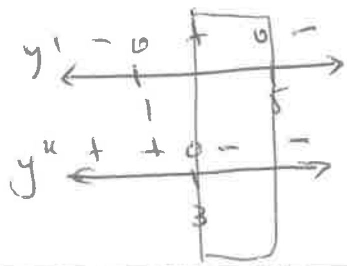
AB Calculus '22-23
Dx Apps I Test Form G
No Calculator

Name Solution Key

1. On which of the following interval(s) is the function $y = -\frac{t^3}{3} + 3t^2 - 5t$ both increasing and concave down?

a) $(-\infty, 1)$ b) $(1,5)$ c) $(3,\infty)$ d) $(3,5)$ e) $(5,\infty)$



$\frac{dy}{dx} = -t^2 + 6t - 5 = -(t-5)(t-1)$

$\frac{d^2y}{dx^2} =$

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2. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that they have values given on the table below.

x	$f'(x)$	$f''(x)$	$g'(x)$	$g''(x)$
2	-1	2	-8	-5
4	8	-11	4	3
8	-3	-12	-1	4

DEC CON UP

Then at $x = 8$, $g(x)$ is

a) increasing and concave down

b) increasing and concave up

c) decreasing and concave down

d) decreasing and concave up

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3. Suppose $f'(x)=\dfrac{(x+1)^{2}(x-4)^{5}}{(x^{3}+8)}$. Which of the following statements must be **true**?

f' + DNE − 0 − 0 +

−2 −1 4

I. $f(x)$ has a relative maximum at $x=-1$ F

II. $f(x)$ is increasing on $x\in(-\infty,-4)$ T

III. $f(x)$ has a relative minimum at $x=4$ T

a) I only b) II only c) III only d) I and II e) II and III only

ab) I and III only ac) I, II, and III ad) None of these

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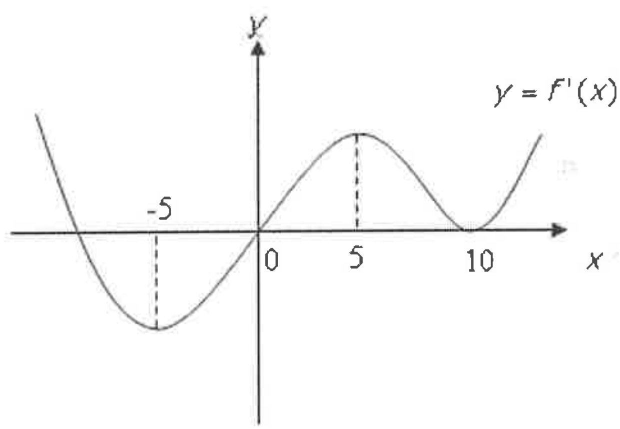
4. Below is the graph of $f'(x)$. For what value(s) of x does $f(x)$ have a minimum?



a) 0 only b) 0 and 10 c) -5 and 5

d) -5 and 10 e) None of these

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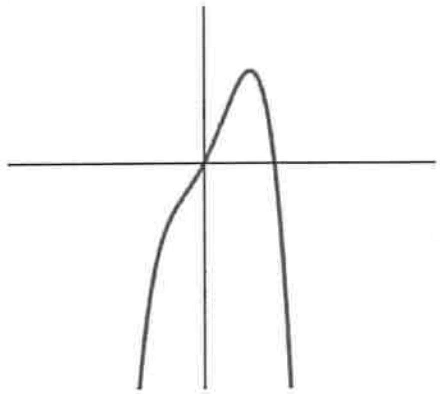
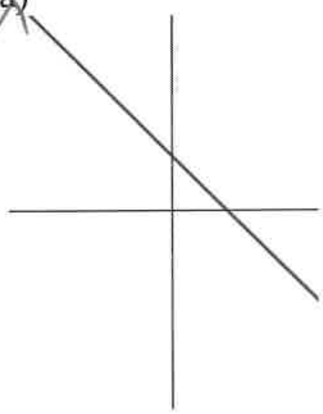
5. The function $h(x)$ is graphed below.



