AP Calculus AB '22-23 Name: Solution Key
Integral Test
Form G

Score: 9

NO CALCULATOR ALLOWED

1. Find $\int_0^3 x^2\,dx = \frac{x^3}{3}\Big]_0^3 = \overset{9}{\cancel{\frac{27}{3}}} - 0$

a) -18 b) -9 c) 0 d) 9 e) 18

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2. $\frac{1}{3}\int_0^1 3x^2 \sin x^3\,dx = \frac{1}{3}\left(-\cos u\right)\Big|_0^1 = -\frac{1}{3}\cos 1 + \frac{1}{3}\cos 0 =$

$\cos 0 = 1$

a) $-\frac{1}{3}\cos(1)$

b) $-\frac{1}{3}\cos(1) - \frac{1}{3}$

c) $-\frac{1}{3}\cos(1) + \frac{1}{3}$

d) $\frac{1}{3}\cos(1)$

e) $\frac{1}{3}\cos(1) - \frac{1}{3}$

$\int_{-2}^{5} f(x)dx = -2$	$\int_{1}^{-2} f(x)dx = 3$
$\int_{-2}^{1} g(x)dx = 4$	$\int_{5}^{1} g(x)dx = 9$

3. Based on the information above, $\int_{1}^{5}[f(x)-g(x)]dx = \int_1^5 f - \int_1^5 g$

$1-(-9)=10$

a) 3 b) 4 c) 7 d) 10 e) 14

$\int_1^5 + \int_{-2}^{1} = \int_{-2}^{5} \rightarrow \int_1^5 = 1$

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4. $\frac{1}{4}\int_{0}^{1}\frac{4x^3dx}{x^4+1} = \frac{1}{4}\int_1^2 \frac{1}{u}du = \frac{1}{4}\ln|u|\Big|_1^2$

a) $-\frac{1}{8}$ b) $-\frac{1}{32}$ c) $\frac{1}{4}\ln 2$ d) $\ln 2$ e) 16

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5. For $t \geq 0$ hours, H is a differentiable function of t that gives the rate of change in temperature, in degrees Celsius per hour, at an Arctic weather station. In what units would $\frac{1}{t}\int_0^t H(x)dx$ be measured?

DEGREES

(A) degrees Celsius
(B) degrees Celsius per hour
(C) degrees Celsius per hour per hour.
(D) hours per degrees Celsius
(E) hours

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6. The basement of a house is flooding. The water pours in at a rate of $f(t)$ gallons per hour and is being pumped out at a rate of $r(t)$. When the pump is started, at time t = 0, there are 1200 gallons of water in the basement. Which of the following expresses how fast the rate of change of the number of gallons of water in the basement is changing between 0 and t hours?

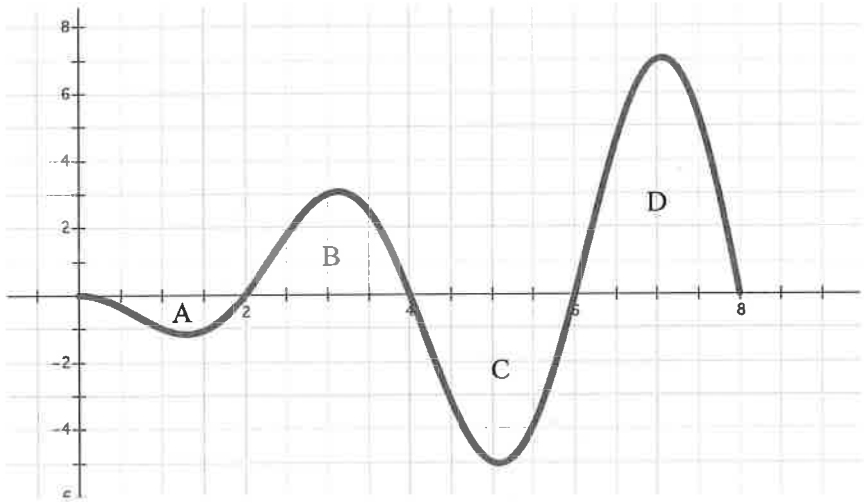
a) $1200 + \int_0^t [f(x) - r(x)]\, dx$

b) $\int_0^t [f(x) - r(x)]\, dx$

c) $f(t) - r(t)$

d) $\frac{1}{t}\int_0^t [f(x) - r(x)]\, dx$

e) $f'(t) - r'(t)$



7. In the above graph, A, B, C, and D are positive numbers that represent the areas between the curve $y = f(x)$ and the x-axis. If A=4, B=8, C=11, and D=15, then $\int_0^8 f(x)\,dx =$

