AP Calculus AB ’22-23

Spring Final Part IIA v1

Calculator Allowed

Name:

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1. The snowfall in Donner Summit is tracked by the US Weather Service. For the month of March, 2022, $S(t)$ represents the rate of snowfall in inches per day and its data is presented in the table below. $M(t) = 0.65 - 0.35\cos\left(\frac{5x^{0.95}}{6}\right)$ represents the rate at which the snow melts in inches per day, where t is measured in days.

t in days	1	3	4	7	11	15	21
$S(t)$ in inches per day	1.4	4.9	4.4	0.1	4.6	0.2	2.7

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a) Find $\int_1^{21} M(t)\, dt$. Using the correct units, explain the meaning of $\frac{1}{21-1}\int_1^{21} M(t)\, dt$.

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b) Using a Midpoint Reimann Sum, find $\int_1^{21} S(t)\, dt$. State the correct units.

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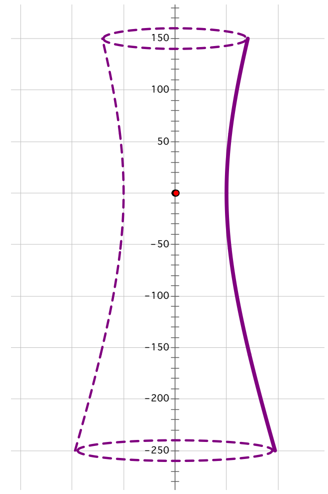
c) Approximate $S'(5)$. Using the correct units, explain $S'(5)$ in context of the problem.

d) Assume $S(t) = 2.5 - 2.5\cos\left(\frac{10x^{0.9}}{9}\right)$ would model the snow fall. If there were 9 inches of snow on the ground at the beginning of Day 1, find the minimum amount of snow on the ground between $t = 1$ and $t = 7$.

2. Many power plants cool their reactions with convective air flow through a hyperboliod tower. The shape increases air flow while minimizing construction material. Consider the shape of a tower formed by revolving the hyperbola $f(y) = 50\sqrt{\frac{1}{22500}y^2 + 1}$ on $y \in [-250,\ 150]$ about the *y*-axis, where *y* is measured in feet.



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a) Find the volume of the interior of the tower. Indicate the units.

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b) Assume that the inner wall is of the shape formed by revolving $f(y)$ about the *y*-axis and the outer wall is of the the shape formed by revolving $g(y) = 50.583\sqrt{\frac{1}{22500}y^2 + 1}$ about the *y*-axis. Find the volume of material needed to make the tower.

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c) The temperature *S* in the tower varies according to the function $S(y) = 70e^{-0.001(y-150)}$, where *y* is measured in feet from the narrowest part of the tower. An object is dropped into the tower from the top. It falls at a rate of $R = -32t\,{}^{\text{ft}}/_{\text{sec}}$ and its height $y = -16t^2 + 150$ ft. How fast is the temperature *S* changing when the object is at a level after falling for 3 seconds? Indicate the units.

End of

AP Calculus AB ’22-23

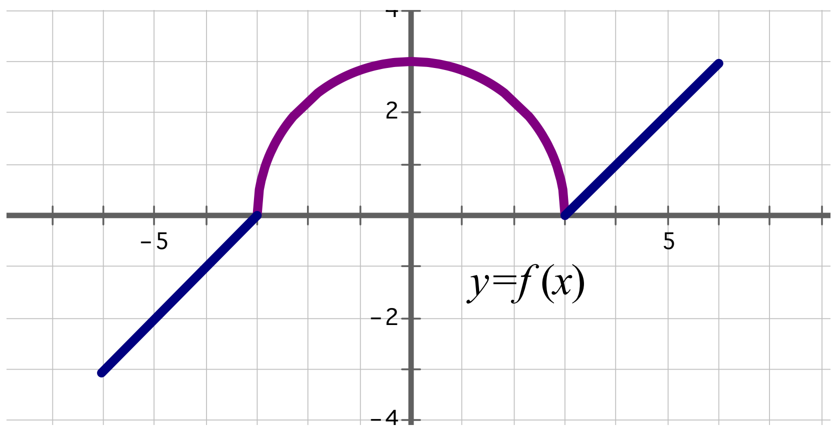
Spring Final Part IIA v1

AP Calculus AB ’22-23

Spring Final Part IIB v1

NO Calculator Allowed

Name: ________________________________



3. The graph above, $f(x)$ on $-6 \le x \le 6$, is comprised of two line segments and a semi-circle. Let $g(x) = 2 + \int_{-3}^{x} f(t)\, dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) At what x-value(s) on $-6 \le x \le 6$ does $g(x)$ have a relative minimum. Explain your reasoning.

