AP Calculus AB '23-24 Name Solution Key
Dr. Quattrin – 1st & 2nd Periods
Anti-Derivative Test Score 9

Part I: Multiple choice (20 minutes) – Circle correct answer.

1. $\int \cos(4x)dx$ $u=4x$ $du=4dx$ $\int = \frac{1}{4}\int \cos u \, du$

~~a)~~ $-4\sin(4x)+c$ ~~b)~~ $-\frac{1}{4}\sin(4x)+c$ c) $\frac{1}{4}\sin(4x)+c$

d) $4\sin(4x)+c$ ~~e)~~ $\frac{1}{4}\cos(4x)+c$

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2. Which of the following statements are **false**?

a) $\int \frac{1}{x\sqrt{16-x^2}}dx = \frac{1}{4}\sec^{-1}\frac{x}{4}+c$ T

b) $\int \cot x dx = \ln|\sin x|+c$ T

c) $\int \left(\frac{e^x}{\tan e^x}\right)dx = \ln|\sec e^x|+c$ F $\int = \int \cot e^x (e^x dx) = \ln|\sin u| + c$

d) $\int (5^x+2e^{-x})\, dx = \frac{1}{\ln 5}5^x - 2e^{-x}+c$

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3. $\int \frac{x^2 - 1}{\sqrt{x}} dx =$ $\int (x^{3/2} - x^{-1/2}) dx$

a) $\frac{2}{5}x^{5/2} - 2x^{1/2} + c$ b) $\frac{1}{2}(x^2 - 1)^2 + c$ c) $\frac{2}{5}x^{5/2} + 2x^{1/2} + c$

d) $\frac{2}{3}x^{3/2} - x^{-1/2} + c$ e) $\frac{2}{3}x^{3/2} + x^{-1/2} + c$

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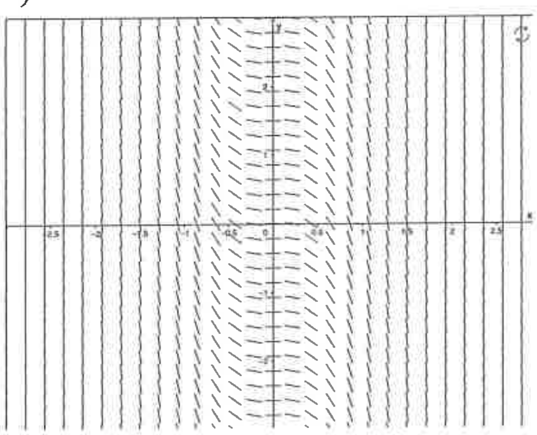
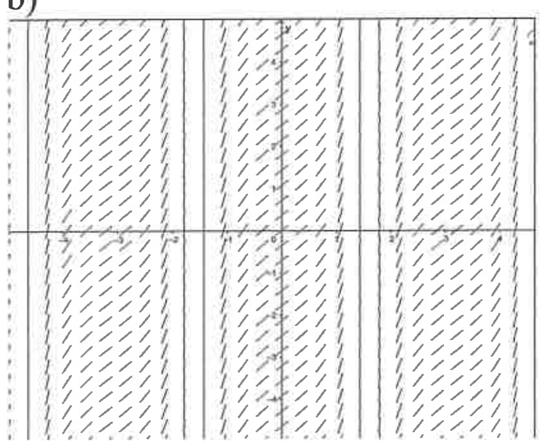
4. Which of the following is the slope field that has the solution $y = 2x^3$?

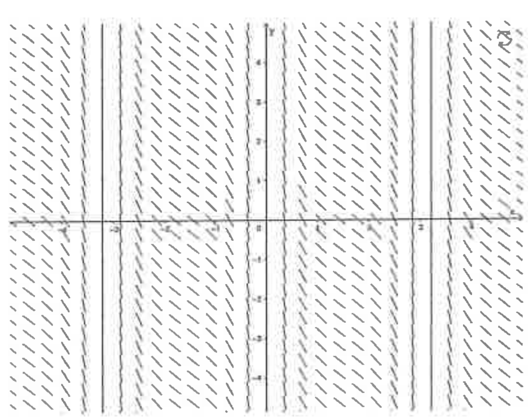
a)



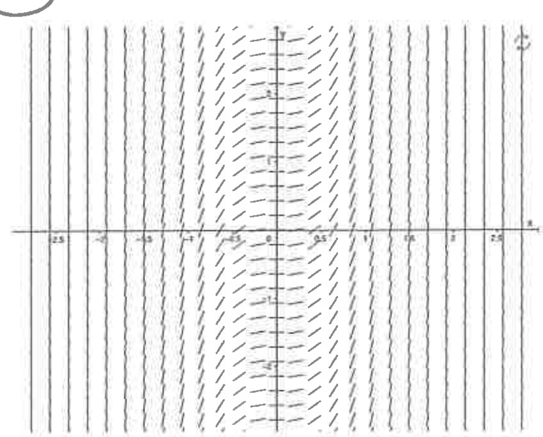
b)



c)



d)



5. Which of the following is the solution to the differential equation $\frac{dy}{dx} = x\csc y$ with the initial condition $y(1) = \frac{3\pi}{2}$?