a) 2 b) 4 c) 8 d) $\frac{19}{4}$ e) 38

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8. The average value of $y = 1 - e^{3x}$ on $x \in \left[0, \frac{1}{3}\right]$ is

$\cancel{\frac{1}{1/3}}\ \frac{1}{1/3}\int_0^{1/3} 1-e^{3x}$

a) $1-3e$ b) $\frac{1}{3}-e$ c) $2-e$ d) $\frac{e-1}{3}$ e) $-e$

$\int_0^{1/3} 1-e^{3x}(3dx)$

$= u - e^{3u}\Big]_0^1$

$= \frac{(1-e)-(0-e^0)}{1}$

$= 2-e$

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t in hours	0	12	24	36	48
$v(t)$ in km/hr	21	26.3	31.4	36.8	41.5

9. A Gravitational Slingshot Effect is sometimes used by space probes like Voyager 2 in order to increase its velocity without expending fuel. By flying close to the planet Saturn in a parabolic arc, the velocities on the table above were achieved by a probe. Which of the following is the setup for a trapezoidal sum which approximates $\int_0^{48} v(t)\, dt$

a) $12[21+26.3+31.4+36.8+41.5]$

b) $12[26.3+31.4+36.8+41.5]$

c) $12[21+26.3+31.4+36.8]$

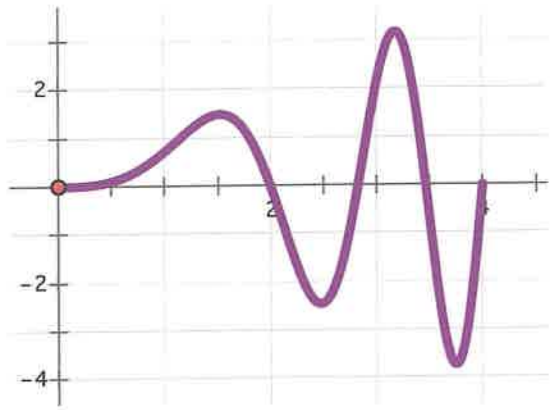
d) $24[26.3+36.8]$

e) $12[21+2(26.3)+2(31.4)+2(36.8)+41.5]$

AP Calculus AB '22-23
Definite Integral Test
Form G
Calculator required.

Name Solution Key

Score 21



1. Consider the function $f(x) = x\sin\left[\frac{\pi}{4}x^2\right]$ on $x \in [0, 4]$. Note that $y = f(x)$ has zeros at $x = 0, 2, 2\sqrt{3}, 2\sqrt{3}$, *and* 4.

2

a) Show the u-sub setup for $\int_0^4 f(x)\,dx$ and solve it by calculator.

$u = \frac{\pi}{4}x^2$

$du = \frac{\pi}{2}x$

$u(0) = 0$

$u(4) = 4\pi$

$= \frac{2}{\pi}\int_0^{4\pi} \sin u \, du = 0$

1

b) Set up, but do not solve, an integral expression to determine the area between $y = f(x)$ and the x-axis on $x \in [0, 4]$.

$A = \int_0^2 f(x)dx - \int_2^{2\sqrt{2}} f(x)dx + \int_{2\sqrt{2}}^{2\sqrt{3}} f(x)dx - \int_{2\sqrt{3}}^4 f(x)dx$

2. The Peninsula Humane Society (PHS) is dedicated to the care and adoption of as many animals who they receive as possible. Since cats breed seasonally, the number of cats and kittens they receive into their facility in a given year varies roughly sinusoidally with time. The data available from 2019 shows the rate $R(t)$, measured in healthy cats per month, varies with time, measured in months after New Year's Day, according to the equation

$$R(t)=120-88\cos\left[\frac{\pi}{6}(t-2)\right].$$

The rate $A(t)$ at which adoption occur, measured in cats per month, varies with time, measured in months after New Year's Day, according to the equation

$$A(t)=125-85\cos\left[\frac{\pi}{6}(t-3)\right].$$

On New Years' Day ($t=0$), there were 131 cats in the PHS Nursery waiting to be adopted.

