AP Calculus AB '22-23
Dr. Quattrin
Derivative Test Form H

Name SOLUTION KEY

score ________

Part I: Multiple choice – Circle correct answer.

1. Let $f(x)$ be the function given by $f(x)=\sqrt{x+3}$. What is the y-intercept of the line tangent to $f(x)$ at $(1,2)$?

$f'(x) = \frac{1}{2}(x+3)^{-1/2}$

$m = 1/4$

$y-2 = \frac{1}{4}(x-1)$

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) $\frac{5}{4}$ e) $\frac{7}{4}$

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2. If $g(x)=\cos^2(2x)$, then $g'(x)$ is $2\cos 2x(-\sin 2x)(2)$

a) $-2\cos 2x \sin 2x$ b) $-4\sin 2x \cos 2x$

c) $-\sin^2(2x)$ d) $-2\sin(2x)$ e) $4\cos(2x)$

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3. Which of the following statements must be **true**?

I. $\frac{d}{dx}\sqrt{e^x+5}=\frac{1}{2\sqrt{e^x+5}}$ F

II. $\frac{d}{dx}(\ln\csc x)=-\cot x$ T

T III. $\frac{d}{dx}\left(6x^3-\pi+\sqrt[3]{x^8}-\frac{2}{x^3}\right)=18x^2+\frac{8}{3}\sqrt[3]{x^5}+\frac{6}{x^4}$ $= -2x^{-3}$

a) I only b) II only c) III only

d) II and III only e) I, II, and III

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x	1	2	4	8
$f(x)$	-3	4	9	-1
$g(x)$	0	6	2	1
$f'(x)$	9	-4	3	2
$g'(x)$	10	1	3	5

4. Let $h(x) = g(x) \cdot f(x^3)$. What is the value of $h'(2)$?

$h' = g(2) f'(8)(3x^2) + f(8) g'(2)$

$= 6(2)(12) + (-1)(1)$

(A) -6 (B) 2 (C) 11 (D) 24 (E) 143

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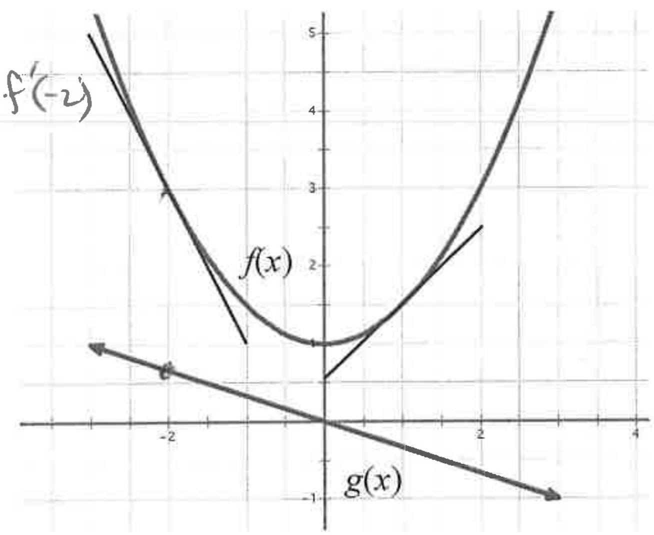
5. The figure below shows the graph of the functions *f* and *g*. The graphs of the lines tangent to the graph of *f* at $x = -2$ and $x = 1$ are also shown. If $B(x) = f(x) \cdot g(x)$, what is $B'(-2)$?

$f(-2) g'(-2) + g(-2) f'(-2)$

$(3)\left(\frac{-1}{3}\right) + \left(\frac{2}{3}\right)(-2)$

$-1 - \frac{4}{3}$



a) $-\frac{5}{6}$ b) $-\frac{1}{2}$ c) $-\frac{7}{3}$ d) $-\frac{11}{3}$ e) $\frac{2}{3}$

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6. If $f(x)=\sin[g(x)]$, then $\frac{d}{dx}[f(x)]$ is $\cos(g(x))\cdot g'(x)$

a) $\sin x \cdot g'(x)+g(x)\cdot \cos x$

b) $\sin x \cdot g'(x)-g(x)\cdot \cos x$

c) $\cos x \cdot g'(x)$

d) $\cos[g(x)]\cdot g'(x)$

e) None of these

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7. If $y=\sin^{-1} e^x$, then $\frac{dy}{dx}=$ $\frac{1}{\sqrt{1-(e^x)^2}}$ e^x

a) $-e^x \cos^{-2} e^x$

b) $e^x \sin e^x + e^x \cos e^x$

c) $e^x \sin^{-1} e^x + e^x \frac{1}{\sqrt{1-x^2}}$

d) $\frac{e^x}{\sqrt{1-e^{2x}}}$

e) $\frac{1}{\sqrt{1-e^{2x}}}$

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8. If $h(t)=\ln(t^2+1)$, then $h''(3)=$

a) $\ln 10$ b) $\frac{1}{10}$ c) $\frac{3}{5}$ d) $\frac{3}{50}$ e) $-\frac{4}{25}$

$h' = \frac{2t}{t^2+1}$

$h'' = \frac{(t^2+1)(2)-(2t)(2t)}{(t^2+1)^2}$

$h''(3) = \frac{20-36}{100} = -\frac{16}{100}$

9. Let f be a differentiable function with $f(2)=3$ and $f'(2)=-5$, and let g be a function defined by $g(x)=xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x=2$?