$\frac{1}{\csc y} dy = x\, dx$

$\sin y\, dy = x\, dx$

$-\cos y = \frac{x^2}{2} + c$

$\cos y = -\frac{x^2}{2} + c$

a) $y = \cos^{-1}\left(\frac{x^2-1}{2}\right)$

b) $y = \sin^{-1}\left(\frac{x^2-1}{2}\right)$

c) $y = \cos^{-1}\left(\frac{-1-x^2}{2}\right)$

d) $y = \sin^{-1}\left(\frac{1-x^2}{2}\right)$

e) $y = \cos^{-1}\left(\frac{1-x^2}{2}\right)$

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6. $\int \frac{2x - e^{x^2}}{e^{x^2}} dx = \int 2xe^{-x^2} dx - \int 1\, dx$

$u = -x^2$ $\quad -x$

$du = -2x$

a) $-e^{-x^2} + c$

b) $e^{x^2} + c$

c) $x + e^{-x^2} + c$

d) $-x + e^{-x^2} + c$

e) $-x - e^{-x^2} + c$

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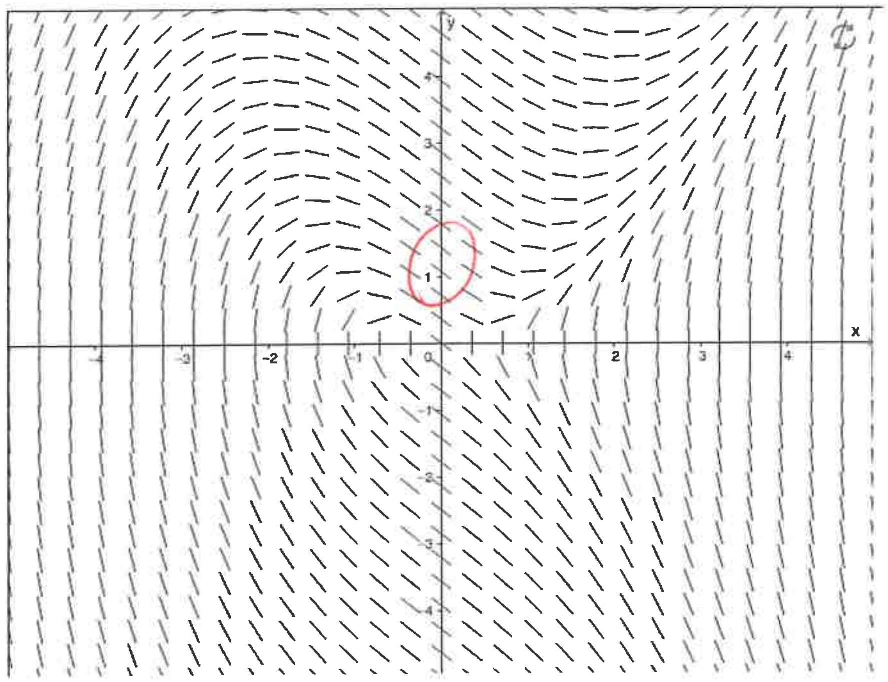
7. Which of the following differential equations best matches this slope field?



a) $\frac{dy}{dx} = 1 - \frac{x^2}{y}$

$x=0$ $\frac{dy}{dx}$ EXISTS

~~b)~~ $\frac{dy}{dx} = 1 - \frac{y^2}{x}$

$(0,1) \rightarrow \frac{dy}{dx} < 0$

c) $\frac{dy}{dx} = \frac{x^2}{y} - 1$

~~d)~~ $\frac{dy}{dx} = \frac{y^2}{x} - 1$

8. For $\int \sin x \cos^2 x \, dx$, the correct u-substitution is

a) $u = \sin x$

b) $u = \cos x$

c) either $u = \sin x$ or $u = \cos x$

d) neither $u = \sin x$ nor $u = \cos x$

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9. A particle moves along the x-axis with velocity at any time t given as $v(t) = \sin 2t$. If the particle's initial position is 0 when $t = 0$, what is the position function?

