AB Calculus '23-24 Name ______________________________
Dx Apps I Test Form H
No Calculator

1. On which of the following interval(s) is the function $y = -\frac{t^3}{3} + 3t^2 - 5t$ both decreasing and concave down?

a) $(-\infty, 1)$ b) $(1,5)$ c) $(3,\infty)$ d) $(3,5)$ e) $(5,\infty)$

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2. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that they have values given on the table below.

x	$f'(x)$	$f''(x)$	$g'(x)$	$g''(x)$
2	-1	2	-8	-5
4	8	-11	4	3
8	-3	-12	-1	4

Then at $x = 4$, $g(x)$ is

a) increasing and concave down
b) increasing and concave up
c) decreasing and concave down
d) decreasing and concave up

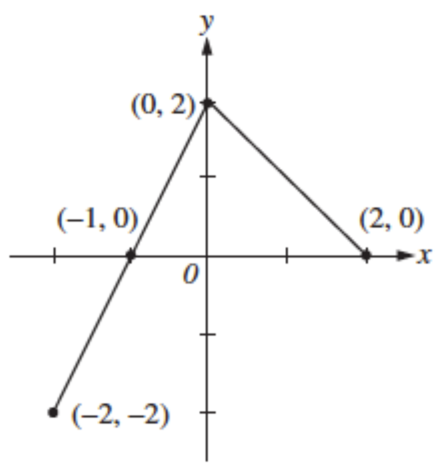
3. Suppose $f'(x) = \frac{(x+1)^2(x-4)^5}{(x^2+4)}$. Which of the following statements must be **true**?

a) $f(x)$ has a point of inflection at $x = -1$

b) $f(x)$ is increasing on $x \in (-\infty, -1)$

c) $f(x)$ has a relative maximum at $x = 4$

d) $f(x)$ has a relative minimum at $x = -1$



Graph of *f*

4. The graph of the function *f* shown above consists of two line segments. If *g* is the function defined by $g(x) = \int_{-1}^{x} f(t)\, dt$, then the maximum value of $g(x)$ occurs at $x =$

a) -2 b) -1 c) 0 d) 1 e) 2

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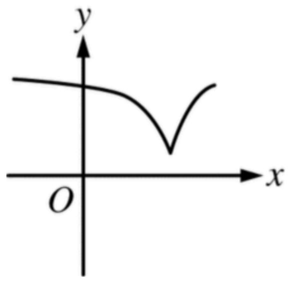
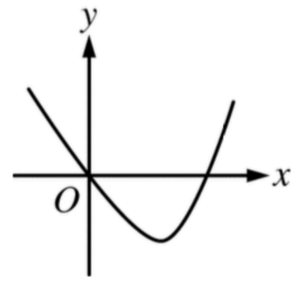
5. The function f is differentiable and increasing for all real numbers x, and the graph of f has exactly one point of inflection. Of the following, which could be the graph of $f'(x)$, the derivative of f?

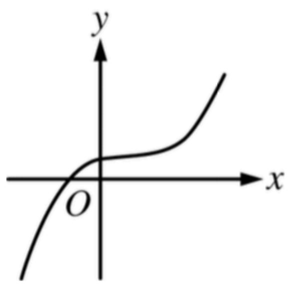
(A)



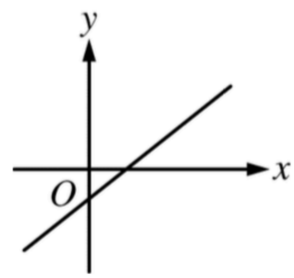
(B)



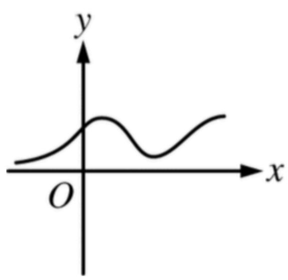
(C)



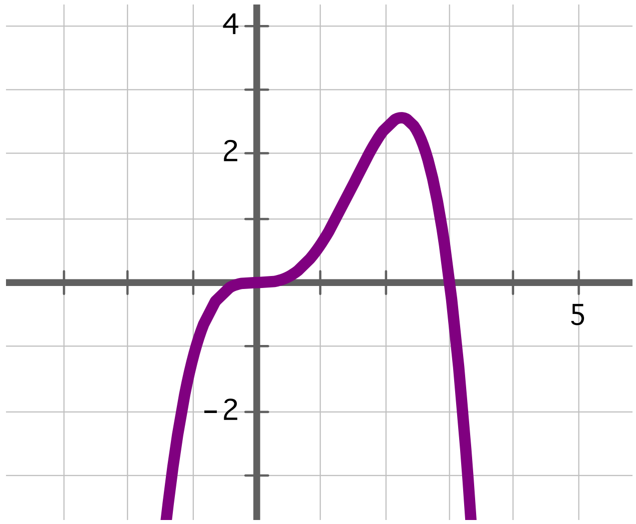
(D)



