AP Calculus AB ’22-23

Fall Final Part I

Calculator Allowed

Name:

Solution Key

1. $\int\left(2e^{2x} - 5^x\right) dx$ $= \frac{2e^{2x}}{2} - \frac{5^x}{\ln 5} + c$

a) $2e^{2x} - 5^x \ln 5 + c$ b) $2e^{2x} - \frac{5^x}{\ln 5} + c$ c) $e^{2x} - 5^x \ln 5 + c$

d) $4e^{2x} - x5^{x-1} + c$ e) $e^{2x} - \frac{5^x}{\ln 5} + c$

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2. Given $f(3) = 5,\ f'(3) = 1.1,\ g(3) = -4$ and $g'(3) = 0.7$, find the value of $(f+g)'(3)$.

a) 1.8 b) 0.4 c) −0.9 d) −1.8 e) 0.9

$f'(3) + g'(3) = 1.1 + .7$

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3. Find the average rate of change of $y = \cos 3x$ on $x \in \left[0, \frac{\pi}{6}\right]$.

a) $-\frac{1}{3}$ b) -1 c) -3 d) $-\frac{6}{\pi}$ e) $-\frac{2}{\pi}$

$$\frac{\cos\frac{\pi}{2} - \cos 0}{\pi/6 - 0} = \frac{-1}{\pi/6}$$

4. If $f(x) = \cos[\sin x]$, then $\frac{d}{dx}[f(x)]$ is

a) $-\cos^2 x + \sin^2 x$ b) $\cos^2 x - \sin^2 x$

c) $\sin(\sin x) \cdot (\cos x)$ d) $\cos(\cos x)$

e) $-\sin(\sin x) \cdot (\cos x)$

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5. $\int \left(\frac{4y^3 - 2y^2 - 5y}{\sqrt{y}} \right) dy =$ $\int \left(4y^{5/2} - 2y^{3/2} - 5y^{1/2}\right) dy$

a) $\left(y^4 - \frac{2}{3}y^3 - \frac{5}{2}y^2\right)\left(2y^{1/2}\right) + c$

b) $4y^{5/2} - 2y^{3/2} - 5y^{1/2} + c$

c) $\frac{8}{7}y^{7/2} - \frac{4}{5}y^{5/2} - \frac{10}{3}y^{3/2} + c$

d) $10y^{3/2} - 3y^{1/2} - \frac{5}{2}y^{-1/2} + c$

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6. Which of the following statements must be **false**?

a) $\frac{d}{dx}\left[\frac{4x}{x^2+4}\right]=\frac{16-4x^2}{(x^2+4)^2}$ T

b) $\frac{d}{dx}\left[e^{\cot x}\right]=-e^{\cot x}\csc^2 x$ T

c) $\frac{d}{dx}\left[\ln(1-x^3)\right]=-\frac{1}{1-x^3}$ F

d) $\frac{d}{dx}\left[\cos^{-1}e^{2x}\right]=\frac{-2e^{2x}}{\sqrt{1-e^{4x}}}$

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7. Consider a particle moving such that its position is described by the function $x(t)=t^4-\frac{t^5}{5}$. When does the particle attain its minimum velocity?

a) $t=0$ b) $t=2$ c) $t=3$

d) $t=4$ e) $t=5$ ae) There is no minimum

$V = 4t^3 - t^4$

$V' = 12t^2 - 4t^3 = 0$

$4t^2(3-t) = 0$

V' + 0 + 0 −

t 0 3

NEITHER MAX

8. Which of the following is the solution to the differential equation

$\frac{dy}{dx} = \frac{x-1}{y}$ with the initial condition $y(0) = -2$?

$y\,dy = (x-1)dx$

$\frac{y^2}{2}dy = x^2 - x + c$

$2 = C$

a) $y = -2e^{x^2-2x}$ b) $y = -2 + e^{x^2-2x}$

c) $y = \sqrt{x^2-2x-2}$ d) $y = -\sqrt{x^2-2x+2}$

e) $y = -\sqrt{x^2-2x-2}$

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9. If h is the function defined by $h(x) = x^2 - 5x + 3$, what is the equation of the tangent line to the function when $h'(x) = -1$? $= 2x - 5 \rightarrow x = 2,\ y = -3$

a) $y = -x - 1$ b) $y = -x + 24$ c) $y = -x - 5$

d) $y = -7x + 2$ e) $y = -7x$

$y + 3 = -(x - 2)$

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10. If $f(x) = 3\sin x + 4\cos^2 x$, then $f''(\pi) =$

a) 8 b) -10 c) 4 d) -8 e) -6

$f' = 3\cos x + 8\cos x(-\sin x)$

$f'' = -3\sin x + 8\cos x(-\cos x) - 8\sin x(-\sin x)$

$f''(\pi) = 0 + 8(-1)(-(-1)) + 0 = -8$

$=$

11. Given that $f(x) = 2x - 1$ and $g(x) = -\dfrac{x}{x^2+1}$, then $\dfrac{d}{dx}\left[f(g(-2))\right]=$

a) $-\dfrac{3}{25}$ b) $-\dfrac{3}{5}$ c) $\dfrac{3}{5}$ d) $\dfrac{3}{25}$ e) $\dfrac{6}{25}$

