AB Calculus '23-24
Dx Apps I Test Form H
No Calculator

Name Solution Key

1. On which of the following interval(s) is the function $y = -\frac{t^3}{3} + 3t^2 - 5t$ both decreasing and concave down?

$y' < 0$ $y'' < 0$

$y' = -t^2 + 6t - 5$ $y'' = -2t + 6$

a) $(-\infty, 1)$ b) $(1,5)$ c) $(3,\infty)$ d) $(3,5)$ e) $(5,\infty)$

y': $-$ 1 $+$ 5 $-$ y'': $+$ 3 $-$

2. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that they have values given on the table below.

x	$f'(x)$	$f''(x)$	$g'(x)$	$g''(x)$
2	-1	2	-8	-5
4	8	-11	4	3
8	-3	-12	-1	4

Then at $x = 4$, $g(x)$ is

a) increasing and concave down

b) increasing and concave up

c) decreasing and concave down

d) decreasing and concave up

3. Suppose $f'(x)=\dfrac{(x+1)^2(x-4)^5}{(x^2+4)}$. Which of the following statements must be **true**?

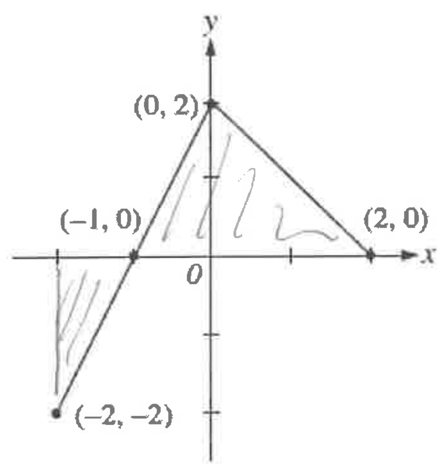
f' − 0 − 0 +
−1 4

a) $f(x)$ has a point of inflection at $x=-1$

b) $f(x)$ is increasing on $x\in(-\infty,-1)$

c) $f(x)$ has a relative maximum at $x=4$

d) $f(x)$ has a relative minimum at $x=-1$



Graph of f

$g'=f$

f − 0 + MAX
−2 MAX −1 MIN 2

4. The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x)=\int_{-1}^{x} f(t)\,dt$, then the maximum value of $g(x)$ occurs at $x=$

a) -2 b) -1 c) 0 d) 1 e) 2

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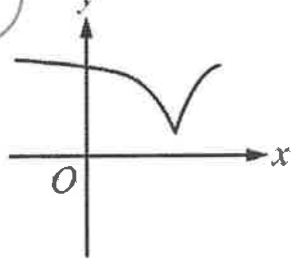
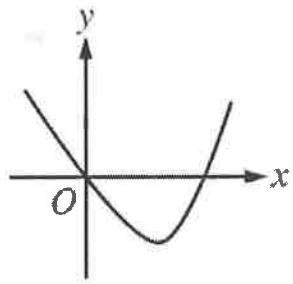
5. The function f is differentiable and increasing for all real numbers x, and the graph of f has exactly one point of inflection. Of the following, which could be the graph of $f'(x)$, the derivative of f?

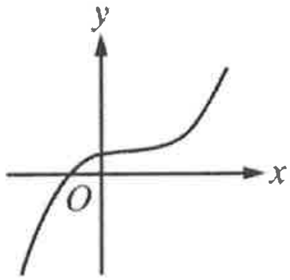
(A)



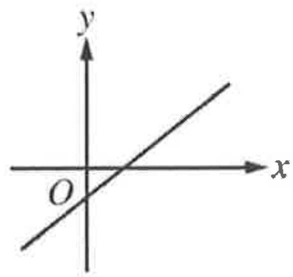
(B)



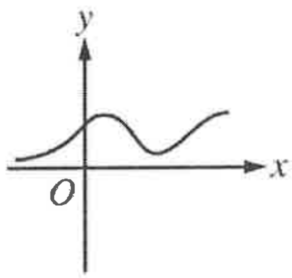
(C)



(D)



