AB Calculus ’22-23
Limit Test Form F
No Calculator

Name Solution Key

1. Let $f(x) = \begin{cases} \ln(1-x), if x \le 0 \\ \tan x, \ if 0 < x \end{cases}$. Which of the following statements is **false** about f?

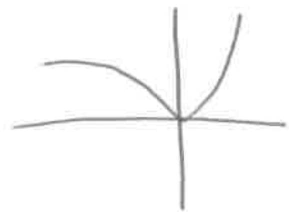
$f' = \begin{cases} \frac{-1}{1-x} & \text{IF } x<0 \\ \sec^2 x & \text{IF } x>0 \end{cases}$

(a) f is continuous at $x = 0$. T

(b) f is not differentiable at $x = 0$. T

(c) f has a local maximum at $x = 0$. F

d) f does not have a point of inflection at $x = 0$.



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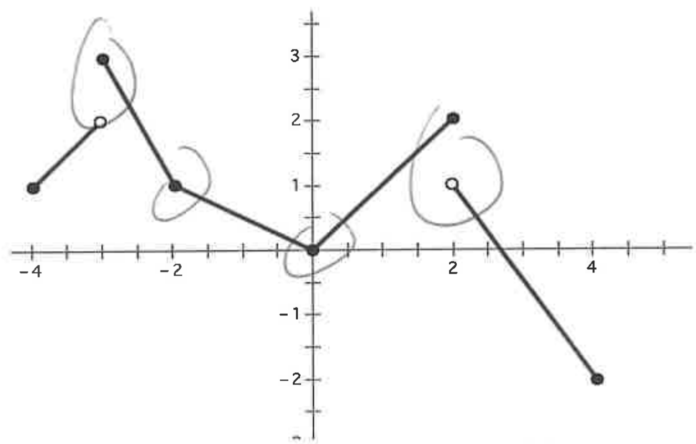
2. The function f is defined on the interval $x \in (-4, 4)$ and has the graph shown below.



For which of the following values is f not differentiable?

a) -3 and 2 only b) 0 only c) -2 and 0 only

d) -4, -2, and 0 only (e) -3, -2, 0, and 2

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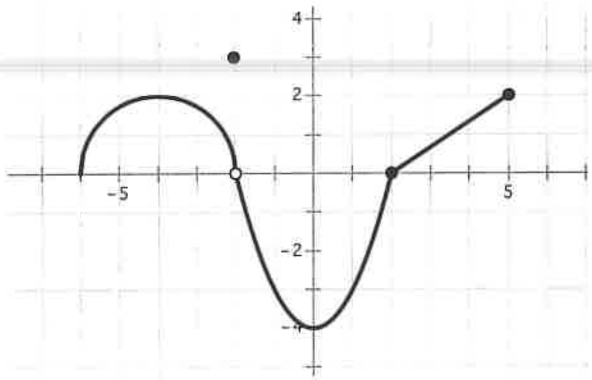
3. The function f is shown below. Which of the following statements about the function f, shown below, is false? TRUE



a) $\lim_{x\to 0} f(x)$ does not exist F

b) $\lim_{x\to 2} f(x)$ exists T

c) f is continuous at $x=-2$ F

d) $\lim_{h\to 0} \frac{f(1-h)+3}{h}$ exists F

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4. $\lim_{h\to 0} \frac{2\left(\frac{1}{3}+h\right)^3 - 2\left(\frac{1}{3}\right)^3}{h} =$ $6\left(\frac{1}{3}\right)^2 = \frac{6}{9}$

(a) 0 (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ (e) DNE

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5. $\lim_{x \to \infty}\left(\tan^{-1}\left(\frac{x}{e^x}+1\right)\right) =$ TAN^{-1} 1

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 1 (e) DNE

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x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
3	0	5	0	5

6. Given that $f(x)$ is a thrice differentiable, continuous function on the interval $(0, 4)$ with the table values given above. $\lim_{x \to 3}\frac{(x-3)^3}{f(x)} =$ $\lim_{x \to 3}\frac{3(x-3)^2}{f'(x)}$

(a) 0 (b) $\frac{7}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{6}$ (e) dne