(E)



6. The graph below is of $g''(x)$, the **second** derivative of $g(x)$. Which of these statements is **false** about $g(x)$?



a) $g(x)$ is concave up on the interval $(0,\ 3)$

b) $g(x)$ has a relative maximum at $x = 3$

c) The derivative of $g(x)$ is increasing on $(2,\ 3)$

d) $g(x)$ has a point of inflection at $x = 0$

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7. Given $g(t) = t\sqrt{t+6}$ on $x \in [0,\ 10]$ is both continuous and differentiable, the Mean Value Theorem guarantees that $g'(t) =$

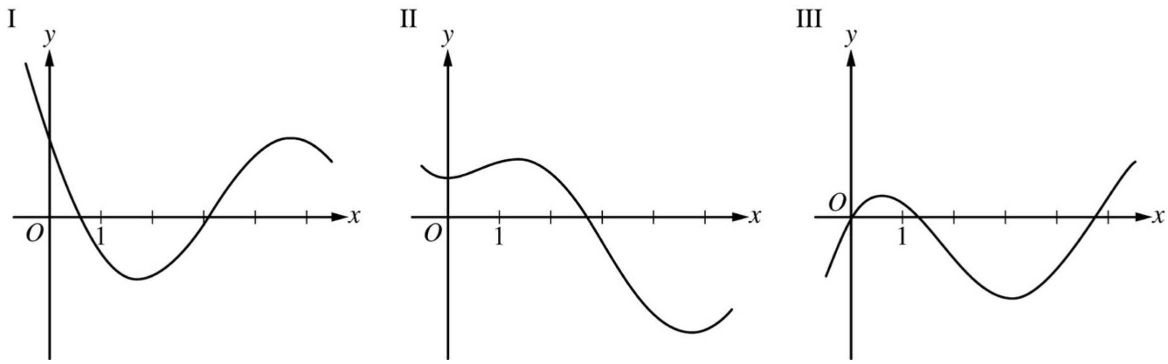
a) 0

b) $2\sqrt{2}$

c) $-2\sqrt{2}$

d) -4

e) 4



8. Three graphs labeled I, II, and III are shown above. One is the graph of $f(x)$, one is the graph of $f'(x)$, and one is the graph of $f''(x)$. Which of the following correctly identities each of the three graphs?

a) $f(x) = \text{I},\ f'(x) = \text{II},\ f''(x) = \text{III}$
b) $f(x) = \text{II},\ f'(x) = \text{I},\ f''(x) = \text{III}$
c) $f(x) = \text{II},\ f'(x) = \text{III},\ f''(x) = \text{I}$
d) $f(x) = \text{III},\ f'(x) = \text{I},\ f''(x) = \text{II}$
e) $f(x) = \text{III},\ f'(x) = \text{II},\ f''(x) = \text{I}$

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9. Find the absolute minimum value of $y = 4x - x^2$ on $0 \le x \le 3$.

a) -2 b) 0 c) 2 d) 4 e) 18

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10. Suppose $f'(x) = (1-x)(3-x)^4(x-5)^3$. Of the following, which best describes the graph of $f(x)$?

a) $f(x)$ has relative minimum at $x = 1$, a relative maximum at $x = 3$, and a points of inflection at $x = 5$

b) $f(x)$ has relative minimum at $x = 3$, a relative maximum at $x = 1$, and a points of inflection at $x = 5$

c) $f(x)$ has relative minimum at $x = 5$, a relative maximum at $x = 3$, and a points of inflection at $x = 1$

d) $f(x)$ has relative minimum at $x = 1$, a relative maximum at $x = 5$, and a points of inflection at $x = 3$

e) $f(x)$ has relative minimum at$x = 3$, a relative maximum at$x = 5$, and a points of inflection at $x = 1$