~~a)~~ $y=3x$

~~b)~~ $y-3=-5(x-2)$

c) $y-6=-5(x-2)$

d) $y-6=-7(x-2)$

e) $y-6=-10(x-2)$

$g' = x f'(x) + f(x)(1)$

$u\,Dv + v\,Du$

$m = g'(2) = 2(-5) + 3 = -7$

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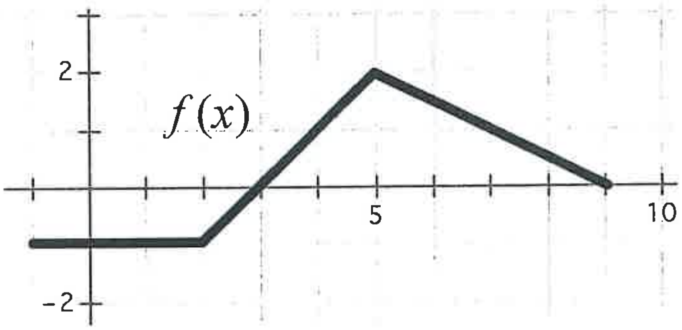
Part II: Free Response – Show all work.



x	$g(x)$	$g'(x)$
0	-1	1
2	1	3
4	3	6
6	6	12
8	4	8

1. Let $f(x)$ be the function whose graph is given above and let $g(x)$ be a differentiable function with selected values for $g(x)$ and $g'(x)$ given on the table above.

a) Find the equation of the line tangent to $g(x)$ at $x = 4$.

$g(4) = 3$

$g'(4) = 6$

$y - 3 = 6(x - 4)$

b) Let K be the function defined by $K(x) = g(g(x))$. Find $K'(\cancel{2})$. 8

$K(2) = g'(g(8)) \cdot g'(8)$

$= g'(4) \cdot g'(8)$

$= 6 \cdot 8 = 48$

c) Let M be the function defined by $M(x)=g(x)\cdot f(x)$. Find $M'(4)$.

$$m'(4) = g(4)\cdot f'(4) + f(4)\,g'(4)$$

$$= 3\,(4\,1) + 1\,(6)$$

$$= \cancel{3}\; 9$$

d) Let J be the function defined by $J(x)=\dfrac{g(x)}{f\left(\frac{1}{2}x\right)}$. Find $J'(1)$.

$$J'(1) = \frac{f(1)\,g'(2)\,(2) - g^{x}(2)\cdot f'(1)}{[g(2)]^2}$$

$$= \frac{(-1)(3)(2) - 1\,(0)}{1^2} = -6$$

2a. $\frac{d}{dx}\left(\cos^{-1}\left(e^{5x}\right)\right)$

$$\frac{-1}{\sqrt{1-e^{10x}}}\, e^{5x}(5)$$

$$\frac{-5e^{5x}}{\sqrt{1-e^{10x}}}$$

2b. $\frac{d}{dx}\left(\sqrt{\csc\left(1-x^2\right)}\right) = \frac{1}{2}\csc^{-1/2}(1-x^2)\left(-\csc(1-x^2)\cot(1-x^2)\right)(-2x)$

$$= +x\csc^{1/2}(1-x^2)\cot(1-x^2)$$

3. If $g(x)=\sec^{-1}3x^2$, find $g''(x)$

$$g'(x)=\frac{1}{3x^2\sqrt{9x^4-1}}(6x)$$

$$=\frac{2}{|x|\sqrt{9x^4-1}}$$

$$=2x^{-1}(9x^4-1)^{-1/2}$$

$$g''(x)=\cancel{2}x^{-1}\left(\frac{-1}{\cancel{2}}(9x^4-1)^{-3/2}(36x^3)\right)+(9x^4-1)^{-1/2}(-2x^{-2})$$

$$=\frac{-36x^2}{(9x^4-1)^{3/2}}-\frac{2}{x^2(9x^4-1)^{1/2}}$$

$$=\frac{-36x^4}{x^2(9x^4-1)^{3/2}}-\frac{2(9x^4-1)}{x^2(9x^4-1)^{3/2}}$$

$$=\frac{-54x^4+2}{x^2(9x^4-1)^{3/2}}$$

4. Write the equation of the line tangent to $f(x) = x \cdot \sqrt[3]{1-x^2}$ at $x = 3$. Show all work

$f(3) = 3(-8)^{1/3} = -6$

$f'(x) = x\left(\frac{1}{3}(1-x^2)^{-2/3}(-2x)\right) + (1-x^2)^{1/3}$

$= \dfrac{-2x^2}{3(1-x^2)^{2/3}} + \dfrac{3(1-x^2)}{3(1-x^2)^{2/3}}$

$= \dfrac{3-5x^2}{3(1-x^2)^{2/3}}$

$m = \dfrac{-42}{3(-8)^{2/3}} = -\dfrac{7}{2}$

$y - 6 = -\frac{7}{2}(x-3)$