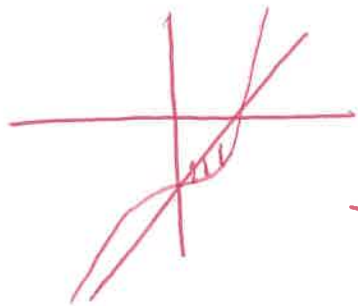
AB Calculus '21-22
Volume Test v1
Calculator Allowed

Name Solution Key

Score: ____________

1. The region R in Quadrant IV is enclosed by the graphs of $y = x^3 - 1$ and $y = x - 1$. The area of region R is

(A) 0.250 (B) 0.191 (C) 0.598 (D) 3.032 (E) 6.462



$\int_0^1 (x-1)-(x^3-1)\,dx$

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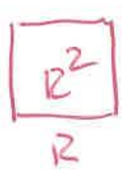
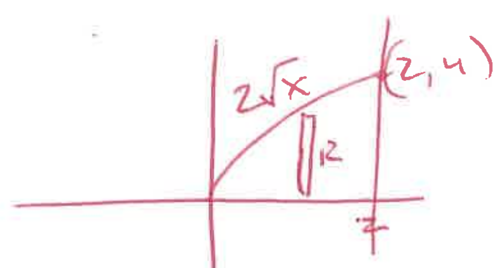
2. The base of a solid is the region bounded by $y = 2\sqrt{x}$, the $x$-axis, and $x = 2$. Each cross-section of the solid perpendicular to the $x$-axis is a square, with one side on the $xy$-plane. Which of the following expressions represents the volume of the solid?

(a) $\int_0^2 \left(2\sqrt{x}\right)dx$ (b) $\int_0^2 (4x)\,dx$ (c) $\int_0^2 (2x)\,dx$

(d) $\int_0^1 \left(2\sqrt{x}\right)dx$ (e) $\int_0^1 (4x)\,dx$




$R = 2\sqrt{x}$

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3. Which of the following integrals gives the length of the graph of $f(x)=e^{3x}$ for $x\in[0,2]$

$\frac{dy}{dx} = 3e^{3x}$

$1+\left(\frac{dy}{dx}\right)^2 = 1+9e^{6x}$

a) $\int_0^2 \sqrt{1+9e^{6x}}\,dx$

b) $\int_0^2 \sqrt{1+e^{6x}}\,dx$

c) $\int_0^2 \sqrt{x+9e^{6x}}\,dx$

d) $\int_0^2 \sqrt{x+e^{6x}}\,dx$

e) $\int_0^2 \sqrt{e^{3x}+9e^{6x}}\,dx$

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4. A region is bounded by $y=\frac{1}{\sqrt[3]{x}}$, the $x$-axis, the line $x=m$, and the line $x=2m$, where $m>0$. A solid is formed by revolving the region about the $x$-axis. The volume of the solid

(a) is independent of $m$.

(b) increases as $m$ increases.

(c) decreases as $m$ increases.

(d) increases until $m=\frac{1}{2}$, then decreases.

(e) is none of the above

$V = \pi \int_m^{2m} \left(\frac{1}{\sqrt[3]{x}}\right)^2 dx$

$= \pi \int_m^{2m} x^{-2/3}\,dx$

$= \pi\, 3x^{1/3}\Big]_m^{2m}$

$= 3\pi\left[(2m)^{1/3} - m^{1/3}\right]$

$= 3\pi\left(2^{1/3}-1\right)m^{1/3}$

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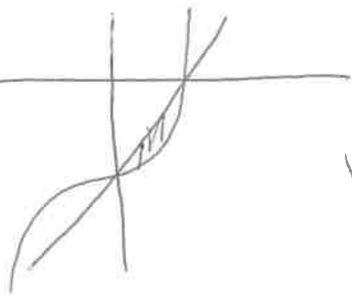
5. The region in Quadrant IV enclosed by the graphs of $y = x^3 - 1$ and $y = x - 1$ is revolved about the $x$-axis. The volume of this solid is

(A) 1.944 (B) 0.972 (C) 0.190 (D) 3.032 (E) 6.462



$V = \pi \int_0^1 (x-1)^2 - (x^3-1)^2 \, dx$

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6. The region bounded by the following graph $y = 3\sin x$ and the $x$-axis on $0 \le x \le \frac{\pi}{2}$, is rotated about the line $x = -\frac{\pi}{2}$. The volume of the solid is represented by

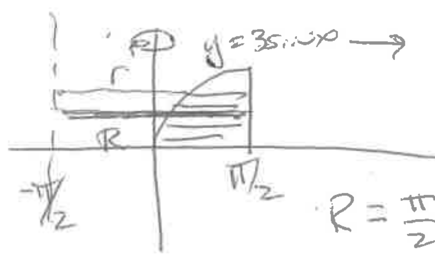
(a) $\pi \int_0^{\frac{\pi}{2}} \left( (3\sin x)^2 - \frac{\pi^2}{4} \right) dx$

