AP Calculus AB '22-23
Anti-Derivative Test
Form G

Name Solution Key

Score__________

1. $\int \sin(4x)dx = \frac{1}{4}\int \sin u\, du$

a) $-4\cos(4x)+c$

b) $-\frac{1}{4}\cos(4x)+c$

c) $\frac{1}{4}\cos(4x)+c$

d) $4\cos(4x)+c$

e) $\frac{1}{4}\cos(4x)+c$

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2. **Which of the following statements is true?**

a) $\int \sec 2x dx = 2\sec 2x \tan 2x + c$ F

b) $\int \left(\frac{3x^2+6x-4}{(x^3+3x^2-4x+2)^2}\right)dx = \ln|x^3+3x^2-4x+2|^2+c$ F

c) $\int \left((x^3+x)\sqrt[4]{x^4+2x^2-5}\right)dx = \frac{1}{5}(x^4+2x^2-5)^{5/4}+c$ T

d) $\int \left(\cot^3 x \csc^2 x\right)dx = \frac{1}{4}\cot^4 x + c$ F

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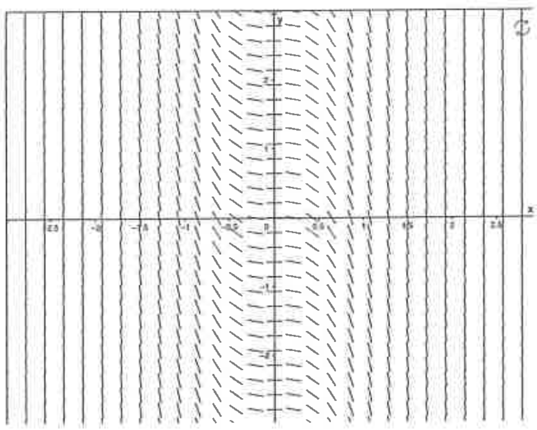
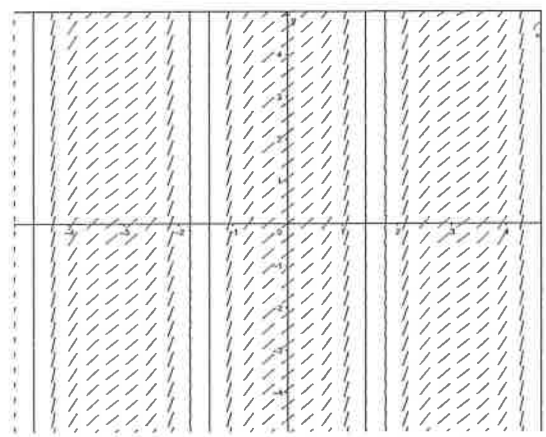
3. Which of the following is the slope field that has the solution $y = \tan x$?

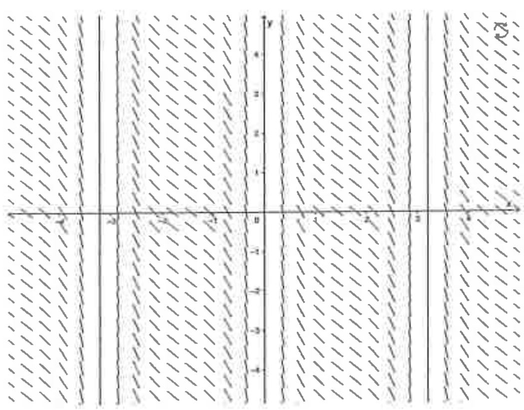
a)



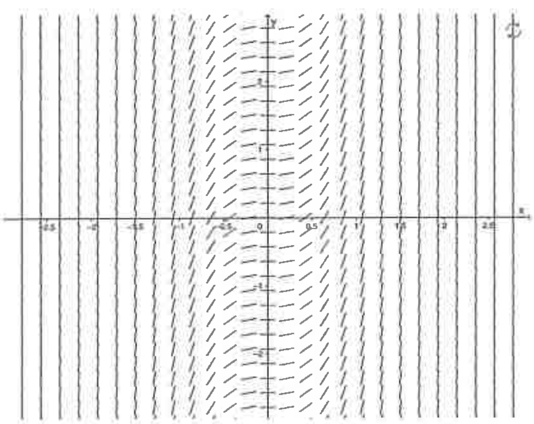
b)



c)



d)