$g' = -\dfrac{(x^2+1)(1) - x(2}{(x^2+1)^2}$

$= f'(g(-2))\cdot g'(-2)$

$= f'\left(\tfrac{2}{5}\right)\cdot g'(-2) = 2\left(-\dfrac{5-8}{25}\right) = \dfrac{+6}{25}$

$\int_{-2}^{1} + \int_{1}^{5} = \int_{-2}^{5} \rightarrow \int_{-2}^{1} + (3) = -2 \rightarrow \int_{-2}^{1} -5$

$\int_{-2}^{5} f(x)dx = -2$	$\int_{1}^{5} f(x)dx = 3$
$\int_{-2}^{1} g(x)dx = -4$	$\int_{5}^{1} g(x)dx = 9$

12. Based on the information above, $\int_{1}^{-2}\left[g(x) + f(x)\right]dx =$

a) -9 b) -1 c) 0 d) 1 e) 9

$\int_{1}^{-2} g+f = -\int_{-2}^{1} g - \int_{-2}^{1} f$

$= -(-4) - (-5) = 9$

13. $2\int\left(\frac{\sin\sqrt{x}}{2\sqrt{x}}\right)dx = 2\int \sin u\, du$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

a) $-\cos\sqrt{x}+c$

b) $\frac{1}{2}\cos\sqrt{x}+c$

c) $-\frac{1}{2}\cos\sqrt{x}+c$

d) $2\cos\sqrt{x}+c$

e) $-2\cos\sqrt{x}+c$

$u = 2+x^{1/3}$

$du = \frac{1}{3}x^{-2/3}dx$

14. $3\int_1^8 \frac{dx}{3(2+\sqrt[3]{x})\sqrt[3]{x^2}}$

a) $\frac{7}{2}$

b) $\frac{21}{2}$

c) $\frac{1}{3}\ln\frac{3}{2}$

d) $\ln\frac{3}{2}$

e) $3\ln\frac{4}{3}$

$\int_1^8 = 3\int_3^4 \frac{1}{u}\,du = 3[\ln 4 - \ln 3]$

15. An object moves with velocity $v(t) = \sec^2(2t)$. It is known that the particle's position at time 0 is 2. What is the particle's position function?

a) $s(t) = \tan(2t) + 2$

$X = \int \sec^2 2t \, dt$

b) $s(t) = \frac{1}{2}\tan(2t) + 2$

$= \frac{1}{2}\int \sec^2 u \, du$

c) $s(t) = \sec^2(2t)\tan^2(2t) + 2$

$= \frac{1}{2} \tan u$

d) $s(t) = \ln|\sec(2t)| + 2$

e) $s(t) = \frac{1}{2}\ln|\sec(2t)| + 2$

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16. A ski resort uses a snow machine to control the snow level on a ski slope. Over a 24-hour period the volume of snow added to the slope per hour is modeled by the equation $S(t)$. The rate that the snow melts is modeled by $M(t)$. Both $M(t)$ and $S(t)$ are measured in $\frac{yd^3}{h}$ and t is measured in hours for $0 \le t \le 24$. At time t = 0, the slope holds $50yd^3$ of snow. Which of the following expresses the total change in the number of cubic yards of snow in the slope at t hours?