(b) $\pi \int_0^3 \left[ \left( \pi^2 - \sin^{-1} \frac{y}{3} \right)^2 \right] dx$

(c) $\pi \int_0^3 \left[ \pi^2 - \left( \sin^{-1} \frac{y}{3} + \frac{\pi}{2} \right)^2 \right] dx$

(d) $\pi \int_0^3 \left[ \left( \sin^{-1} \frac{y}{3} + \frac{\pi}{2} \right)^2 - \pi^2 \right] dx$

(e) $\pi \int_0^{\frac{\pi}{2}} \left( 3\sin x - \frac{\pi}{2} \right) dx$




$y = 3\sin x \rightarrow x = \sin^{-1} \frac{y}{3}$

$R = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi^2$

$r = \sin^{-1} \frac{y}{3} - \left(-\frac{\pi}{2}\right.$

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7. Let R be the region in Quadrant I bounded by the graph of $y=e^{-x^2/2}$, the line $x=3$. The region R is the base of a solid for which each cross-section perpendicular to the $x$-axis is a square. What is the volume of the solid?

(a) 0.886 (b) 0.906 (c) 1.078 (d) 1.245 (e) 2.784


$$\int_0^3 s^2\,dx = \int_0^3 \left(e^{-x^2/2}\right)^2 dx$$

# AP Calculus AB '21-22

# Volume FRQ Test v1

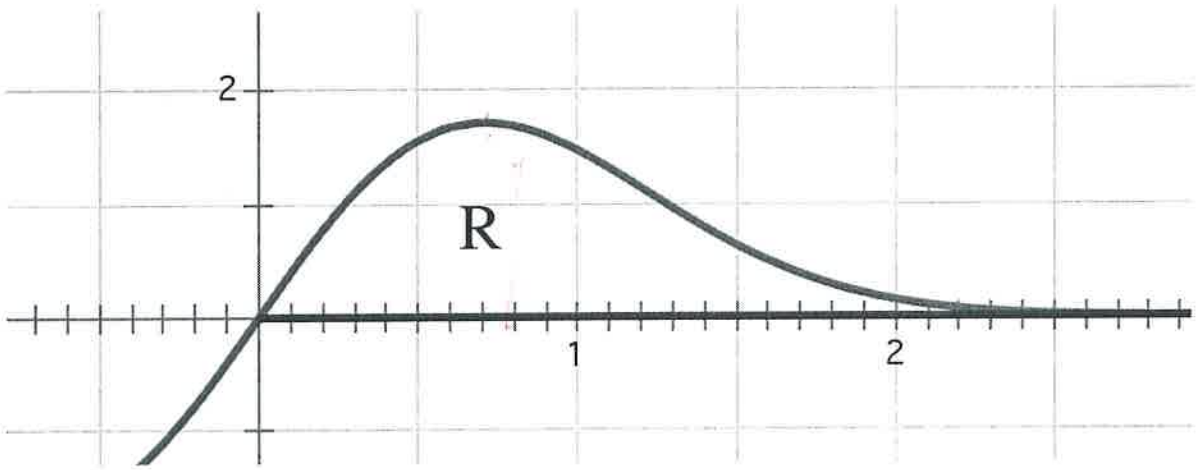
# Calculator Allowed

Name: Solution Key

AB Calculus '20-21
Volume Test v4
Calculator Allowed

Name: Solution Key

Score: 18




1. The picture above is the graph of $f(x)=4xe^{-x^2}$. Consider $R$ to be the region in Quadrant I bounded by $f(x)=4xe^{-x^2}$, the $x$-axis, $x=0$ and $x=2$.

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4 (a) Find the area of region $R$. Show the set-up and the antiderivative.

1 1

2 $A=\int_0^2 4xe^{-x^2}\,dx$ $\quad u=-x^2$, $du=-2x\,dx$

$=-2\int_0^{-4} e^u\,du$

1

$=-2e^u\Big]_0^{-4} = -2e^{-4}-(-2e^0)$

1

$=2-2e^{-4}=$ 1.963

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(b) Find the volume of the solid generated when $R$ is revolved about the $x$-axis.

