1.1 Free Response Solutions

1. $f(x) = x^2 + 3x - 4 \to f'(x) = 2x + 3$

3. $y = x^{-2/3} \to \dfrac{dy}{dx} = -\dfrac{2}{3}x^{-5/3}$

5. $v(r) = \dfrac{4}{3}\pi r^3 \to v'(r) = 4\pi r^2$

7. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}} = x^{3/2} + 4x^{1/2} + 3x^{-1/2}$

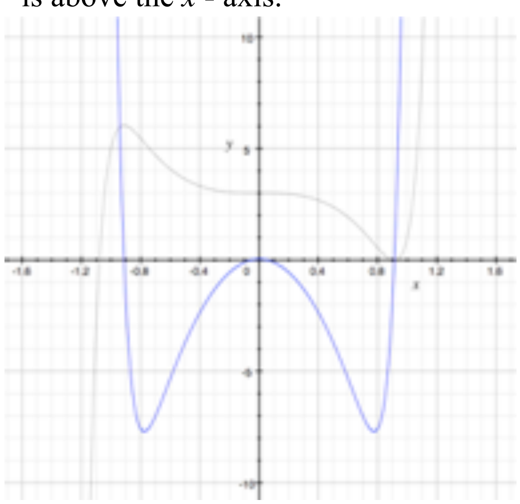
$$\frac{dy}{dx} = \frac{3}{2}x^{1/2} + 2x^{-1/2} - \frac{3}{2}x^{-3/2} = \frac{3\sqrt{x}}{2} + \frac{2}{\sqrt{x}} - \frac{3}{2\sqrt{x^3}}$$

9. $z = \dfrac{A}{y^{10}} + Be^y = Ay^{-10} + Be^y$

$z' = -10Ay^{-11} + Be^y$

11. If $f(x) = 3x^5 - 5x^3 + 3$, find $f'(x)$ $f'(x) = 15x^4 - 15x^2$

You can see that when *f* is decreasing the graph of f' is below the *x* - axis, i.e. f' is negative - if *f* is increasing the graph of f' is above the *x* - axis.



13. $$\frac{d}{dx}\left[x^6 - 3\sqrt[6]{x^7} + 5^x - \frac{1}{\sqrt[3]{x^5}} + \frac{1}{2x}\right]$$
$$= \frac{d}{dx}\left[x^6 - 3x^{7/6} + 5^x - x^{-5/3} + \frac{1}{2}x^{-1}\right]$$
$$= 6x^5 - \frac{7}{2}x^{1/6} + 5^x \ln 5 + \frac{5}{3}x^{-8/3} - \frac{1}{2}x^{-2}$$

15. $$\frac{d}{dx}\left[(x-1)x^{1/2}\right] = \frac{d}{dx}\left[\left(x^{3/2} - x^{1/2}\right)\right] = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$$

17. $$\frac{d}{dx}\left[\left(x^2 - 4x + 3\right)x^{5/2}\right] = \frac{d}{dx}\left[x^{9/2} - 4x^{7/2} + 3x^{5/2}\right] = \frac{9}{2}x^{7/2} - 14x^{5/2} + \frac{15}{2}x^{3/2}$$

19. $$\frac{d}{dy}\left(\frac{4y^3 - 2y^2 - 5y}{\sqrt{y}}\right) = \frac{d}{dy}\left(4y^{5/2} - 2y^{3/2} - 5y^{1/2}\right) = 10y^{3/2} - 3y^{1/2} - \frac{5}{2}y^{-1/2}$$

21. $$\frac{d}{dw}\left(\frac{7w^2 - 4w + 1}{5w^3}\right) = \frac{d}{dw}\left(\frac{7}{5}w^{-1} - \frac{4}{5}w^{-2} + \frac{1}{5}w^{-3}\right) = -\frac{7}{5}w^{-2} + \frac{8}{5}w^{-3} - \frac{3}{5}w^{-4}$$

1.1 Multiple Choice Solutions

1. $f'(x)=\frac{3}{2}x^{1/2}\to f'(4)=\frac{3}{2}(4x)^{1/2}=3$

The correct answer is C.

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3. $f(x)=\frac{1}{2}x^{-1}+x^{-2}\to f'(x)=-\frac{1}{2}x^{-2}-2x^{-3}=-\frac{1}{2x^2}-\frac{2}{x^3}$

The correct answer is A.

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5. If h is the function defined by $h'(x)=e^{5x}(5)+1\to h'(0)=e^{0}(5)+1=6$, then $h'(0)$ is

The correct answer is D.

