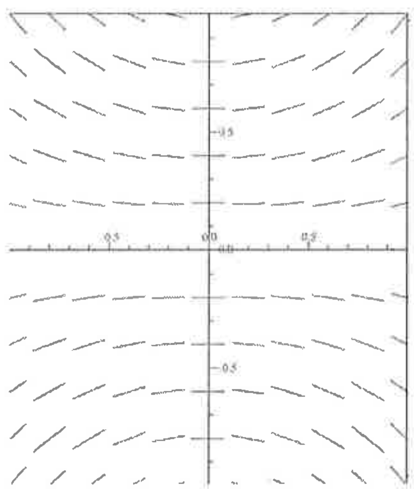
29. Which of the slope field shown below corresponds to $\dfrac{dy}{dx}=yx$?

a) 

b) 

c) 

d) 

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30. $\lim_{x\to 3^+} \frac{4x-\pi}{3-x} =$ $\frac{12-\pi}{0^-}$

a) ∞ b) $-\infty$ c) 1 d) -1