3 $V = \pi \int_0^2 \left[4xe^{-x^2}\right]^2 dx$ 1 1

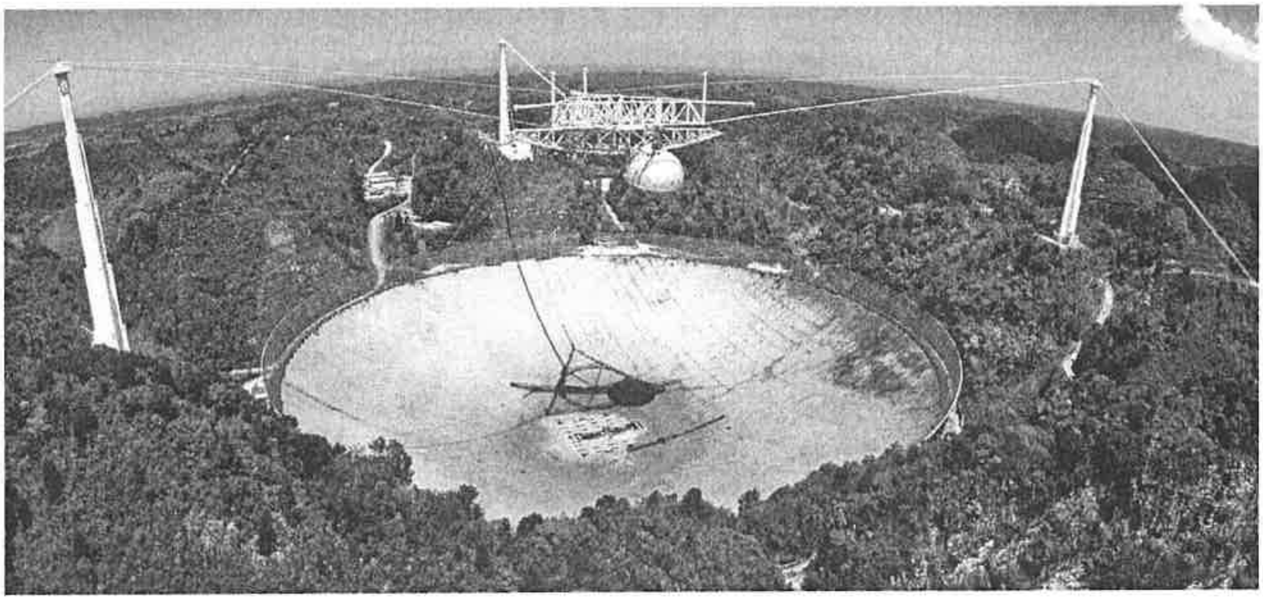
$= 2.503\pi$

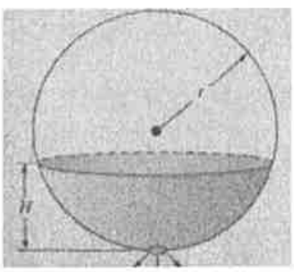
$= \cancel{31.4917}\ 7.866$

1

(c) Let the base of the solid be the region R. Find the volume of the solid where the cross-sections perpendicular to the $x$-axis are squares.

2 $V = \int_0^2 \left[4xe^{-x^2}\right]^2 dx = \cancel{10.086}$ $2.503$





2. Arecibo Observatory in Puerto Rico was built in 1963 to provide early warning of incoming missiles to the United States. The radio telescope has a 1000' diameter spherical reflector built into the ground. The bowl in which the reflector rests conforms to a shape formed by revolving the equation $w=\sqrt{277889-h^2}$ revolved about the $y$-axis from $h=0$ to $h=167$ ft.

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3 a) Find the volume, to the nearest $ft^3$, of the bowl. Show the setup and the antiderivative.

1 $V=\pi\int_0^{167}\left(\sqrt{277889-h^2}\right)^2 dh$

$=\pi\int_0^{167}(277889-h^2)\,dy$

1 $=\pi\left[277889h-\frac{1}{3}h^3\right]_0^{167} = 140{,}091{,}061\ ft^3$ 1

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4 b) The bowl is line with 6' aluminum sheets to create the reflector surface. Use the arc length formula on $w \in [-500, 500]$ to determine how many sheets are needed to cross the widest part of the surface. ~~(Note: Isolate $h$ in the equation.)~~

1 $\frac{dw}{dh} = \frac{1}{2}(277889 - h^2)^{-1/2}(-2h)$

$L = \int_{-500}^{500} \sqrt{1 + \left(\frac{-h}{(277889 - h^2)^{1/2}}\right)^2}$ 1 $= 1316.241$ FT

1

$= 219.373$ SHEETS

1

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2 c) The volume of water in a spherical container at a given $h$ is $V = \frac{\pi}{3}h^2(3(277889) - h)$. During a certain storm at Arecibo, the average rainfall is 0.103 feet per hour. How fast is the volume changing when the height of the water in the biowl is 4.1 feet? Indicate the correct units.

~~$\frac{dV}{dt} = \frac{d}{} \frac{2\pi}{3} h$~~

$V = 277889\pi h^2 - \frac{\pi}{3} h^3$

$\frac{dV}{dt} = (555778\pi h - \pi h^2) \frac{dh}{dt}$

$h = 4.1 \rightarrow \frac{dv}{dt} = (555778\pi(4.1) - \pi(4.1)^2)(.103)$

$= 737343.219$

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