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4. $\displaystyle\int \frac{4}{1+x^2}\,dx =$

a) $4tan^{-1}x + C$ b) $\dfrac{4}{x}tan^{-1}x + C$ c) $\dfrac{1}{2}\ln(1+x^2) + C$

d) $2\ln(1+x^2) + C$ e) $2x^2 + 4\ln|x| + C$

5. Which of the following is the solution to the differential equation $\frac{dy}{dx} = x\sec y$with the initial condition $y\left(\frac{3\pi}{2}\right) = 1$?

a) $y = \cos^{-1}\left(\frac{x^2-3}{2}\right)$

b) $y = \sin^{-1}\left(\frac{x^2-3}{2}\right)$

c) $y = -\cos^{-1}\left(\frac{x^2-1}{2}\right)$

d) $y = -\sin^{-1}\left(\frac{x^2-3}{2}\right)$

e) $y = -\sin^{-1}\left(\frac{x^2-1}{2}\right)$

$\cos y \, dy = x \, dx$

$\sin y = \frac{x^2}{2}$

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6. $\int \frac{x^2 - 3e^{x^3}}{e^{x^3}} dx =$ $\int \left(x^2 e^{-x^3} - 3\right) dx$

$-\frac{1}{3}\int e^u du$

a) $-\frac{1}{3}e^{-x^3} - 3x + c$

b) $\frac{1}{3}e^{-x^3} + 3x + c$

c) $e^{-x^3} - x + c$

d) $e^{-x^3} + 3x + c$

e) $\frac{1}{3}e^{-x^3} - 3x + c$

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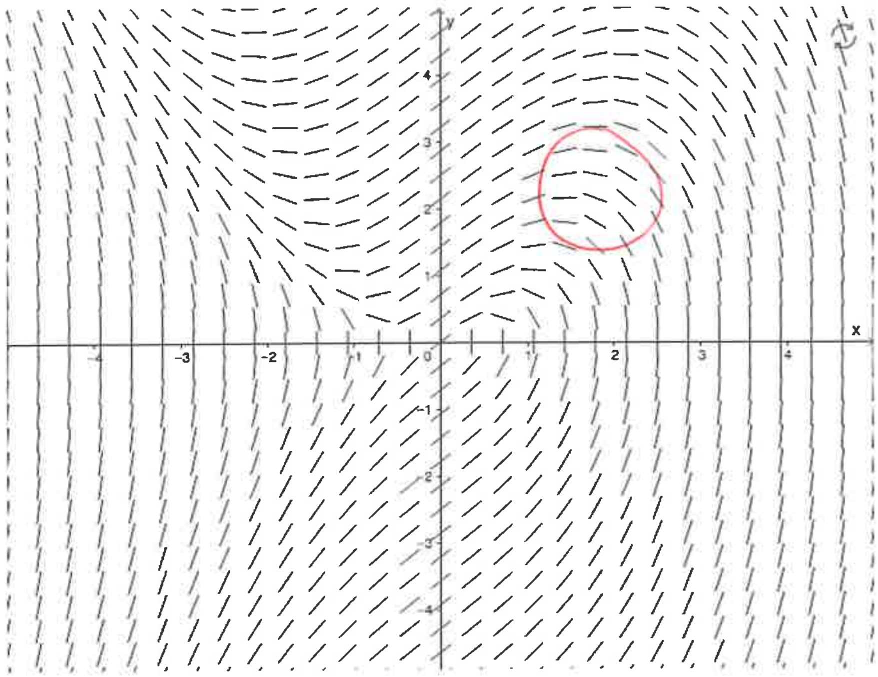
7. Which of the following differential equations best matches this slope field?



a) $\frac{dy}{dx} = 1 - \frac{x^2}{y}$ $(2,2) \Rightarrow m = -1$

b) $\frac{dy}{dx} = 1 - \frac{y^2}{x}$

c) $\frac{dy}{dx} = \frac{x^2}{y} - 1$

d) $\frac{dy}{dx} = \frac{y^2}{x} - 1$

8. For $\int \sin^2 x \cos x \, dx$, the correct u-substitution is

a) $u = \sin x$

b) $u = \cos x$

c) either $u = \sin x$ or $u = \cos x$

d) neither $u = \sin x$ nor $u = \cos x$

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9. A particle moves along the x-axis with acceleration at any time t given as $a(t) = t^2 - \frac{1}{\sqrt{t}}$. If the particle's velocity is 2 and its initial position is -4 when $t = 0$, what is the position function?

