AB Calculus '23-24 Name Solution Key

Dr Quattrin – 4th period

Dx Apps II Form H Part I

Calculator allowed Score___________

20 minutes

1. A particle moves along the x-axis and its position for time $t \geq 0$ is $x(t) = \cos(2t) + \sec t$. When $t = \pi$, the acceleration of the particle is

(a) -6 (b) -5 (c) -4 (d) -3 (e) none of these

$v(t) = -2\sin 2t + \sec t \tan t$

$a(t) = -4\cos 2t + \sec^3 t + \sec t \tan^2 t$

$a(\pi) = -4 - 1 + 0 =$

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2. An object moves along the y-axis with coordinate position $y(t)$ and velocity $v(t) = \left(8 - \dfrac{e^{0.47t}}{t+6}\right) - \left(7 - .46t\cos(t)\right)$ for $0 \leq t \leq 10$. At time $t = 3$, the object is

a) moving downward with negative acceleration.

b) moving upward with negative acceleration.

c) moving downward with positive acceleration.

d) moving upward with positive acceleration.

e) at rest.

$v(3) < 0$

$a(3) < 0$

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3. A Golden Rectangle is one where the ratio (called ϕ)of the length to the short side w to the long side l is equal to the ratio of the long side to the sum of the two sides. In other words, $l = 1.236w$. If a Golden Rectangle changes such that w is growing at 2 in/min, how fast is the area changing when w is 5 inches?

a) $1.236\ {}^{\text{in}^2}\!/_{\text{min}}$

b) $12.36\ {}^{\text{in}^2}\!/_{\text{min}}$

c) $2.472\ {}^{\text{in}^2}\!/_{\text{min}}$

d) $24.72\ {}^{\text{in}^2}\!/_{\text{min}}$

e) $30.9\ {}^{\text{in}^2}\!/_{\text{min}}$

$A = 1.236w^2$

$\frac{dA}{dt} = 2.472w \frac{dw}{dt}$

$= 2.472(5)(2)$

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4. The growth rate of a population $y(t)$ of dolphins is modeled by the logistic growth equation $\frac{dy}{dt} = \frac{y}{2}(120 - y)$. If $y(0) = 30$, which of these describes the future behavior of the population?

A

a) The population will increase towards 60 dolphins
b) The population will increase towards 120 dolphins
c) The population will decrease towards 120 dolphins
d) The population will decrease towards 60 dolphins

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5. A particle travels along a straight line with a velocity of $v(t)=\left(8-\frac{e^{0.47t}}{t+6}\right)-(7-.46t\cos(t))$ feet per second. What is the total displacement, in feet, traveled by the particle during the time interval $1 \le t \le 6$ seconds?

a) 0.672 b) 0.877 c) 1.857 d) 4.097

$\int_1^6 v(t)\,dt$

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6. Choose the integral expression that would result in the total distance traveled on the interval [0, 3] if the velocity is given by $v(t)=e^t-6$. $=0$

$e^t=6 \rightarrow t=\ln 6$

(a) $\int_0^{\ln 6}(e^t-6)\,dt-\int_{\ln 6}^{3}(e^t-6)\,dt$ (b) $\int_3^{\ln 6}(e^t-6)\,dt-\int_{\ln 6}^{0}(e^t-6)\,dt$

(c) $\int_0^{\ln 6}(e^t-6)\,dt+\int_{\ln 6}^{3}(e^t-6)\,dt$ (d) $\int_{\ln 6}^{3}(e^t-6)\,dt-\int_0^{\ln 6}(e^t-6)\,dt$