(c) At what x-value(s) on $-6 \le x \le 6$ does $g(x)$ have a point of inflection. Explain your reasoning.

(d) On what interval(s) is $g(x)$ both increasing and concave down? Explain why.

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4. Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 8 - 2e^{-2t}$. Particle Q moves along the y-axis, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{6}{t^3}$. At time $t = 1$, the position of particle Q is $y_Q(1) = -3$.

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a) Find $v_P(t)$, the velocity of particle P at time t.

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b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all the times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

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c) Find $y_Q(t)$, the position of particle Q at time t.

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d) As $t \to \infty$, which particle will eventually be farther from the origin. Give a reason for your answer.

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5. Consider the function $y^2 - y + e^x = \cos x$.

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a) Prove that $\dfrac{dy}{dx} = \dfrac{e^x + \sin x}{1 - 2y}$.

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b) Find the equation of the tangent line at (0, 1).

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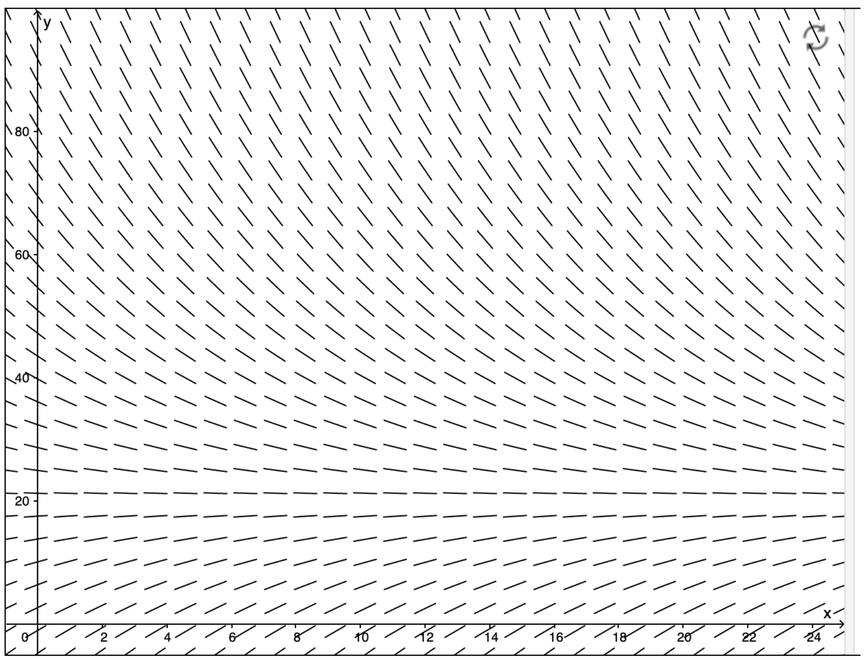
c) Find the value of $\frac{d^2y}{dx^2}$ at (0, 1). Does the curve have a relative maximum, a relative minimum, or neither at (0, 1)? Justify your answer.

6. A cup of coffee is made with boiling water at a temperature of 100 C°, in a room at temperature 20 C°. After two minutes, it has cooled to 80 C°. According to Newton's Law of Cooling, the temperature of the coffee follows the differential equation

$$\frac{dy}{dt} = -0.14(y-20),$$

where y is the temperature of the coffee at time t minutes.



(a) Above is given a partial slope field for the temperature differential equation. Draw the solution to the differential equation at (5, 60).

(b) If $y(0) = 100$, find the equation of the line tangent to the temperature curve and use the tangent line equation to approximate $y(5)$. Explain what this estimate means.

(c) Find the particular solution to $\frac{dy}{dt} = -0.14(y - 20)$ with the initial condition $y(0) = 100$.

End of

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