AP Calculus AB '23-24
Definite Integral Test – Dr Quattrin
Form H
20 minutes
NO CALCULATOR ALLOWED

Name Solution Key

Score____________

1. Find $\int_{-3}^{0} x^2\,dx$ $= \left.\frac{x^3}{3}\right]_{-3}^{0} = 0-(-9)$

a) -18 b) -9 c) 0 d) 9 e) 18

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2. $\displaystyle\int_0^1 \frac{x^3dx}{(x^4+1)^2}$

$u = x^4+1$ $u(1)=2$
$du = 4x^3dx$ $u(0)=1$

$= \frac{1}{4}\int_1^2 u^{-2}\,du = \left.\frac{1}{4}\frac{u^{-1}}{(-1)}\right]_1^2 = -\frac{1}{8}-\left(-\frac{1}{4}\right)$

a) $\frac{1}{8}$ b) $\frac{1}{32}$ c) $\frac{1}{4}\ln 2$ d) 1 e) 16

$\int_{-2}^{5} f(x)dx = -2$	$\int_{1}^{5} f(x)dx = 3$
$\int_{-2}^{1} g(x)dx = 4$	$\int_{5}^{1} g(x)dx = 9$

3. Based on the information above, $\int_{-2}^{5}[f(x)+g(x)]dx =$ $(-2)+(-5)$

a) 11 b) -7 c) 3 d) 0 e) 14

$\int_{-2}^{5} g(x) = \int_{-2}^{1} g - \int_{1}^{5} g$

$4 - 9 = -5$

4. $\int_0^1 x^2 \cos x^3 \, dx$

$u=x^3$ $\quad$ $du=3x^2$ $\quad$ $u(0)=0$ $\quad$ $u(1)=1$

$\frac{1}{3}\int_0^1 \cos u \, du = \frac{1}{3} \sin u \Big]_0^1$

$= \frac{1}{3}\sin 1 - 0$

a) $-\frac{1}{3}\sin(1)$ $\qquad$ b) $-\frac{1}{3}\sin(1)-\frac{1}{3}$

c) $-\frac{1}{3}\sin(1)+\frac{1}{3}$ $\qquad$ d) $\frac{1}{3}\sin(1)$

e) $\frac{1}{3}\sin(1)-\frac{1}{3}$

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5. For $t \geq 0$ hours, H is a differentiable function of t that gives the rate of change in temperature, in degrees Celsius per hour, at an Arctic weather station. In what units would $\int_0^t H(x)dx$ be measured?

$\int \frac{C^\circ}{HR} \cdot dHR = C^\circ$

a) degrees Celsius
b) degrees Celsius per hour
c) degrees Celsius per hour per hour.
d) hours per degrees Celsius
e) hours

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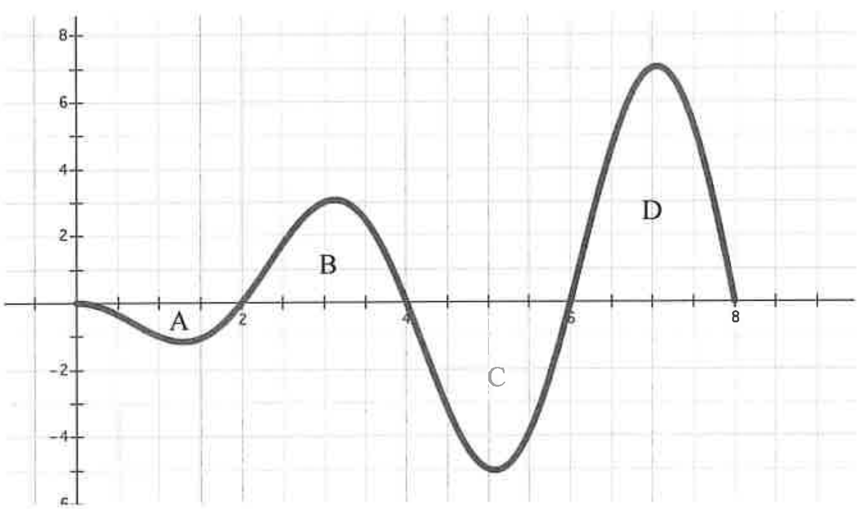
6. The basement of a house is flooding. The water pours in at a rate of $f(t)$ gallons per hour and is being pumped out at a rate of $r(t)$. When the pump is started, at time t = 0, there are 1200 gallons of water in the basement. Which of the following expresses the rate of change in the number of gallons of water in the basement at *t* hours?