(a) How many cats and kittens were received at PHS in 2019?

1 PT

$\int_0^{12} R(t)\,dt = 1440$

(b) Find $A'(10.3)$. Using the correct units, explain the meaning of $A'(10.3)$ in context of the problem.

2 PTS

$A'(10.3) = -28.008$

AT $t=10.3$ MONTHS, THE RATE AT WHICH THE CATS ARE BEING ADOPTED IS DECREASING BY 28.008 CATS/MONTH^2

3 PTS

(c) **Find the number of healthy cats and kittens predicted by the models to be in the PHS facility at the end of 2019.**

$$\text{TOTAL} = 131 + \int_0^{12} R(t) - A(t)\,dt$$

$$= 71 \text{ CATS}$$

(d) **Cats breed seasonally, with most births occurring between April 1st ($t=\cancel{0}\ 3$) and October 31st ($t=10$). Find the total change the number of healthy cats and kittens in the PHS facility during "Kitten Season." Include the units.**, USING EXPLAIN THE MEANING

$$\int_{3}^{10} R(t) - A(t)\,dt = -55.780 \text{ CATS}$$

$$-55.780$$

DURING KITTEN SEASON, 58 ~~CATS~~ MORE CATS WERE ADOPTED OUT THAN WERE TAKEN IN

Bay Area Housing Price Problem

t in Month	1	2	4	6	7	9	12
$R(t)$ in thousands of dollars per month	25	-15	-100	135	110	95	-50

The table above shows the rate of chance of the median price, in thousands of dollars per month, of single family detached homes in the Bay Area from September 2021 to August 2022, according to the Association of California Realtors.

(a) Based on the data on the table, estimate $R'(7)$. Based on the estimate of $R'(7)$, is the median housing price increasing at a decreasing rate?

$$R'(7) = \frac{95-135}{9-6} = -13.333$$

$R(7)>0$ AND $R'(7)<0$ $\therefore$ THE MEDIAN PRICE, IN THOUSANDS OF DOLLARS OF SINGLE FAMILY HOMES IS INCREASING AT A DECREASING RATE WHEN $t=7$

(b) Using a Midpoint Reimann Sum, find $\int_1^{12} R(t)\,dt$. Using the correct units, explain $\int_1^{12} R(t)\,dt$. $\approx 3(-15)+3(135)+5(95)=835$

~~THE TOTAL CHANGE OF~~ THE MEDIAN PRICE OF BAY AREA SINGLE FAMILY HOMES, IN THOUSANDS OF DOLLARS, ~~$835~~ WENT UP \$835 THOUSANDS BETWEEN SEPT 2021 AND AUGUST 2022

(c) Set up a right-hand Reimann Sum to approximate $\int_1^{12} R(t)\,dt$. Using the correct units, explain the meaning of $\frac{1}{11}\int_1^{12} R(t)\,dt$.

$\int_1^{12} R(t)\,dt \approx 1(-5)+2(-130)+2(135)+1(110)+2(95)+3(-50) = 215$

$\frac{1}{11}\int_1^{12} R(t)\,dt$ IS THE AVERAGE CHANGE, IN THOUSANDS OF DOLLARS PER MONTH, OF THE MEDIAN PRICE OF BAY AREA SINGLE FAMILY HOMES ~~BETWEEN~~ FROM SEPT 2021 ~~AND~~ TO AUG 2022.

(d) Assume $P'(t) = -1.69t^3 + 29.79t^2 - 128.9t + 121.8$ models the median single family detached home price in the Bay Area between September 2021 to August 2022. To the nearest dollar, determine the total change in median single family detached home price in the Bay Area between September 2021 to August 2022.

$\int_1^{12} P'(t)\,dt = \$512,023$