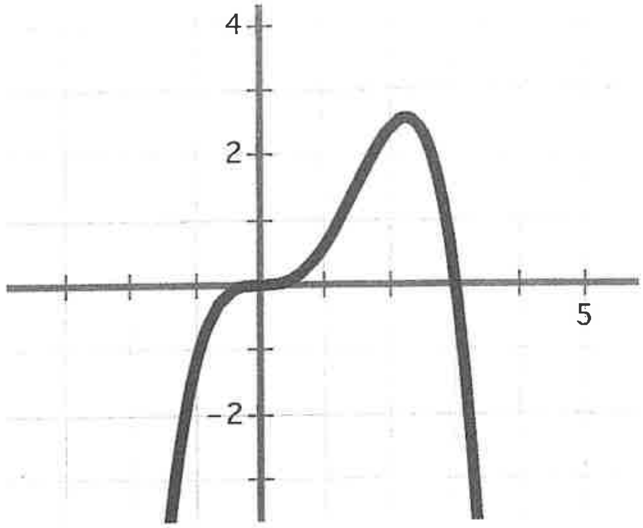
(E)



f INCREASING → f' POSITIVE

ONE POI MEANS f' HAS ONE EXT

6. The graph below is of $g''(x)$, the **second** derivative of $g(x)$. Which of these statements is **false** about $g(x)$?



a) g(x)is concave up on the interval $(0,\ 3)$ T

b) $g(x)$ has a relative maximum at $x = 3$ F

c) The derivative of $g(x)$ is increasing on$(2,\ 3)$ T

d) $g(x)$ has a point of inflection at $x = 0$ T

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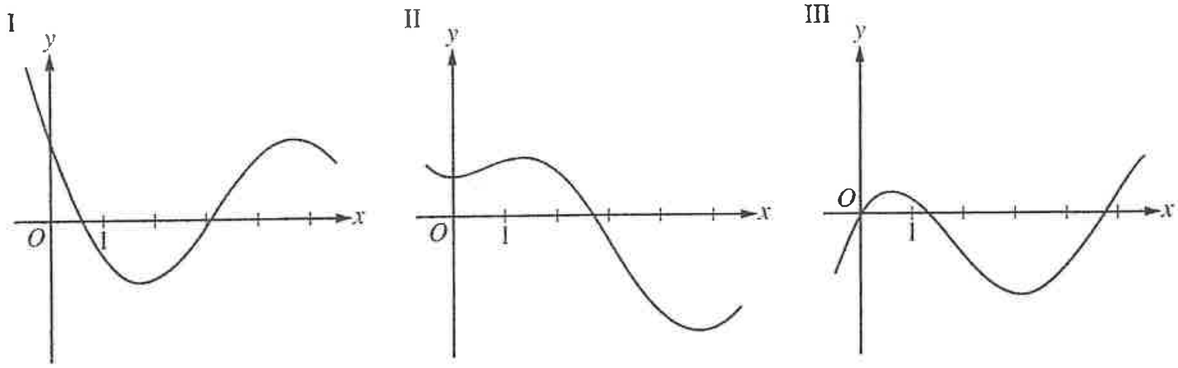
7. Given $g(t) = t\sqrt{t+6}$ on $x \in [0,\ 10]$ is both continuous and differentiable, the Mean Value Theorem guarantees that $g'(t) =$

a) 0 b) $2\sqrt{2}$ c) $-2\sqrt{2}$

d) -4 e) 4

$$g'(c) = \frac{g(10) - g(0)}{10-0} = \frac{40-0}{10-0} = 4$$



8. Three graphs labeled I, II, and III are shown above. One is the graph of $f(x)$, one is the graph of $f'(x)$, and one is the graph of $f''(x)$. Which of the following correctly identities each of the three graphs?

a) $f(x)=\text{I},\ f'(x)=\text{II},\ f''(x)=\text{III}$
b) $f(x)=\text{II},\ f'(x)=\text{I},\ f''(x)=\text{III}$
c) $f(x)=\text{II},\ f'(x)=\text{III},\ f''(x)=\text{I}$
d) $f(x)=\text{III},\ f'(x)=\text{I},\ f''(x)=\text{II}$
e) $f(x)=\text{III},\ f'(x)=\text{II},\ f''(x)=\text{I}$

$\text{III}' = \text{I}$

$\text{II}' = \text{III}$

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9. Find the $\boxed{\text{absolute}}$ minimum value of $y=4x-x^2$ on $0 \le x \le 3$.

