AB Calculus '22-23
Dx Apps II Form G
Calculator allowed

Name Solution Key

Score__________

1. For $t \geq 0$, the position of a particle is given by $x(t) = \sin t + \cos t$ What is the acceleration of the particle at the point where the velocity is first equal to 0?

a) $-\sqrt{2}$ b) -1 c) 0 d) 1 e) $\sqrt{2}$

$V = \cos t - \sin t = 0 \Rightarrow t = \pi/4$

$V'(\pi/4) = -\sin\frac{\pi}{4} - \cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2}$

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2. An object moves along the y-axis with coordinate position $y(t)$ and velocity $v(t) = -1.5 + \frac{4.1t}{\sqrt{t^2+1}} - 3.4\sin(0.3t)$ for $0 \leq t \leq 10$. At time $t = 1$, the object is

a) moving downward with negative acceleration.

b) moving upward with negative acceleration.

c) moving downward with positive acceleration.

d) moving upward with positive acceleration.

e) at rest.

$V(1) > 0$

$a(1) > 0$

~~3.~~ A Golden Rectangle is one where the ratio of the length to the short side to the long side is equal to the ratio of the long side to the sum of the two sides. This ratio is called ϕ, and is approximately equal to 0.618. If a Golden Rectangle changes such that its short side is growing at 2 in/min. How fast is the area changing when the long side is 5 inches?

a) $6.18\ \text{in}^2/\text{min}$ b) $12.36\ \text{in}^2/\text{min}$

c) $3.09\ \text{in}^2/\text{min}$ d) $2.472\ \text{in}^2/\text{min}$

$\frac{d(\text{area})}{dt} = \cancel{.618}(2)$

$A = \frac{x^2}{.618}$

$\frac{dA}{dt} = \frac{2x}{.618}\frac{dx}{dt}$

$= 1.236(5)(1.236)$

$= \cancel{32.36}\ 2$

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4. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = 10 - 5y$ with $v(0) = 50$. Which of the following must be **false**?

✓ a) $v = 10 - e^{5t+c}$ ✓ b) $-\frac{1}{5}\ln|10-5v| = t + c$

c) $\ln|10 - v| = -5t + c$ ✓ d) $v = 10 - ke^{-5t}$

e) None are false.

$-\frac{1}{5}\int \frac{1}{10-5v}(-5)\,dv = \int dt$

$-\frac{1}{5}\ln|10-5v| = t + c$

$\ln|10-5v| = -5t + c$

3. A Golden Rectangle is one where the ratio of the length to the short side to the long side is equal to the ratio of the long side to the sum of the two sides. This ratio is called ϕ, and is approximately equal to ~~1.618.~~ If a Golden Rectangle changes such that its short side is growing at 2 in/min, how fast is the area changing when the short side is 5 inches? 1.236

a) $6.18\ {}^{\text{in}^2}\!/_{\text{min}}$ b) $12.36\ {}^{\text{in}^2}\!/_{\text{min}}$

c) $3.09\ {}^{\text{in}^2}\!/_{\text{min}}$ d) $2.472\ {}^{\text{in}^2}\!/_{\text{min}}$

$A = 1.236w^2$

$\frac{dA}{dt} = 2.472w\,\frac{dw}{dt}$

$=$

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4. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = 10 - 5v$ with $v(0) = 50$. Which of the following must be **true**?

a) $v = 10 - e^{5t+c}$ b) $-\frac{1}{5}\ln|10-5v| = t + c$

c) $\ln|10 - v| = -5t + c$ d) $v = 10 - ke^{-5t}$

e) None are true.

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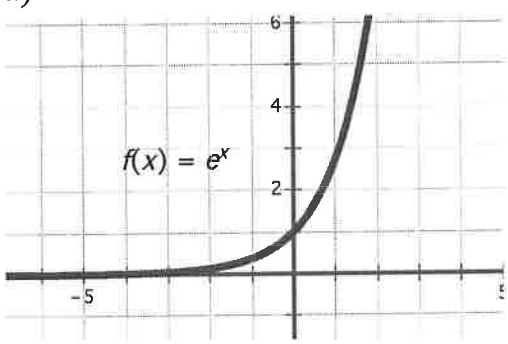
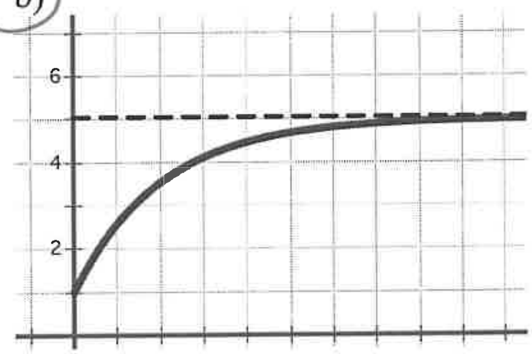
7. According to the Ebbinghaus model, the rate at which a student forgets material is proportional to the difference between the amount y (material which is currently forgotten) and the total amount of material learned. Based on this, which of the following might be the graph of the model?