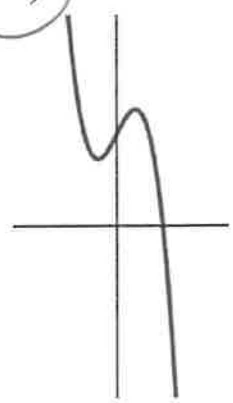
Which of these functions represents $h'(x)$?

a)

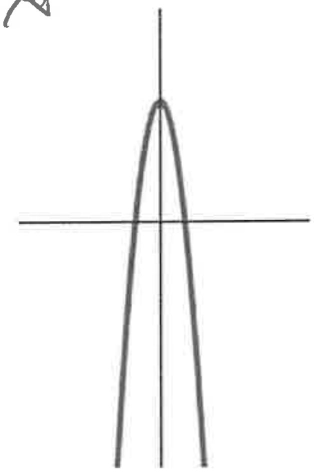


b)

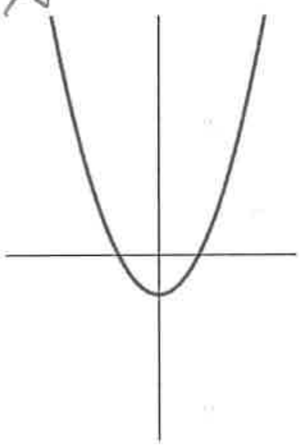
c)



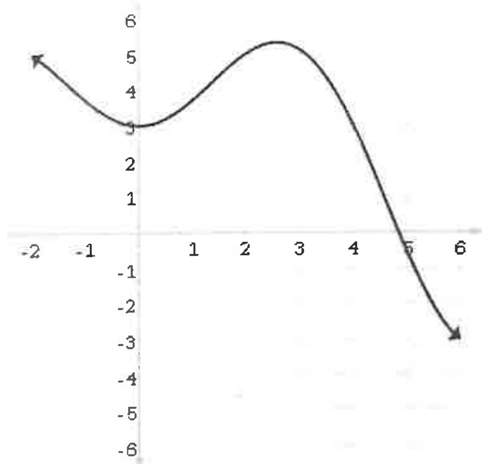
d)



e)



6. The graph below is of $g''(x)$, the **second** derivative of $g(x)$. Which of these statements is true about $g(x)$?



I. $g(x)$ is concave up on the interval $(3,4)$ g'' + T

II. $g(x)$ has a point of inflection at x=0 g'' HAS SIGN CHANGE F

III. The derivative of $g(x)$ is increasing on $(3,4)$ g' T

a) I only b) II only c) III only d) I and II only

e) II and III only f) I and III only g) I, II, and III

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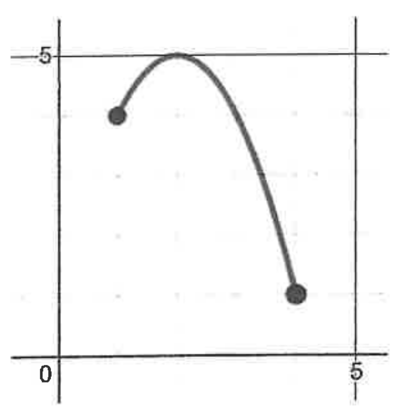
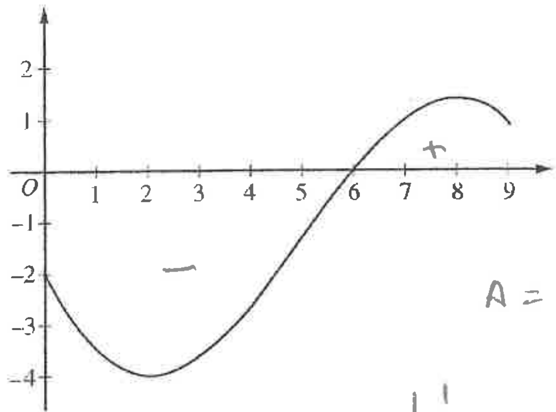
7. The function $f(x)$ is shown below on the closed interval $x \in [1,4]$. The c value guaranteed by the Mean Value Theorem for $f(x)$ on this interval is closest to what number?