1.2 Free Response Solutions

1. $\frac{d}{dx}\left[x^3+4x-\pi\right]^{-7}=-7\left(x^3+4x-\pi\right)^{-8}\left(3x^2+4\right)$

3. $f(x)=\sqrt[5]{\left(\frac{1}{x}+2x+e^x\right)^3}=\left(\frac{1}{x}+2x+e^x\right)^{3/5}$

$$f'(x)=\frac{3}{5}\left(x^{-1}+2x+e^x\right)^{-2/5}\left(-x^{-2}+2+e^x\right)=\frac{-3x^{-2}+6+3e^x}{5\left(x^{-1}+2x+e^x\right)^{2/5}}$$

5. If $g(2)=3$ and $g'(2)=-4$, find $f'(2)$ if $f(x)=e^{(g(x))}$.

$f'(x)=e^{g(x)}\cdot g'(x)\Rightarrow f'(2)=e^{g(2)}\cdot g'(2)\Rightarrow f'(2)=e^{3}\cdot -4=-4e^{3}$

7. $\frac{d}{dx}\left[\sqrt{3x^2-4x+9}\right]=\frac{d}{dx}\left[\left(3x^2-4x+9\right)^{1/2}\right]$

$$=\frac{1}{2}\left(3x^2-4x+9\right)^{-1/2}\cdot(6x-4)=\frac{3x-2}{\sqrt{3x^2-4x+9}}$$

9. $y = e^{\sqrt{9-x^2}} \to \dfrac{dy}{dx} = e^{\sqrt{9-x^2}}\left(\dfrac{d}{dx}\left[\sqrt{9-x^2}\right]\right)$

$= e^{\sqrt{9-x^2}}\left(\dfrac{1}{2}\left(9-x^2\right)^{-1/2}\right)(-2x) = \dfrac{-x}{\sqrt{9-x^2}} e^{\sqrt{9-x^2}}$, find $\dfrac{dy}{dx}$.

11. $v(t) = \sqrt{\left[\left(\dfrac{E(t)}{3}+3t\right)^{3/7}-4\right]} = \left[\left(\dfrac{E(t)}{3}+3t\right)^{3/7}-4\right]^{1/2}$

$v'(t) = \dfrac{1}{2}\left[\left(\dfrac{E(t)}{3}+3t\right)^{3/7}-4\right]^{-1/2}\left[\dfrac{3}{7}\left(\dfrac{E(t)}{3}+3t\right)^{-4/7}\right]\left(\dfrac{1}{3}E'(t)+3\right)$

1.2 Multiple Choice Solutions

1. If $y = \left(x^4+4\right)^2$, then $\dfrac{dy}{dx} = 2\left(x^4+4\right)^1\left(4x^3\right) = 8x^3\left(x^4+4\right)^1$

The correct answer is E

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3. I. True: $\dfrac{d}{dx}\left(e^x+3\right)^{1/2} = \dfrac{1}{2}\left(e^x+3\right)^{-1/2}\left(e^x\right)$

II. True: $\dfrac{d}{dx}\left(5^{3x^2}\right) = a^u \cdot \ln a\left(D_u\right) = 5^{3x^2} \cdot \ln 5(6x) = 6x\ln 5\left(5^{3x^2}\right)$

III. True: $\dfrac{d}{dx}\left(6x^3 - \pi + x^{8/3} - 2x^{-3}\right) = 18x^2 + \dfrac{8}{3}x^{8/3} + 6x^{-4}$

The correct answer is E.

1.3 Free Response Solutions

1. $y = \sin 4x$
$y' = 4\cos 4x$

3. $f(t) = \sqrt[3]{1+\tan t}$
$f'(t) = \dfrac{1}{3}(1+\tan t)^{-2/3} \cdot \sec^2 t = \dfrac{\sec^2 t}{(1+\tan t)^{2/3}}$

5. $y = a^3 + \cos^3 x$
$y' = 3\cos^2 x \cdot (-\sin x) = -3\cos^2 x \sin x$

7. $f(x) = \cos(\ln x)$
$f'(x) = -\sin(\ln x) \cdot \frac{1}{x} = -\frac{\sin(\ln x)}{x}$

9. $f(x) = \log_{10}(2 + \sin x)$
$f'(x) = \frac{1}{\ln 10(2 + \sin x)} \cdot \cos x = \frac{\cos x}{\ln 10(2 + \sin x)}$

11. $y = \sin^{-1}(e^x)$

$$y' = \frac{1}{\sqrt{1 - e^{2x}}} \cdot e^x = \frac{e^x}{\sqrt{1 - e^{2x}}}$$

13. $y = \tan^{-1}(\sqrt{x})$
$y' = \frac{1}{1 + x} \cdot \frac{1}{2} x^{-1/2} = \frac{1}{2(x^{1/2} + x^{3/2})}$