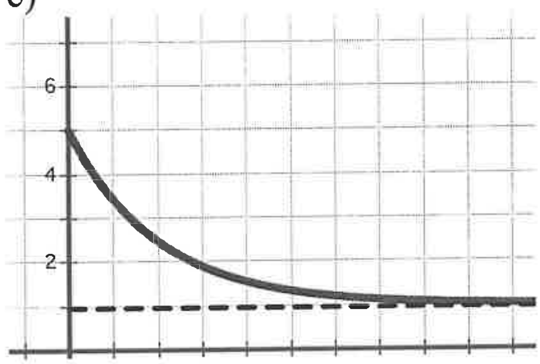
a)



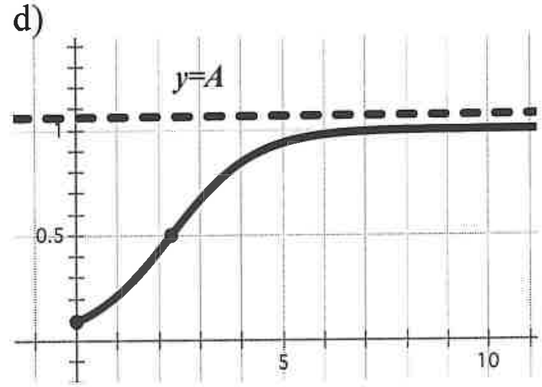
b)



c)



d)



8. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that they have values given on the table below.

x	$f'(x)$	$f''(x)$	$g'(x)$	$g''(x)$
2	0	2	-8	0
4	8	0	0	3
8	0	-12	0	4

Which of the following statements is true?

a) $f(x)$ has a relative maximum at $x=8$ T $f'=0$ $f''<0$

b) $g(x)$ has a relative minimum at $x=2$

c) $f(x)$ has a relative minimum at $x=4$

d) $g(x)$ has a point of inflection at $x=4$

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9. Let $y=f(x)$ be a twice-differentiable function such that $f(-1)=-2$ and $\frac{dy}{dx}=xy^2$. What is $\frac{d^2y}{dx^2}$ at $x=-1$?

a) -12 b) -7 c) 0 d) 12 e) 20

$\frac{d^2y}{dx^2} = x\left(2y\frac{dy}{dx}\right) + y^2(1) = 2xy\left(xy^2\right) + y^2$

@ $x=-1 \rightarrow \frac{d^2y}{dx^2} = -12$

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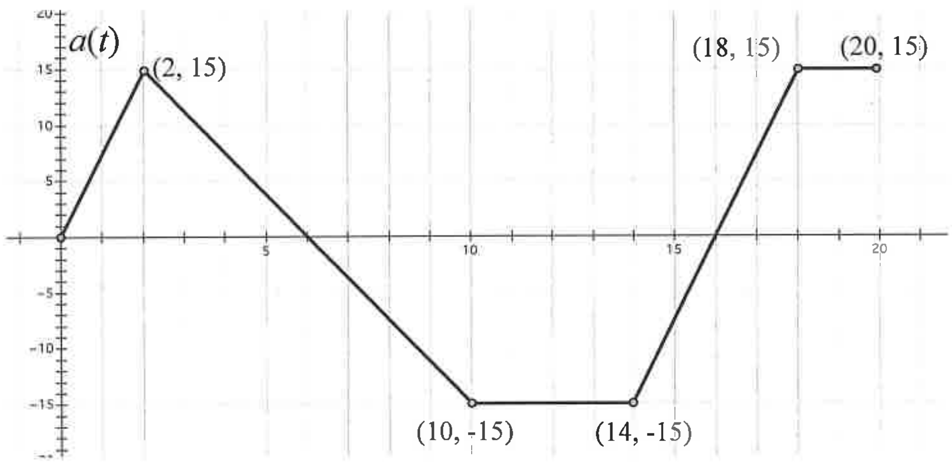
Directions: Do problem #1 and EITHER #2 OR #3



1. A pickup truck is traveling along a straight country road with a velocity of $50 \, ft/sec$ at time $t = 0$ second. For $0 \le t \le 20$ seconds, the truck's acceleration $a(t)$, in ft/sec^2, is given by the piecewise linear function defined by the graph above.

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a) Is the velocity increasing at $t = 2$ seconds? Why or why not?

$V'(2) = a(2) = 15 > 0 \therefore$ yes, the velocity is increasing

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b) On what time in the interval $0 \le t \le 20$, other than at $t = 0$, is the velocity of the truck $50 \, ft/sec$? Why?