$m = \frac{4-1}{1-4} = -1 = f'(c)$

a) 1 b) 2 c) 3 d) 4 e) 5



Graph of f $= h'$

8. The graph of differentiable equation f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

AREA = $h(6) < 0$

$f(6) = h'(6) = 0.$

$f'(6) = h''(6) > 0$

a) $h(6) < h'(6) < h''(6)$
b) $h(6) < h''(6) < h'(6)$
c) $h'(6) < h(6) < h''(6)$
d) $h''(6) < h(6) < h'(6)$
e) $h''(6) < h'(6) < h(6)$

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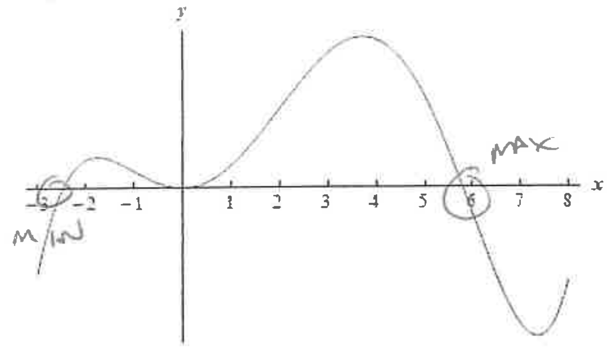
9. An object moves with velocity $v(t) = \sec^2(2t)$. It is known that the particle's position at time 0 is 2. What is the particle's position function?

$\frac{1}{2}\int \sec^2 2t\,(2)\,dt$

a) $s(t) = \tan(2t) + 2$
b) $s(t) = \frac{1}{2}\tan(2t) + 2$
c) $s(t) = \sec^2(2t)\tan^2(2t) + 2$
d) $s(t) = \ln|\sec(2t)| + 2$
e) $s(t) = \frac{1}{2}\ln|\sec(2t)| + 2$

10. Below is the graph of $f'(x)$, the derivative of $f(x)$. Which of the following statements is true about $f(x)$ on the interval $-3 < x < 8$?



4 EXT = 4 POI
on f' on f

a) $f(x)$ has two relative minima, one relative maximum, and three points of inflection.
b) $f(x)$ has two relative minima, one relative maximum, and two points of inflection.
c) $f(x)$ has one relative minimum, one relative maximum, and three points of inflection.
d) $f(x)$ has one relative minimum, two relative maxima, and four points of inflection.
e) $f(x)$ has one relative minimum, one relative maximum, and four points of inflection.

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11. Find the maximum value of $y = x^2 - 4x$ on $0 \le x \le 3$.

a) -4
b) -3
c) 0
d) 2
e) No maximum value exists

$\frac{dy}{dx} = 2x - 4 = 0 \rightarrow x = 2$

x	y
0	0
2	-4
3	-3

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12. Find the average rate of change of $w(x) = \cos(x)$ on $x \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$.

a) $\frac{-3}{\pi}$ b) $\frac{3}{\pi}$ c) $3\pi\sqrt{3}$ d) $\frac{6-3\sqrt{3}}{\pi}$ e) $\frac{-\pi}{12}$

$$\frac{\cos \pi/2 - \cos \pi/3}{\pi/2 - \pi/3} = \frac{-3}{\pi}$$

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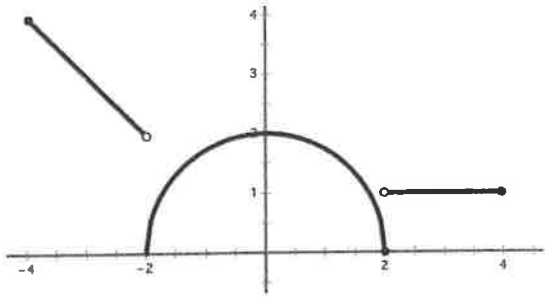
Score__________

Directions: Show all work.