a) -2 b) 0 c) 2 d) 4 e) 18

$\frac{dy}{dx} = \cancel{4x}\ 4-2x=0$

$x=2$

cv	y
0	0
2	4
3	3

EXT POI EXT

10. Suppose $f'(x)=(1-x)(3-x)^4(x-5)^3$. Of the following, which best describes the graph of $f(x)$?

a) $f(x)$ has relative minimum at $x=1$, a relative maximum at $x=3$, and a points of inflection at $\cancel{x=5}$

b) $f(x)$ has relative minimum at $x=3$, a relative maximum at $x=1$, and a points of inflection at $\cancel{x=5}$

c) $f(x)$ has relative minimum at $x=5$, a relative maximum at $x=3$, and a points of inflection at $x=1$

d) $f(x)$ has relative minimum at $x=1$, a relative maximum at $x=5$, and a points of inflection at $x=3$

e) $f(x)$ has relative minimum at $x=3$, a relative maximum at $x=5$, and a points of inflection at $x=1$

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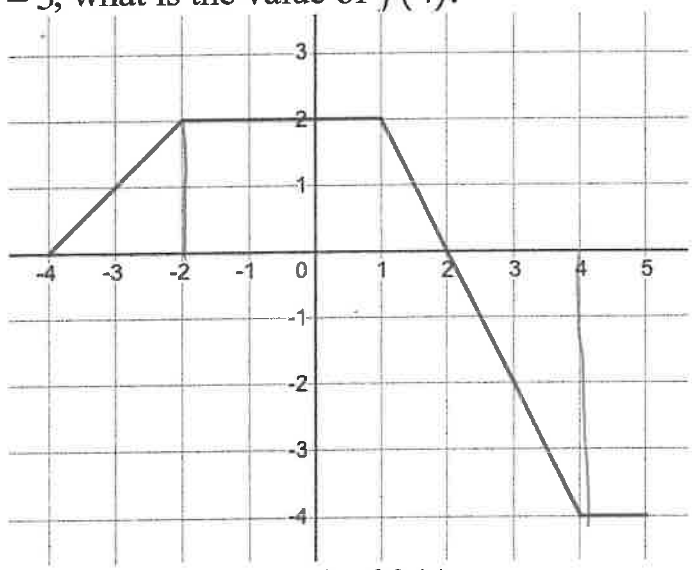
11. The graph below gives the graph of $f'(x)$, the derivative of $f(x)$. If it is known that $f(-2)=3$, what is the value of $f(4)$?

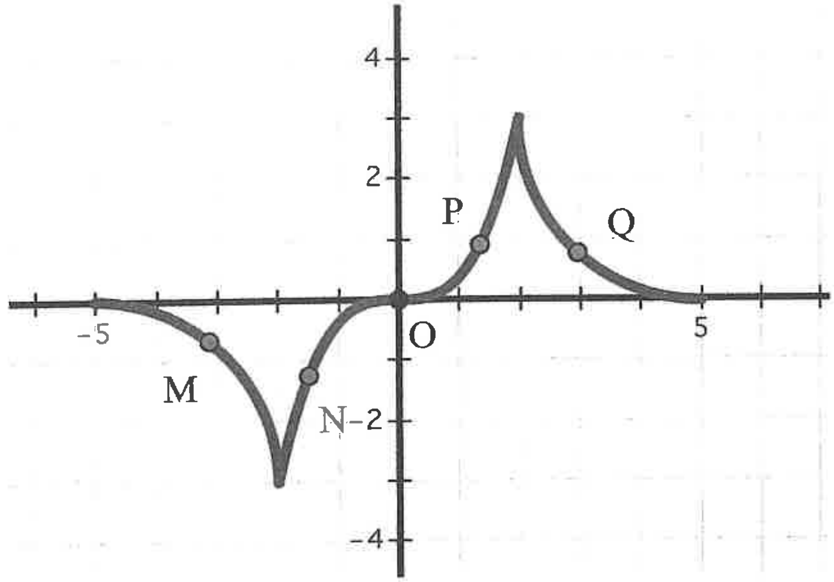


graph of f'(x)

$f(4) = 3 + \int_{-2}^{4} f(t)\,dt$

$= 3 + 7 - 4$

a) 3 b) 4 c) 6 d) 7 e) 9



12. At what point on the above curve is $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} < 0$

DEC CON ↓

a) M b) N c) P d) Q

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Calculator allowed

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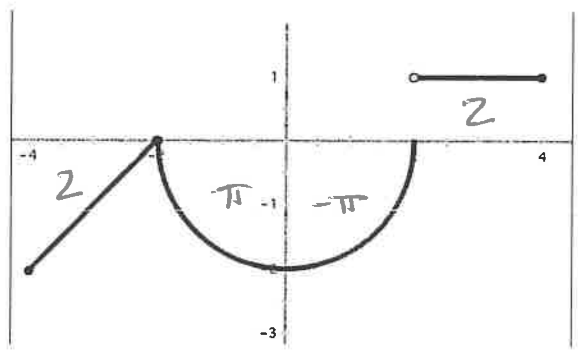
Score__________

Directions: Show all work.