15. $y = \tan^{-1} x^2$. $y' = \frac{1}{1 + (x^2)^2}(2x) = \frac{2x}{1 + x^4}$

17. $\frac{d}{dx}\left(3e^{x^2 + 2x}\right) = 3e^{x^2 + 2x}(2x + 2) = 6(x + 1)e^{x^2 + 2x}$

19. $\frac{d}{dx}\left(\sqrt[3]{16 + x^3}\right) = \frac{d}{dx}(16 + x^3)^{1/3} = \frac{1}{3}(16 + x^3)^{-2/3}(3x^2) = \frac{x^2}{(16 + x^3)^{2/3}}$

21. $g(x) = \ln(x^2 + 16)$. $g'(x) = \frac{1}{x^2 + 16}(2x) = \frac{2x}{x^2 + 16}$

23. $\frac{d}{dx}(\ln(\sec x)) = \frac{1}{\sec x} \cdot \sec x \tan x = \tan x$

25. $f(x) = \ln\left(x^2+3\right)$. $f'(x) = \frac{1}{x^2+3}(2x) = \frac{2x}{x^2+3}$

27. $h(x) = \sqrt{x^2+5} = \left(x^2+5\right)^{1/2}$. $h'(x) = \frac{1}{2}\left(x^2+5\right)^{-1/2}(2x) = \frac{x}{\left(x^2+5\right)^{1/2}}$

29. $y = \sin^{-1}(\cos x)$. $y' = \frac{1}{\sqrt{1-(\cos x)^2}}(-\sin x) = \frac{-\sin x}{\sqrt{1-\cos^2 x}} = \frac{-\sin x}{\sqrt{\sin^2 x}} = -1$

31. $\frac{d}{dx}\left(5e^{\tan(7x)}\right) = 5e^{\tan(7x)}\left(\sec^2 7x\right)7 = 35e^{\tan(7x)}\left(\sec^2 7x\right)$

33. $\frac{d}{dx}\left(\ln^3\left(x^2+1\right)\right) = 3\ln^2\left(x^2+1\right)\frac{1}{x^2+1}(2x) = \frac{6x\ln^2\left(x^2+1\right)}{x^2+1}$

35. $y = \tan^2(3\theta) \rightarrow \quad y' = 2\tan(3\theta) \cdot \sec^2(3\theta) \cdot 3 = 6\tan(3\theta) \cdot \sec^2(3\theta)$

37. $h'(1) = \frac{1}{h'(g(1))} = \frac{1}{h'(2)} = 5$

1.3 Multiple Choice Solutions

1. If $y = \sin^{-1} e^{3\theta}$, then $\frac{dy}{d\theta} = \frac{1}{\sqrt{1-\left(e^{3\theta}\right)^2}}\left(e^{3\theta}\right)(3)$

The correct answer is E

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3. $h'(x) = \ln(x^2)\left(\frac{1}{1+x^2}\right) + \tan^{-1}(x)\left(\frac{2}{x}\right) \to h'(1) = \ln(1)\left(\frac{1}{2}\right) + \tan^{-1}(1)\left(\frac{2}{1}\right) = 0 + 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$

The correct answer is C

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5. $h'(x) = 5e^{5x} + 1 \to h'(0) = 5e^{0} + 1 = 6$

The correct answer is D

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7. $g'(x) = 2\cos(2x)(-\sin(2x))(2) = -4\sin(2x)\cos(2x)$

The correct answer is B

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9. $f'(x) = \sec^2(3^x)(3^x \ln 3) \to f'(1.042) = 3.451$

The correct answer is C.

1.4 Free Response Solutions

1. Use the tangent line equation to $g(x) = x^3 - x^2 + 4x - 4$ at $x = -3$ to approximate the value of $g(-2.9)$

$$g(-3) = (-3)^3 - (-3)^2 + 4(-3) - 4 = 3$$

$$g'(x) = 3x^2 - 2x + 4 \Rightarrow m = g'(-3) = 3(-3)^2 - 2(-3) + 4 = 25$$

Tangent Line: $y - 3 = 25(x + 3)$

$$g(-2.9) \approx y(-2.9) = 25(-2.9 + 3) + 3 = 5.5$$

3. Use the equation of the tangent line to $f(x) = \sqrt{6 - x}$ at $x = 2$ to approximate $\sqrt{4.1}$.

$$f(2) = \sqrt{4} = 2$$

$$f'(x) = \frac{1}{2}(6-x)^{-1/2}(-1) \Rightarrow m = f'(2) = \frac{-1}{2(6-2)^{1/2}} = -\frac{1}{4}$$