$t = 11$ because $\int_0^6 a(t) = 45 = -\int_6^{11} a(t)\,dt$

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c) Find the truck's absolute maximum velocity on $0 \le t \le 20$. Justify your answer.

CV: $t = 0, 6, 16, 20$

MAX @ $t = 6$ & 20

$V(20) = 50 + \int_0^{20} a(t) = 35$

$v(6) = 50 + \int_0^6 a(t)dt = 95$

MAX VELOCITY IS $95 \frac{FT}{SEC}$

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d) On the time in the interval $0 \le t \le 20$, if any, is the truck's. velocity equal to zero? Justify your answer.

MIN @ $t = 0, 16$

$v(0) = 50$

$v(16) = 50 + \int_0^{16} a(t)dt = -10$

SINCE $v(0) > 0$ & $v(16) < 0$ THERE MUST BE A TIME BETWEEN $t = 0$ & $t = 16$ WHEN $V = 0$

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3. According to a simple physiological model, an athletic adult needs 20 calories per day per pound of body weight to maintain his weight. If he consumes more or fewer calories than those required to maintain his weight, his weight W changes at a rate proportional to the difference between the number of calories consumed and the number needed to maintain his current weight. Let $\frac{dW}{dt} = \frac{1}{3500}(3600 - 20W)$, where W is the person's weight at time t (measured in days).

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1 a) What is the limit to this person's weight $W(t)$?

$\frac{dw}{dt} = \frac{1}{175}(180 - w) \rightarrow$ LIMIT $= 180$

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1 b) Assume that the 3600 in the equation is the person's daily caloric intake. If that is reduced to 3000, what would be the limit to this person's weight?

$$\frac{dw}{dt} = \frac{1}{3500}(3000 - 20w)$$

$$= \frac{1}{175}(150 - w)$$

LIMIT $= 150$

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c) Find the particular solution to $\frac{dW}{dt} = \frac{1}{3500}(3600 - 20W)$ if the person weights 150 pounds when $t = 0$.

6

5

$$\frac{dw}{dt} = \frac{1}{175}(180-w)$$

$$-\int \frac{1}{180-w}(-dw) = +\frac{1}{175}\,dt$$

$$-\ln|180-w| = \frac{1}{175}t + c$$

$$\ln|180-w| = -\frac{1}{175}t + c$$

$$180 - w = e^{-\frac{1}{175}t + c} = Ke^{-\frac{1}{175}t}$$

$$(0, 150) \rightarrow K = 30$$

$$W = 180 - 30e^{-\frac{1}{175}t}$$

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2 d) Use your solution in c) to determine how long it would take this person's weight to reach 170 pounds. [Show the set up, but solve by graphing.]

$$170 = 180 - 30e^{-\frac{1}{175}t}$$

$$\ln\frac{1}{3} = -\frac{1}{175}t$$

$$t = 192.257 \text{ DAYS}$$

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3. Consider the curve given by $2x^2 - 4xy + 5y^2 = 27$.

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a) Show that $\frac{dy}{dx} = \frac{2x-2y}{2x-5y}$.

2

$$\frac{d}{dx}\left[2x^2 - 4xy + 5y^2 = 27\right]$$

$$4x - 4x\frac{dy}{dx} - 4y(1) + 10y\frac{dy}{dx} = 0$$

$$(10y - 4x)\frac{dy}{dx} = 4y - 4x$$

$$\frac{dy}{dx} = \frac{4y-4x}{10y-4x} = \frac{2x-2y}{2x-5y}$$

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b) Find point(s) P where the tangent line is horizontal.

3

HORIZONTAL TANGENT: $\frac{dy}{dx} = 0 \rightarrow x = y$

$$2x^2 - 4x(x) + 5(x)^2 = 27$$

$$3x^2 = 27$$

$$x = \pm 3$$

$(3, 3)$

$(-3, -3)$

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c) Find $\frac{d^2y}{dx^2}$.

$$\frac{d}{dx}\left[\frac{dy}{dx} = \frac{2x-2y}{2x-5y}\right]$$

$$\frac{d^2y}{dx^2} = \frac{(2x-5y)\left(2-2\frac{dy}{dx}\right) - (2x-2y)\left(2-5\frac{dy}{dx}\right)}{(2x-5y)^2}$$

$$= \frac{(2x-5y)\left(2-2\left(\frac{2x-2y}{2x-5y}\right)\right) - (2x-2y)\left(2-5\left(\frac{2x-2y}{2x-5y}\right)\right)}{(2x-5y)^2}$$

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d) Determine if each value found in b) is at a maximum, a minimum, or neither. Justify your answer.

$$\left.\frac{dy}{dx}\right|_{x=\pm 3} = 0$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=3} = \frac{(-9)(2)-0}{81} = <0 \therefore \text{MAX}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=-3} = \frac{9(2)-0}{81} > 0 \therefore \text{MIN}$$