a) $1200+\int_0^t\left[f(x)-r(x)\right]dx$ b) $\int_0^t\left[f(x)-r(x)\right]dx$

c) $f(t)-r(t)$ d) $\frac{1}{t}\int_0^t\left[f(x)-r(x)\right]dx$

e) $f'(t)-r'(t)$



7. In the above graph, A, B, C, and D are positive numbers that represent the areas between the curve $y = f(x)$ and the *x*-axis. If A=4, B=8, C=11, and D=15, then $\int_2^8 f(x)\,dx =$ $B - C + D = 8 - 11 + 15$

a) 2 b) 4 c) 8 d) $\frac{19}{4}$ e) 12

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8. Find the average rate of change of $y = x^2 + 5x + 14$ on $x \in [-1, 2]$

a) 3 b) 6 c) 9 d) $\frac{65}{6}$ e) 17.5

$\frac{y(2) - y(-1)}{2 - (-1)} = \frac{28 - 10}{2 - (-1)}$

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t in hours	0	12	24	36	48
$v(t)$ in km/hr	21	26.3	31.4	36.8	41.5

9. A Gravitational Slingshot Effect is sometimes used by space probes like Voyager 2 in order to increase its velocity without expending fuel. By flying close to the planet Saturn in a parabolic arc, the velocities on the table above were achieved by a probe. Which of the following is the setup for a right-hand Riemann sum which approximates $\int_0^{48} v(t)\,dt$

a) $12[\cancel{21} + 26.3 + 31.4 + 36.8 + 41.5]$

b) $12[26.3 + 31.4 + 36.8 + 41.5]$

c) $12[\cancel{21} + 26.3 + 31.4 + 36.8]$

d) $\cancel{24[26.3 + 36.8]}$

e) $12\left[\frac{21 + 26.3}{2} + \frac{26.3 + 31.4}{2} + \frac{31.4 + 36.8}{2} + \frac{36.8 + 41.5}{2}\right]$

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55 minutes
Calculator required.

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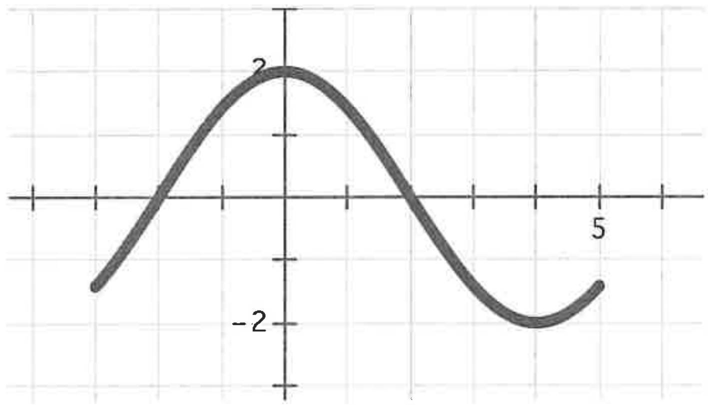

1. Consider the function $g(x) = 2\cos\left(\frac{\pi}{4}x\right)$ which has the graph above.

2

a) Show the u-sub setup for $\int_{-3}^{5} g(x)\,dx$ and solve it by calculator.

$u = \frac{\pi}{4}x$

$du = \frac{\pi}{4}dx$

$u(-3) = \frac{-3\pi}{4}$

$u(5) = \frac{5\pi}{4}$

$= \frac{8}{\pi}\int_{-3\pi/4}^{5\pi/4} \cos u\,du = 0$

1

b) Set up, but do not solve, an integral expression to determine the area between $y = g(x)$ and the x-axis on $x \in [-3, 5]$.

