AP Calculus AB ’22-23

Fall Final Part II

Calculator Allowed

Name:

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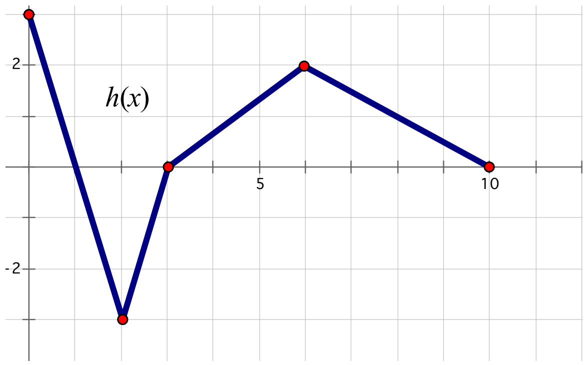

x	$g(x)$	$g'(x)$
0	-2	12
2	0	-3
4	5	5
6	3	8
8	-4	11

1. Let $f(x)$ be the function defined by the equation above, let $h(x)$ be the function whose graph is given above, and let $g(x)$ be a differentiable function with selected values for $g(x)$ and $g'(x)$ given on the table above. A third function, $y = f(x)$, is given by $f(x) = \sin\frac{\pi}{2}x$.

(a) Find the equation of the line tangent to $h(x)$ at $x = 8$.

(b) Let K be the function defined by $K(x) = g(f(x))$. Find $K'(6)$.

(c) Let M be the function defined by $M(x) = g(x) \cdot h(x)$. Find $M'(4)$.

(d) Let J be the function defined by $J(x) = \dfrac{h(x)}{g\left(\frac{1}{2}x\right)}$. Find $J'(8)$.

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2. Consider the differential equation $\frac{dy}{dx} = \frac{2y}{x^2+1}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $y(0) = -2$. The function $y = f(x)$ is defined for all real numbers.

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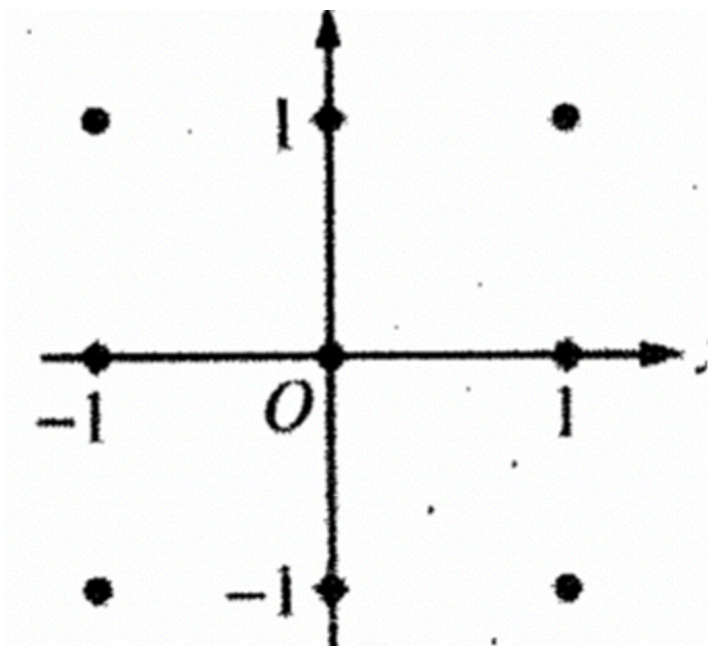
a) On the axis system provided, sketch the slope field for the $\frac{dy}{dx}$ at all points plotted on the graph.

b)



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b) Find the particular solution to $\dfrac{dy}{dx} = \dfrac{2y}{x^2+1}$ with the initial condition $y(0) = -2$.

The Yuma Desalting Plant Problem Ic

The desalting plant at Yuma, AZ, removes alkaline (salt) products from the Colorado River the make the water better for irrigation downstream in Mexico. Water enters the plant at a rate $W(t)$ modeled by

$$W(t) = 2800 + 750\sin\left(\frac{2\pi}{11}t\right),$$

where $W(t)$ measured in foot-acre per month and $0 \le t \le 10$ is measured in months. Data for the rate $P(t)$ of outflow of proceessed water, in foot-acre per month is given in the table below:

t in Month	0	1	2	3	4	5	6	7	8	9	10
$P(t)$ in foot-acres per month	0	1617	1929	2167	2241	2169	2269	2151	2167	2133	1969

Based on supplies available, not all the water gets processed before returning to the Colorado River. Assume the initial value $W(0) = 0$.

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a) Find the volume of water that passes into the plant during these ten months.

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b) Using a Midpoint Reiman Sum, approximate the volume of processed water that leaves the plant during these ten months. Indicate units.

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c) Approximate $P'(5)$. Using the correct units, explain the meaning of your answer.

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d) Assuming $P(t)$ can be modeled by $Q(t) = -0.55t^4 + 15t^3 - 158t^2 + 722t + 1032$, find the maximum amount of unprocessed water flowing though the plant for $0 < t \leq 10$. Justify your answer.

End of

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