$v = \int t^2 - t^{-1/2}$

$v = \frac{t^3}{3} - 2t^{1/2} + c_1$

$v = \frac{1}{3}t^3 - 2t^{1/2} + 2$

$x = \int v = \frac{1}{12}t^4$

a) $x(t) = \frac{1}{12}t^4 - \frac{4}{3}t^{3/2} + 2t - 4$

b) $x(t) = \frac{1}{12}t^4 - \frac{4}{3}t^{3/2} - 2$

c) $x(t) = \frac{1}{12}t^4 - \frac{4}{3}t^{3/2} - 4$

d) $x(t) = \frac{1}{4}t^4 - \frac{4}{3}t^{3/2} + 2t - 4$

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1a. $\int \left(5x^3 + 5^x - \frac{1}{\sqrt[5]{x^3}} + \frac{1}{5x^3}\right) dx$ $= \int \left(5x^3 + 5^x - x^{-3/5} + \frac{1}{5}x^{-3}\right) dx$

$\frac{5x^4}{4} + \frac{5^x}{\ln 5} + \frac{5}{2}x^{2/5} - \frac{1}{10}x^{-2} + C$

1b. $\int \frac{\tan(\ln x)}{x} dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$= \int \tan u \, du$

$= \ln|\sec u| + C$

$= \ln|\sec(\ln x)| + C$

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2. The acceleration of a particle is described by $a(t) = 9\cos 3t$. Find the distance equation for $x(t)$ if $v(0) = 0$ and $x(0) = 3$.

$V = \int 3\cos 3t \; 3dt$ $u = 3t$

$du = 3dt$

$= 3\int \cos u \; du$

$= 3 \sin u + c_1$ $0 = 3(0) + c_1 \Rightarrow c_1 = 0$

$x = \int \sin 3t \,(3dt)$

$= -\cos u + c_2$

$= -\cos 3t + c_2$ $(0,3) \Rightarrow 3 = -\cos 0 + c_2$

$4 = c_2$

$x(t) = -\cos 3t + 4$

3. $\int \left(sec(5x)\tan(5x) + sec^2(3x) + sec(7x)\right) dx$

$$= \frac{1}{5}\int \sec 5x \tan 5x \; 5dx + \frac{1}{3}\int \sec^2 3x \,(3dx) + \frac{1}{7}\int \sec 7x\,(7dx)$$

$u_1 = 5x$, $du_1 = 5dx$ $\quad$ $u_2 = 3x$, $du_2 = 3dx$ $\quad$ $u_3 = 7x$, $du_3 = 7dx$

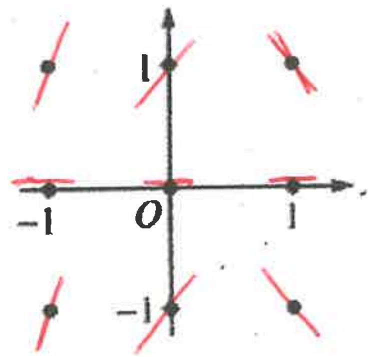
$$= \frac{1}{5}\int \sec u_1 \tan u_1 \, du_1 + \frac{1}{3}\int \sec^2 u_2 \, du_2 + \frac{1}{7}\int \sec u_3 \, du_3$$

$$= \frac{1}{5}\sec u_1 + \frac{1}{3}\tan u_2 + \frac{1}{7}\ln|\sec u_3 + \tan u_3| + C$$

$$= \frac{1}{5}\sec 5x + \frac{1}{3}\tan 3x + \frac{1}{7}\ln|\sec 7x + \tan 7x| + C$$

4. Given the differential equation, $\frac{dw}{dt} = w^2(1-2t)$

a. On the axis system provided, sketch the slope field for the $\frac{dw}{dt}$ at all points plotted on the graph.



b) Find the particular solution $w = f(t)$ that passes through $(1, 2)$.

$$\frac{1}{w^{2}}\,dw = (1 - 2t)\,dt$$

$$\int w^{-2}\,dw = \int (1-2t)\,dt$$

$$\frac{w^{-1}}{-1} = t - t^2 + c$$

$$(1,2) \rightarrow \frac{-1}{2} = 1 - 1 + c$$

$$-\frac{1}{2} = c$$

$$\frac{-1}{w} = t - t^2 - \frac{1}{2}$$

$$\frac{1}{w} = t^2 - t + \frac{1}{2} = \frac{2t^2 - 2t + 1}{2}$$

$$w = \frac{2}{2t^2 - 2t + 1}$$