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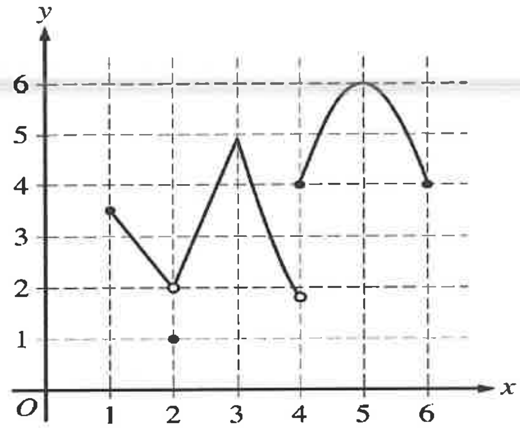
7. The function f is defined on the interval $x \in [-5, 5]$ and has the graph shown below.



Graph of f

Which of the following is true?

a) $\lim_{x \to 2} f(x) = 1$ F

b) $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = dne$ T

c) $\lim_{x \to 3} f(x) = f(6)$ F

d) $\lim_{x \to 4^-} f(x) = 4$

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8. At $x = 0$, the function given by $f(x) = \begin{cases} e^x, if x \leq 0 \\ \sin x, if 0 < x \end{cases}$ is

$e^0 = 1$ $\sin 0 = 0$

(A) Undefined

(B) Continuous but not differentiable

(C) Differentiable but not continuous

(D) Neither continuous nor differentiable

(E) Both continuous and differentiable

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9. Which of the following functions is NOT differentiable at $x = \frac{\pi}{2}$?

(a) $f(x) = x^2$ (b) $f(x) = e^x$ (c) $f(x) = \ln(x+1)$

(d) $f(x) = \sec x$ (e) $f(x) = \cot x$

VA @ $x = \frac{\pi}{2}$

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10. $\lim_{x \to 0} \frac{\int_0^{x^3} \cos t^2 \, dt}{x^3}$ L'H $= \lim_{x \to 0} \frac{\cos x^6 (3x^2)}{3x^2} = \cos 0$

(a) 0 (b) 1 (c) $\frac{1}{3}$ (d) 3 (e) DNE

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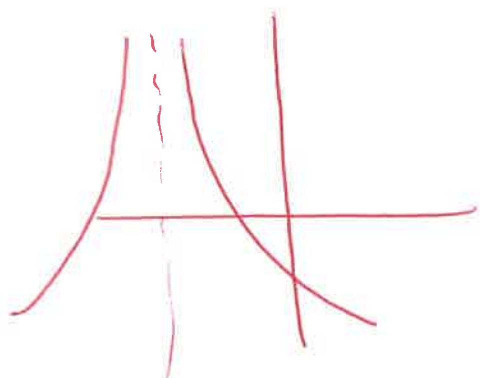
11. A function $f(x)$ has a vertical asymptote at $x = -2$. The derivative of $f(x)$ is positive for all $x < -2$ and negative for all $-2 < x$. Which of the following statements are **true**?

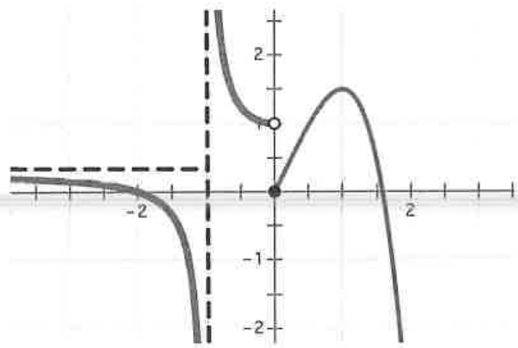
a) $\lim_{x \to -2^-} f(x) = -\infty$ and $\lim_{x \to -2^+} f(x) = -\infty$