a) $\int_0^t [S(x) - M(x)]\, dx$

b) $\frac{1}{t}\int_0^t [S(x) - M(x)]\, dx$

c) $S(t) - M(t)$

d) $S'(t) - M'(t)$

e) $50 + \int_0^t [S(x) - M(x)]\, dx$

17. Which of the following slope fields represents $\dfrac{dy}{dx} = y^2 + 2x^2$?

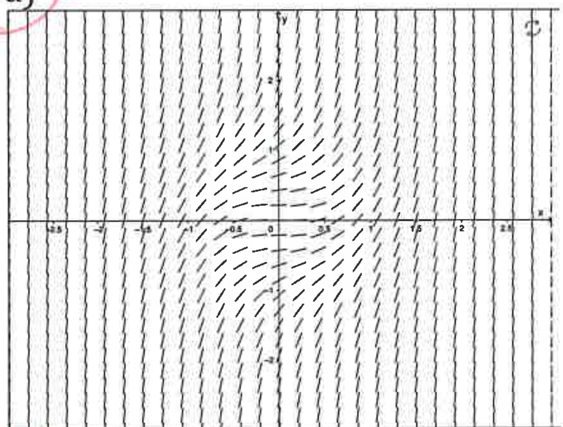
$x=0 \rightarrow \frac{dy}{dx} = y^2$

$y=0 \rightarrow \frac{dy}{dx} = 2x^2$

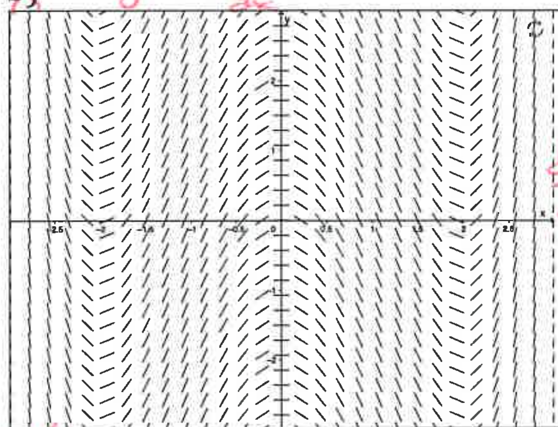
a)

$M = 2x^2$

>0



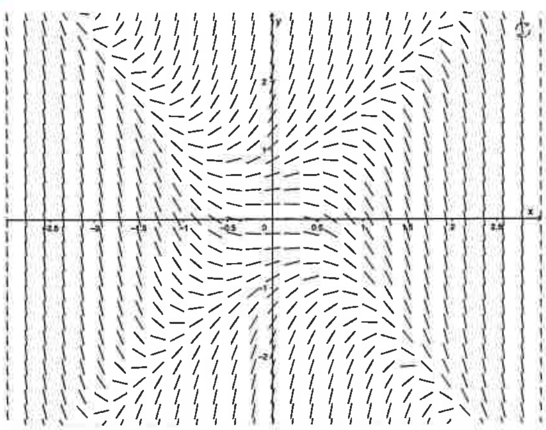
b)



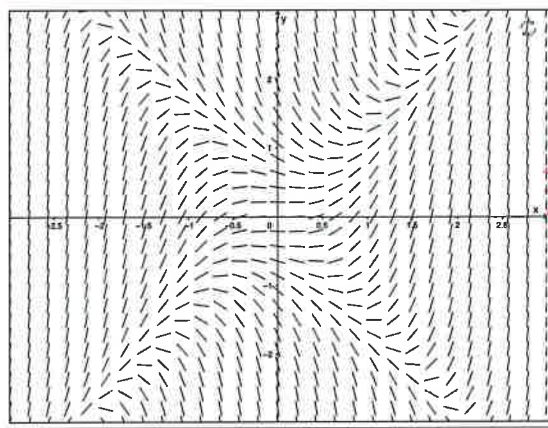
$\frac{dy}{dx} = 0$

BUT $y^2 > 0$

c)



d)



$\frac{dy}{dx} < 0$

BUT $y^2 > 0$

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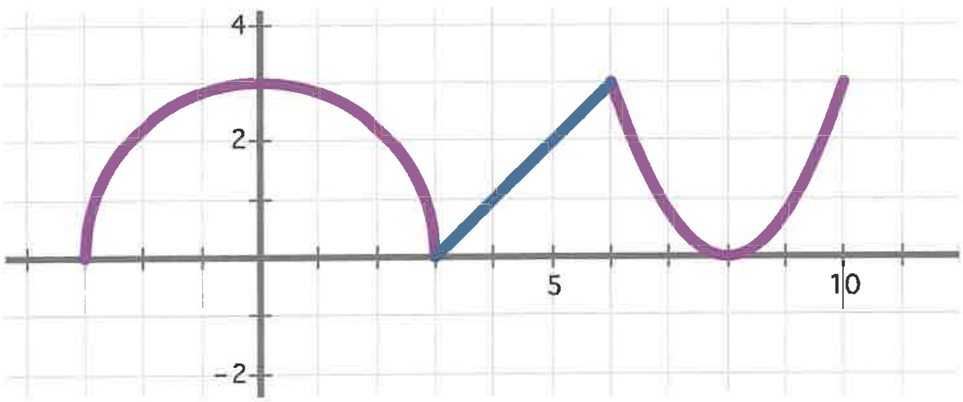
18. For $t \geq 0$ hours, H is a differentiable function of t that gives the rate of change in temperature, in degrees Celsius per hour, at an Arctic weather station. In what units would $\dfrac{1}{t}\displaystyle\int_0^t H'(x)dx$ be measured?

a) degrees Celsius
b) degrees Celsius per hour
c) degrees Celsius per hour per hour.
d) hours per degrees Celsius
e) hours

$\dfrac{\int H'}{t} = H$ in $\dfrac{\frac{C^\circ}{HR}}{HR}$



19. At what point on the above curve is $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$

DEC CON UP

a) -2 b) 2 c) 5 d) 7 e) 9

End of

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