1. Let $h(x) = 2 - \int_0^x f(t)\,dt$ on $x \in [-4,\ 4]$. Let the graph of $f(x)$ be comprised of one semicircle and two line segments as shown below.



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3 (a) Find $h(2)$, $h'(2)$, and $h''(2)$.

$h(2) = 2 - \int_0^2 f(t)\,dt = 2 - \frac{4\pi}{4} = 2 - \pi$

$h'(2) = f(2) = 0$

$h''(2) = f'(2) = \text{DNE}$

2 (b) Find the equation of the line tangent to $h(x)$ at $x = 0$.

$y - 2 = 2(x - 0)$

2 (c) At what x-values is $h(x)$ increasing and concave up? Justify your answer.

$h' = f$ AND ~~h~~ WILL BE INC & CONCAVE UP WHEN:

f IS POSITIVE AND INCREASING

$x \in (-2, 0)$

(d) What is the absolute maximum value of $h(x)$ on the interval $x \in [-4, 4]$?

2

MAX AT $x = 4$ BECAUSE IT IS THE ONLY MAX

$h(4) = \int_0^4 f(t)\,dt = \pi \mp 2$

The 49er Sack Leader Problem

NB's Games completed	0	19	21	41	57
$B(g)$ in sacks per game	0	0.68	0.62	0.98	1.16

2. The table above shows Nick Bosa's sack rate, in sacks per game, over his first four seasons.

(a) Using a Right-hand Reimann Sum, determine the approximate number of sacks Nick Bosa had during these four years. Round to the neares whole number.

1

$$\int_0^{57} B(g)\,dg \approx 19(.68) + 2(.62) + 10(.98) + 16(1.16) = 52.32$$

NICK BOSA HAD APPROXIMATELY 52 SACKS IN HIS FIRST 57 GAMES

(b) Using the data on the table, estimate $B'(32)$. Based on this estimate and the data on the table, was Bosa's sack total increasing at an increasing or decreasing rate? Using the correct units, explain your answer.

3

$$B'(32) = \frac{.98 - .63}{41 - 21} = .0175 \ \frac{\text{SACKS}}{\text{GAME}^2}$$

NICK BOSAS SACK RATE, IN SACKS PER GAME, WAS INCREASING BY APPROXIMATELY .0175 SACKS PER GAME PER GAME DURING THE 32ND GAME HE PLAYED.

(c) Disregarding his injury-shortened 2020 season, $N(g) = 0.178\sqrt{g}$ models Bosa's sack rate per game during the first four years of his career. Find $\frac{1}{57}\int_0^{57} N(g)\,dg$. Using the correct units, explain the the result in context of the problem.

2

$$\frac{1}{57}\int_0^{57} N(g)\,dg = .896$$

BOSA ~~SACKS~~ SACKS THE OPPOSING QB AN AVERAGE OF .896 TIMES PER GAME DURING HIS FIRST 57 GAMES OF HIS CAREER

(d) Assume that $A(g) = -.001g^2 + 0.073g - 0.004$ models the rate of Aldon Smith's sacks over his first 50 games from 2011 to 2014 before he was suspended. During what game, on $0 < g \le 57$, is the difference between Bosa's and Smith's sack totals at a maximum? Show your calculations.

3

$$\text{AMOUNT} = \int_0^t N(g) - A(g)\,dg$$

$N = A \rightarrow t = 7.509$ & 46.932

t	$\int_0^t (N-A)\,dg$
0	0
7.509	-.555
46.932	7.597
57	-1.683

DURING GAME 46, ALDON SMITH HAD 7.597 SACKS MORE THAN NICK BOSA.