Tangent Line: $y - 2 = -\frac{1}{4}(x-2)$

$$\sqrt{4.1} = f(1.9) \approx y(1.9) = -\frac{1}{4}(1.9-2) + 2 = 2.25$$

5. Find the equation of the tangent line at $x = 2$ for $h(x) = \ln(9 - x^3)$. Use this to approximate $g(2.1)$

$$h(2) = \ln 1 = 0$$

$$h'(x) = \frac{-3x^2}{9 - x^3} \Rightarrow m = f'(2) = \frac{0}{1}$$

Tangent Line: $y = 0$

$$h(2.1) \approx y(2.1) = 0$$

7. Find an equation of the line tangent to the curve $y = x^4 + 2e^x$ at the point $(0, 2)$.

$$y' = 4x^3 + 2e^x \Rightarrow y'\big|_{x=0} = 2$$

Tangent Line: $y - 2 = 2(x - 0)$

9. Use the equation of the tangent line to $f(x) = 2x + \cos(x-2)$ at $x = 2$ to approximate $f(1.9)$.

$$f(2) = 2(2) + \cos((2)-2) = 4 + 1 = 5$$

$$f'(x) = 2 - \sin(2-x) \Rightarrow m = |f'(2) = 2 - \sin(0) = 2$$

Tangent Line: $y - 5 = 2(x - 2)$

$$f(1.9) \approx y(1.9) = 2((1.9) - 2) + 5 = 4.8$$

11. Find the equation of the tangent line to $y = x + \cos x$ at the point $(0,1)$.

$$y' = 1 - \sin x \Rightarrow y'\big|_{x=0} = 1$$

Tangent Line: $y-1=1(x-0)$

13. Find the equation of the line tangent to $y=\frac{2}{\pi}x+\cos(4x)$ when $x=\frac{\pi}{2}$.

$$y'=\frac{2}{\pi}-4\sin(4x)\Rightarrow y'\Big|_{x=\pi/2}=\frac{2}{\pi}-4\sin(2\pi)=\frac{2}{\pi}$$

$$y\Big|_{x=\pi/2}=\frac{2}{\pi}\cdot\frac{\pi}{2}+\cos(2\pi)=2$$

Tangent Line: $y-2=\frac{2}{\pi}\left(x-\frac{\pi}{2}\right)$

15. At what point on the graph of $y=x^2-3x-4$ is the tangent parallel to the line $5x-y=3$?

$$y=x^2-3x-4\rightarrow\frac{dy}{dx}=2x-3;\ 5x-y=3\rightarrow m=5;\ \frac{dy}{dx}=2x-3=5=m\rightarrow x=4$$

$y(4)=(4)^2-3(4)-4=0$. The point is $(4,\ 0)$

17. Find the equation of the line tangent to $f(x)=2x^3-9x^2-12x$ where $f'(x)=12$.

$f(x)=2x^3-9x^2-12x\rightarrow f'(x)=6x^2-18x-12=12\rightarrow x^2-3x-4=(x+1)(x-4)=0$

$x=-1,\ 4\rightarrow f(-1)=1;\ \ f(4)=-64$

$y-1=12(x+1)$ and $y+64=12(x-4)$

19. Find all points on the graph of $y=2\sin x+\sin^2 x$ where the tangent line is horizontal.

$f'(x)=2\cos x+2\sin x\cos x=0$

$\cos x=0$ or $\sin x=-1$

$x=\left\{\pm\frac{\pi}{2}\pm2\pi n\right.$ or $x=\left\{\frac{3\pi}{2}\pm2\pi n\right.$

Points where the tangent line is horizontal: $\left(\frac{\pi}{2}\pm2\pi n,\ 3\right),\ \left(-\frac{\pi}{2}\pm2\pi n,\ 1\right)$

21. $x(t)=2t^3-21t^2+60t+4$

$v(t)=6t^2-42t+60=0$

$a(t)=12t-42$

a) $t^2 - 7t + 10 = (t-5)(t-2) = 0 \rightarrow t = 2$ and $t = 5$

b) $\begin{array}{l} v \quad +0 \quad -0+ \\ \quad\quad \longleftrightarrow \\ t \quad\quad 2 \;\; 5 \end{array}$ Left; $v(3) < 0$

c) $x(3) = 2(3)^3 - 21(3)^2 + 60(3) + 4 = 49$. So the particle is 49 units to the right of the origin at $t = 3$.

d) $a(3) = 12(3) - 42 = -6$

e) Speeding up, because $v(3)$ and $a(3)$ are both negative.