b) $\lim_{x \to -2^-} f(x) = -\infty$ and $\lim_{x \to -2^+} f(x) = +\infty$

c) $\lim_{x \to -2^-} f(x) = +\infty$ and $\lim_{x \to -2^+} f(x) = +\infty$

d) $\lim_{x \to -2^-} f(x) = -\infty$ and $\lim_{x \to -2^+} f(x) = -\infty$





12. Given the graph of $f(x)$ above, the reason that $f(x)$ is not continuous at $x=0$ is

a) $f(0)$ does not exist

b) $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

c) $\lim_{x \to 0} f(x) \neq f(0)$

d) $\lim_{x \to 0} f(x)$ does not exist

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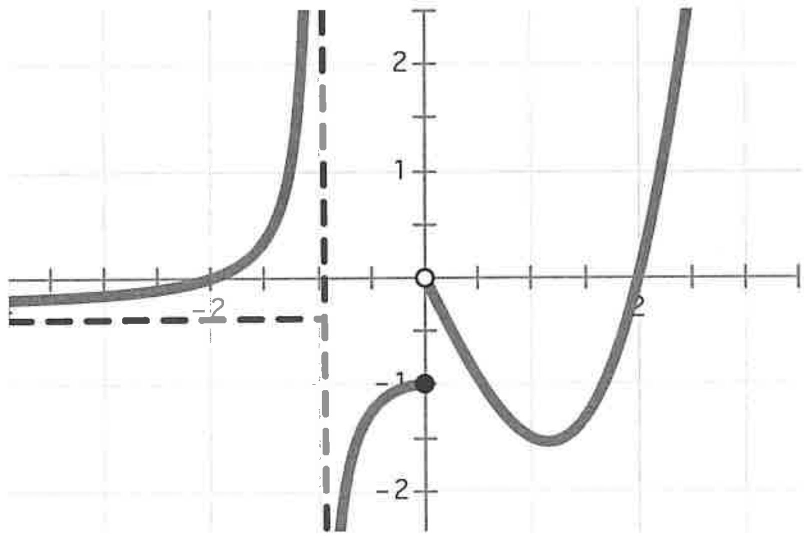
Score__________

Directions: Show all work.



1. For this graph, find

(a) $\lim_{x \to -1^-} f(x) = \infty$

(b) $\lim_{x \to 0^-} f(x) = -1$

(c) $\lim_{x \to 1} f(x) = -1.5$

(d) $\lim_{x \to -1} f(x) =$ DNE

$\lim_{x \to -1^+} f(x) = -\infty$

(e) $\lim_{x \to 0^+} f(x) = 0$

(f)

(g) $f(-1) =$ DNE

(h) $f(0) = -1$

(i) $f(1) = -1.5$

(j) $f(2) = 0$

2. $h(x) = \begin{cases} 10 - x^2, & \text{if } x < -3 \\ e^{x+3}, & \text{if } -3 \leq x \end{cases}$

a) Is $f(x)$ continuous at $x = 0$? Why/Why not?

i) $f(-3)$ EXISTS

ii) $\lim_{x \to -3} f(x)$ EXISTS BECAUSE $\lim_{x \to -3^-} f(x) = 10 - 9 = 1$

AND $\lim_{x \to -3^+} f(x) = e^0 = 1$

iii) $\lim_{x \to -3} f(x) = f(-3)$

$\therefore$ $f(x)$ IS CONTINUOUS AT $x = -3$

(b) Find $f'(-1)$ and $f'(-4)$.

$f'(x) = \begin{cases} -2x & \text{IF } x < -3 \\ e^{x+3} & \text{IF } x > -3 \end{cases}$

$f'(-1) = e^2$

$f'(-4) = 8$

(c) Express $f'(x)$ as a piecewise-defined function. Explain why $f'(0)$ does not exist.

(-3)

$$f'(x) = \begin{cases} -2x & \text{IF } x < -3 \\ e^{x+3} & \text{IF } x > 3 \end{cases}$$

i) $f(x)$ IS CONTINUOUS AT $x = -3$

ii) $\lim_{x \to -3^-} f'(x) = 6 \neq 1 = \lim_{x \to -3} e^{x+3} = \lim_{x \to -3^+} f(x)$

(d) Find $\lim_{x \to 3^+} \dfrac{f(x)}{\ln(x-2)}$. Justify your answer.

$= \dfrac{1}{0} =$

$= \infty$

$\lim_{x \to 3^-} f(x) = 1$

$\lim_{x \to -3^+} \ln 1 = 0$