(d) $\int_0^3(e^t-6)\,dt$

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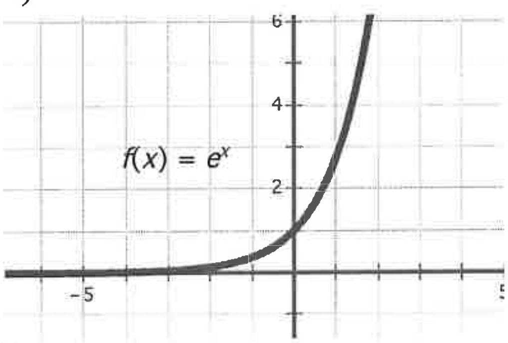
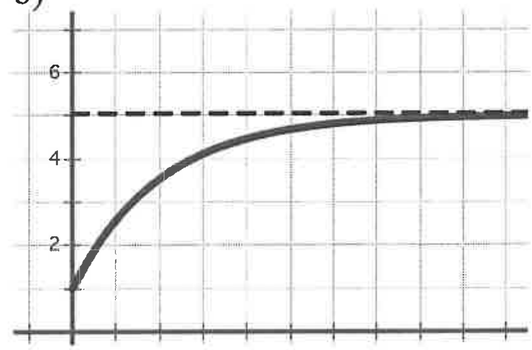
7. According to Newton's Law of Cooling, the temperature of the coffee follows the differential equation $\frac{dy}{dt} = k(y-1)$. Based on this, which of the following might be the graph of the model?

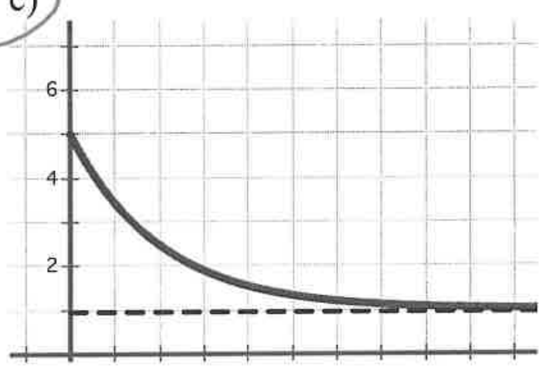
a)



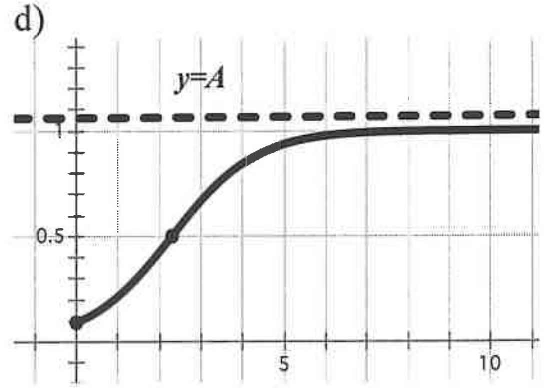
b)



c)



d)



8. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that they have values given on the table below.

x	$f'(x)$	$f''(x)$	$g'(x)$	$g''(x)$
2	0	2	-8	0
4	8	0	0	3
8	0	-12	0	-4

Which of the following statements is true?

~~a)~~ $f(x)$ has a relative minimum at $x=8$ $f'=0$ $f''>0 \to$ MAX

b) $g(x)$ has a relative maximum at $x=4$ $g'(4)=0$ $g''(4)=3$ MAX

c) $f(x)$ has a relative minimum at $x=2$

d) $f(x)$ has a point of inflection at $x=4$

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9. Consider the closed curve in the *x-y* plane given by $2y-x+xy=8$. Which of the following is correct?

a) $\frac{dy}{dx}=\frac{1-y}{2+x}$ b) $\frac{dy}{dx}=\frac{y-1}{2+x}$

c) $\frac{dy}{dx}=\frac{1+y}{2+x}$ d) $\frac{dy}{dx}=-\frac{1+y}{2+x}$

$2\frac{dy}{dx}-1+x\frac{dy}{dx}+y(1)=0$

$(2+x)\frac{dy}{dx}=1-y$

AB Calculus '23-24
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Dx Apps II Form H Part II
Calculator allowed
45 minutes

Name Solution Key

Score 27

1. A particle is moving along the x-axis so that its velocity is given by $v(t) = 2 + 3.6\sqrt{t}\sin t - \frac{e^{0.3t}}{t+4}$ on $0 \le t \le 10$. The position of the particle at $t = 1$ is $x = -4.3$.

a) At what time(s) on $t \in [0, 10]$, if any, does the particle switch directions?