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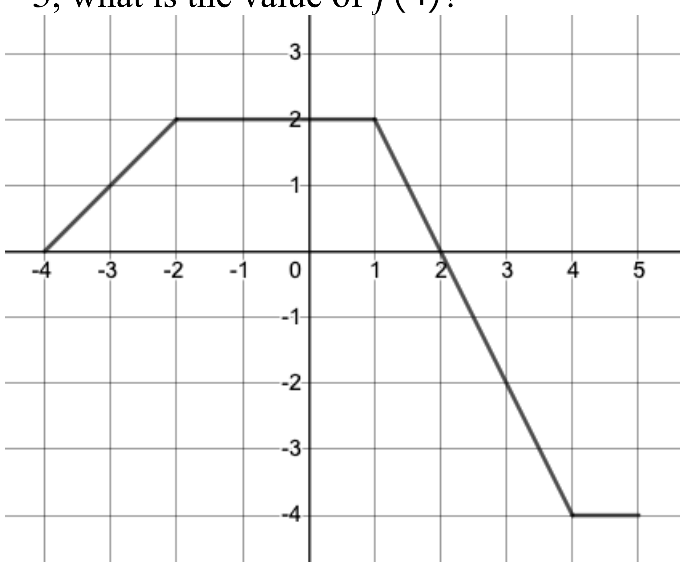
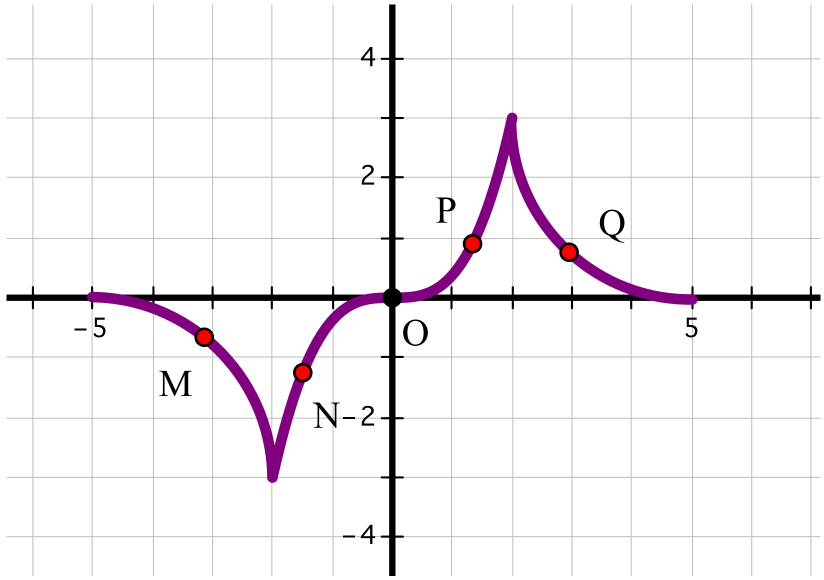
11. The graph below gives the graph of$f'(x)$, the derivative of $f(x)$. If it is known that $f(-2) = 3$, what is the value of $f(4)$?



graph of f'(x)

a) 3 b) 4 c) 6 d) 7 e) 9



12. At what point on the above curve is $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} < 0$

a) M b) N c) P d) Q

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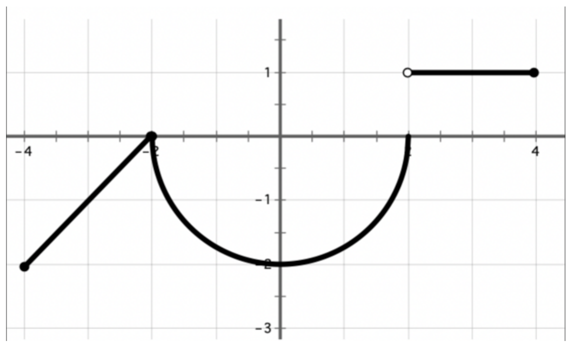
Calculator allowed

Name ____________________________

Score___________

Directions: Show all work.

1. Let $h(x) = 1 + \int_0^t f(t)\, dt$ on $x \in [-4,\ 4]$. Let the graph of be comprised of one semicircle and two line segments as shown below.



(a) Find $h(2)$, $h'(2)$, and $h''(2)$.

(b) Find the equation of the line tangent to $h(x)$ at $x = 0$.

(c) At what x-values is $h(x)$ decreasing and concave up? Justify your answer.

(d) What is the absolute maximum value of $h(x)$ on the interval $x \in [-4, 4]$?

2. The desalting plant at Yuma, AZ, removes alkaline (salt) products from the Colorado River the make the water better for irrigation downstream in Mexico. Data from a Pilot Run of the plant shows that water enters the plant at a rate $W(t)$ as shown on the table below:

t in Month	0	1	2	3	4	5	6	7	8	9	10
$W(t)$ in foot-acres per month	0	2375	3189	3411	3207	2169	2269	2151	2167	3022	2293

The rate $P(t)$ of outflow of processed water, in foot-acre per month is modeled by

$$P(t) = -0.55t^4 + 15t^3 - 158t^2 + 722t + 1032$$

For $0 \le t \le 10$. Based on supplies available, not all the water gets processed before returning to the Colorado River.

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a) Using a Midpoint Reiman Sum, approximate the volume of water that enters the plant during these ten months.

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b) Set up an equation for $U(t)$ which would define the amount of unprocessed water that exits the plant. Using your answer in part a), approximate $U(10)$. Indicate units.

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c) Approximate $W'(6)$. Using the correct units, explain the meaning of your answer.

d) Assuming $W(t)$ can be modeled by $E(t) = 2800 + 750\sin\left(\frac{2\pi}{11}t\right)$, find the time at which there is an absolute maximum amount of unprocessed water flowing through the plant for $0 \leq t \leq 10$. Justify your answer.