$\int \sin 2t \, dt = \frac{1}{2}\int \sin 2t \,(2dt)$

a) $x(t) = 2\cos 2t$

b) $x(t) = -2\cos 2t + 2$

$\frac{1}{2}\int \sin u \, du = -\cos u$

c) $x(t) = \frac{1}{2}\cos 2t$

d) $x(t) = -\frac{1}{2}\cos 2t + \frac{1}{2}$

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Part II: Free Response (35 minutes) – Show all work.

1a. $$\int\left(3x^5+3^x-\frac{1}{\sqrt[5]{x^3}}+\frac{1}{3x^5}\right)dx$$

$= \int 3x^5dx + \int 3^x dx - \int x^{-3/5}dx + \frac{1}{3}\int x^{-5}dx$

$= \frac{1}{2}x^6 + \frac{3^x}{\ln 3} - \frac{5}{2}x^{2/5} - \frac{1}{12}x^{-4} + C$

1b. $\int e^{3x}\sec^2\left(e^{3x}\right)dx$

$u = e^{3x}$

$du = e^{3x}(3dx$

$= \frac{1}{3}\int \sec^2 u\,du$

$= \frac{1}{3}\tan u + C$

$= \frac{1}{3}\tan e^{3x} + C$

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2. The acceleration of a particle is described by $a(t) = 18e^{3t}$. Find the distance equation for $x(t)$ if $v(0) = -1$ and $x(0) = 1$.

$v(t) = \int 18e^{3t}\,dt =$

$= \int 6e^{3t}(3dt)$

$= 6\int e^{u}\,du$

$= 6e^{u} + c = 6e^{3t} + C_1$

$(0,-1) \Rightarrow -1 = 6e^{0} + c_1$

$-6 \quad -6$

$-7 = c_1$

$* \; v(t) = 6e^{3t} - 7$

$x(t) = \int (6e^{3t} - 7)\,dt$

$= 2\int e^{3t}(3dt) + \int(-7)\,dt$

$= 2e^{3t} - 7t + C_2$

$(0,1) \Rightarrow 1 = 2e^{0} - 0 + c_2$

$-1 = c_2$

$x(t) = 2e^{3t} - 7t - 1$

3. $\displaystyle\int\left(\frac{1}{(9+x)^2}+\frac{1}{9+x^2}+\frac{x}{9+x^2}\right)dx$

$u=9+x$

$du=dx$

$$=\int\frac{1}{(9+x)^2}\,dx+\int\frac{1}{9+x^2}\,dx+\int\frac{x\,dx}{9+x^2}$$

$u=9+x^2 \quad du=2x\,dx$

$$=\int\frac{1}{u^2}\,du+\frac{1}{3}\tan^{-1}\frac{x}{3}+\frac{1}{2}\int\frac{2x\,dx}{9+x^2}$$

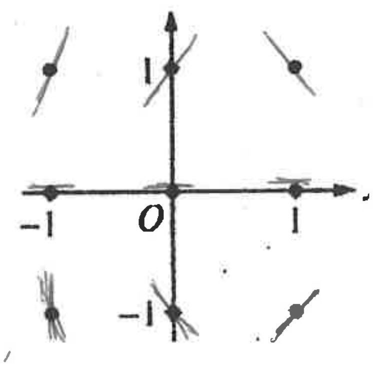
$$=\int u^{-2}\,du+\frac{1}{3}\tan^{-1}\frac{x}{3}+\frac{1}{2}\int\frac{1}{u}\,du$$

$$=\frac{u^{-1}}{-1}+\frac{1}{3}\tan^{-1}\frac{x}{3}+\frac{1}{2}\ln|u|+C$$

$$=\frac{-1}{9+x}+\frac{1}{3}\tan^{-1}\frac{x}{3}+\frac{1}{2}\ln|9+x^2|+C$$

4. Given the differential equation, $\frac{dw}{dt} = w(1-2t)$.

a. On the axis system provided, sketch the slope field for the $\frac{dw}{dt}$ at all points plotted on the graph.



b) Find the particular solution $w = f(t)$ that passes through $(0,\ 2)$.

$\frac{1}{w}\,dw = (1-2t)\,dt$

$\ln|w| = t - \cancel{2}t^2 + C$

$W = Ke^{t-t^2}$

$(0,2) \Rightarrow K=2$

$w = 2e^{t-t^2}$