3

$t = 3.389,\ 6.128,\ 9.490$

GRAPH $v(t)$ & FIND ZEROS

b) Find the acceleration equation at $t = 5.4$.

1

$a(5.4) = 4.607$ MATH 8

2 c) What is the total distance traveled by the particle on $t \in [2, 9]$.

$$\int_2^9 |v|\, dt = 38.869$$

3 d) What is the position of the particle on $t = 6.9$?

$$\text{Position} = -4.3 + \int_1^{6.9} v(t)\, dt = -1.511$$

3. A cup of coffee is made with boiling water at a temperature of 100 C°, in a room at temperature 20 C°. After two minutes, it has cooled to 80 C°. According to Newton's Law of Cooling, the temperature of the coffee follows the differential equation

$$\frac{dy}{dt} = k(y-20),$$

where y is the temperature of the coffee at time t minutes.

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a) Find the particular solution to the differential equation.

6

$$\frac{1}{y-20}\,dy = k\,dt$$

$$\ln|y-20| = kt + c$$

$$y - 20 = e^{kt+c} = Ce^{kt}$$

$(0, 100) \Rightarrow \quad C = 80$

$(2, 80) \Rightarrow \quad 60 = 80e^{k(2)}$

$-.144 = k$

$$y = 20 + 80e^{-.144t}$$

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b) What is its temperature after five minutes?

1

$y = 20 + 80e^{-.144(5)}$

$= 58.960°C$

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c) The coffee will be perceived as "cold" when the temperature drops below 40 C°. At what time *t* will this occur?

2

$40 = 20 + 80e^{-.144t}$

$\ln .25 = -.144t$

$t = 9.634$ MINUTES

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3. Consider the curve given by $2x^2 - xy + y^2 = 28$.

2 a) Show that $\frac{dy}{dx} = \frac{4x-y}{x-2y}$.

$$\frac{d}{dx}\left[2x^2 - xy + y^2 = 28\right]$$

$$4x - x\frac{dy}{dx} - y(1) + 2y\frac{dy}{dx} = 0$$

$$(2y-x)\frac{dy}{dx} = y - 4x$$

$$\frac{dy}{dx} = \frac{y-4x}{2y-x} = \frac{4x-y}{x-2y}$$

3 b) Find point(s) P where the tangent line is horizontal.

HORIZONTAL: $\frac{dy}{dx} = 0 \rightarrow y = 4x$

$$2x^2 - x(4x) + (4x)^2 = 28$$

$$14x^2 = 28$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$(\sqrt{2}, 4\sqrt{2})$

$(-\sqrt{2}, -4\sqrt{2})$

c) Find $\frac{d^2y}{dx^2}$. $= \frac{d}{dx}\left[\frac{4x-y}{x-2y}\right]$

2

$$= \frac{(x-2y)\left(4-\frac{dy}{dx}\right) - (4x-y)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)\left(4-\frac{4x-y}{x-2y}\right) - (4x-y)\left(1-2\,\frac{4x-y}{x-2y}\right)}{(x-2y)^2}$$

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2 d) Determine if each value found in b) is at a maximum, a minimum, or neither. Justify your answer.

$$\left.\frac{d^2y}{dx^2}\right|_{(\sqrt{2},\,4\sqrt{2})} = \frac{(-7\sqrt{2})(4-0)-0}{(-7\sqrt{2})^2} < 0 \therefore \text{MAX}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(-\sqrt{2},\,-4\sqrt{2})} = \frac{7\sqrt{2}(4-0)-0}{(7\sqrt{2})^2} > 0 \therefore \text{min}$$