1. Let $h(x) = 1 + \int_0^{t} f(t)\,dt$ on $x \in [-4,\ 4]$. Let the graph of be comprised of one semicircle and two line segments as shown below.



(a) Find $h(2)$, $h'(2)$, and $h''(2)$.

3

$h(2) = 1 + \int_0^2 f = 1 + (-\pi) = 1 - \pi$

$h'(2) = f(2) = 0$

$h''(2) = f'(2) = \text{DNE}$

2 (b) Find the equation of the line tangent to $h(x)$ at $x = 0$.

$y - 1 = -2(x - 0)$

(c) At what x-values is $h(x)$ decreasing and concave up? Justify your answer.

2

$h' = f$ h IS DEC & CON ↑ → f IS NEG & INC

$x \in (-4, -2) \cup (0, 2)$

2 (d) What is the absolute maximum value of $h(x)$ on the interval $x \in [-4, 4]$?

CV AT $x = \pm 4, 2$ MAX @ $x = 4$ ~~AND~~ -4

~~$x = 2$ IS THE ONLY MAX~~ $h(4) = 2 - \pi$

~~$h(2) = 1 - \pi$~~ $h(-4) = \pi + 2$

MAX IS $\pi + 2$

2. The desalting plant at Yuma, AZ, removes alkaline (salt) products from the Colorado River the make the water better for irrigation downstream in Mexico. Data from a Pilot Run of the plant shows that water enters the plant at a rate $W(t)$ as shown on the table below:

t in Month	0	1	2	3	4	5	6	7	8	9	10
$W(t)$ in foot-acres per month	0	2375	3189	3411	3207	2169	2269	2151	2167	3022	2293

The rate $P(t)$ of outflow of processed water, in foot-acre per month is modeled by

$$P(t) = -0.55t^4 + 15t^3 - 158t^2 + 722t + 1032$$

For $0 \le t \le 10$. Based on supplies available, not all the water gets processed before returning to the Colorado River.

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a) Using a Midpoint Reiman Sum, approximate the volume of water that enters the plant during these ten months.

1

$\int_0^{10} W(t) \approx 2(2375) + 2(3411) + 2(2169) + 2(2151) + 2(3022)$

$= \cancel{[illegible]}$ 26,256 FOOT-ACRES

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b) Set up an equation for $U(t)$ which would define the amount of unprocessed water that exits the plant. Using your answer in part a), approximate $U(10)$. Indicate units.

2

$u(t) = \int_0^t W(x) - P(x)\,dx = \int_0^{10}$

$u(10) = \int_0^{10} w(x) - \int_0^{10} P(x)\,dx = 26456 - 20253.333$

$= 6202.667$ FOOT-ACRES

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c) Approximate $W'(6)$. Using the correct units, explain the meaning of your answer.

2

$W'(6) \approx \frac{2151-2169}{7-5} = -9 \frac{\text{FT-ACRES}}{\text{MO}^2}$

THE RATE AT WHICH WATER IS ENTERING THE PLANT IS DECREASING BY NINE FT-ACRE-~~FT~~ PER MONTH PER MONTH AT $t=6$ MONTHS

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d) Assuming $W(t)$ can be modeled by $E(t) = 2800 + 750\sin\left(\frac{2\pi}{11}t\right)$, find the time at which there is an absolute maximum amount of unprocessed water flowing through the plant for $0 \le t \le 10$. Justify your answer.

4

$u'(t) = 0 \Rightarrow E = P \rightarrow t = 7.309$ AND 8.735

t	$u(t)$
0	0
7.309	7779.057
8.735	7716.759
10	7955.105

ABS MAX OF UNPROCESSED WATER FLOWING THROUGH THE PLANT OCCURS AT $t = 10$ MONTHS