$A = -\int_{-3}^{-2} g(x)\,dx + \int_{-2}^{2} g(x)\,dx - \int_{2}^{5} g(x)\,dx$

The Alcohol Metabolization Problem

2. Metabolism is the body's process of converting ingested substances to other compounds. Alcohol is absorbed into the blood stream, then, through oxidation, it is detoxified and removed from the blood. Research shows that, after consuming three shots of alcohol in rapid succession, an average (180-lb) adult, fasting male has the alcohol metabolized at a rate modeled by $R(t)=0.04\left(t^2-8t+6\right)e^{-t}$, where $R(t)$ is the measured in percentage of alcohol in the blood stream per hour and t is measured in hours after consuming the third drink. The equation is valid for $0 \le t \le 6$.

(a) How much alcohol is in the blood stream at $t = 2.5$ hours?

1

$\int_0^{2.5} R(t)\,dt = .028$ or $.029$

(b) At $t=2$ hours, $R(t)$ is negative, meaning the amount of alcohol in the blood stream in decreasing. Is the amount decreasing at an increasing or a decreasing rate? Using the correct units, explain your answer.

2

$R'(2) = .011 > 0$

$\therefore$ $R(t)$ IS DECREASING AT AN INCREASING RATE

OF .011 %/HR/HR.

(c) A person is considered impaired when the BAC reaches 0.05. The average man in this study reached impairment 17 minutes after starting to drink. Is he still impaired 1.5 hours after drinking? Justify your answer.

$$\int_0^{1.5} R(t)\,dt = .060 > .05$$

3

$\therefore$ YES, HIS IS STILL IMPAIRED.

(d) Find $\frac{1}{6}\int_0^6 R(t)\,dt$. Using the correct units, explain your answer.

3

$$\frac{1}{6}\int_0^6 R(t)\,dt = 0$$

THE AVERAGE RATE OF CHANGE OF THE PERSONS BAC IS 0 %/HOUR.

The Squaw Valley Snowfall Problem

t in days	1	3	4	7	11	15	21
$S(t)$ in inches per day	3	7	6	2	7	2	4
$M(t)$ in inches per day	0.8	0.4	0.2	0	0	0.4	0.8

3. The snowfall in Squaw Valley is tracked by the US Weather Service. For the month of March, 2021, the values on the table above were gathered. $S(t)$ represents the rate of snowfall in inches per day, $M(t)$ represents the rate at which the snow melts in inches per day, and t is measured in days.

(a) Approximate $S'(5)$. Using the correct units, explain $S'(5)$ in context of the problem.

$S'(5) \approx \frac{6-2}{4-7} = -\frac{4}{3}$ IN/DAY^2

2

THE SNOW ~~LEVEL~~ FALL RATE OF CHANGE ~~IS~~ IS DECREASING BY 4/3 IN/DAY/DAY AT $t=5$ DAYS.

(b) Using a Midpoint Reimann Sum, find $\int_1^{21} S(t)dt$. Using the correct unots, explain $\int_1^{21} S(t)dt$.

2 $\int_1^{21} S(t)dt \approx 3(7) + 7(2) + 10(2) = \cancel{62}\ 55$ INCHES

APPROXIMATELY ~~62~~ 55 INCHES OF SNOW HAS FALLEN ON SQUAW VALLEY DURING THESE 21 DAYS

(c) Set up a left-hand Reimann Sum to approximate $\int_1^{21} M(t)dt$. Using the correct units, explain the meaning of $\frac{1}{21-1}\int_1^{21} M(t)dt$.

3 $\int_1^{21} M(t) \approx 2(.8)+1(.4)+3(.2)+4(0)+4(0)+6(.4) = 5$ IN

$\frac{1}{21-1}\int_1^{21} M(t)\,dt$ IS THE AVERAGE RATE IN INCHES PER DAY ~~AMOUNT OF INCHES~~ OF SNOW THAT MELTS ~~PER DAY~~ FROM $t=1$ TO $t=21$

(d) If there were 11 inches of snow on the ground at the beginning of Day 1, how much snow is on the ground on Day 21? Indicate the units.

2 AMOUNT $= 11 + \int_1^{21} S(t) - M(t)\,dt$

$= 11 + 65 - 5$

$= 67$ INCHES OF SNOW