23. $y(t) = 9t^4 - 4t^3 - 240t^2 + 576t - 48$

$v(t) = 36t^3 - 12t^2 - 480t + 576$

$a(t) = 108t^2 - 24t - 480$

a) $v(t) = 36t^3 - 12t^2 - 480t + 576 = 0$

$\rightarrow 3t^3 - t^2 - 40t + 48 = (t+4)(3t-4)(t-3) = 0 \rightarrow$

$t = -4,\ 4/3,\ 3$

b) neither; $v(3) = 0$

c) $y(3) = 9(3)^4 - 4(3)^3 - 240(3)^2 + 576(3) - 48 = 114$. So, the particle is 114 units above the origin at $t = 3$.

d) $a(3) = 108(3)^2 - 24(3) - 480 = 420$

e) neither; $v(3) = 0$

25. $x(t) = t^2 - 5t + 4 \rightarrow v(t) = 2t - 5 = 0 \rightarrow t = 2.5$

$x(2.5) = (2.5)^2 - 5(2.5) + 4 = -2.25$

27. $x(t) = 2t^3 - 21t^2 + 60t + 4$

$v(t) = 6t^2 - 42t + 60$

$a(t) = 12t - 42 = 0 \rightarrow t = 3.5$

$x(3.5) = 42.5$

$v(3.5) = -13.5$

29a. $v(t) = y'(t) = 6t^2 - 2t - 4 = 0 \rightarrow 3t^2 - t - 2 = 0 \rightarrow (3t+2)(t-1) = 0$

$$t=-\frac{2}{3},\ 1 \to x(1)=-1,\ a(1)=10,\ x\left(-\frac{2}{3}\right)=3.630,\ a\left(-\frac{2}{3}\right)=-10$$

29b. $a(t)=12t-2=0 \to t=\frac{1}{6}$

$$v\left(\frac{1}{6}\right)=1.315;\quad x\left(\frac{1}{6}\right)=-4.167$$

30. $y(t)=t^4-2t^2-8 \to v(t)=4t^3-4t \to a(t)=12t^2-4$

a) $v(t)=4t^3-4t=0 \to t=0,\ \pm 1.$

$x(\pm 1)=-9,\ a(\pm 1)=8,\ x(0)=-8,\ a(0)=-4$

b) $a(t)t=12t^2-4=0 \to t=\pm\frac{1}{\sqrt{3}}$

$$y\left(\frac{1}{\sqrt{3}}\right)=-\frac{77}{9}; v\left(\frac{1}{\sqrt{3}}\right)=-\frac{8}{3\sqrt{3}};\ y\left(-\frac{1}{\sqrt{3}}\right)=-\frac{77}{9}; v\left(-\frac{1}{\sqrt{3}}\right)=\frac{8}{3\sqrt{3}}$$

1.4 Multiple Choice Solutions

1. $f'(x)=2e^{4x^2}(8x)=3$. Graph on calculator to solve: $2e^{4x^2}(8x)=3 \to x=0.168$

The correct answer is A

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3. Tangent line: $y-1=f'(2)(x-2) \to y=f'(2)(x-2)+1$

$.7=f'(2)(2.1-2)+1 \to -.3=f'(2)(.1) \to -3=f'(2)$

The correct answer is C

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5. $f(1)=2$ and $f'(x)=\sqrt{x^2+3} \to m=\sqrt{1^2+3}=2.$

Tangent line: $y-2=2(x-1)$

$\mathrm{f}(0.98)\approx 2(0.98-1)+2=-.04+2=1.96$

The correct answer is D

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7. $h'(x)=2x-5=-1 \to x=2 \to y(2)=-3$

$y + 3 = -(x - 2) \rightarrow y = -x - 1$

The correct answer is A.

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9. $f'(x) = 6x^5 + 5x^4 + 2x = -1$. Graph to find the x-value and y-value.

The correct answer is E

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11. $\frac{dy}{dx} = \frac{1}{3}(x^2 - 1)^{-2/3}(2x) \rightarrow m_{\tan} = \frac{1}{3}(3^2 - 1)^{-2/3}(2(3)) = \frac{1}{2} \rightarrow m_{norm} = -2$

The correct answer is E

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13. Graph $v(t) = 3t^4 - 11t^2 + 9t - 2$ for $-3 \le t \le 3$ on a calculator and count the number of zeros.

The correct answer is C

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15. $v(t) = 2t - 6 \rightarrow 2(t) = 2 \neq 0$

The correct answer is E

1.5 Free Response Solutions

1. $y' = t^3 \cdot (-\sin t) + \cos t \cdot 3t^2 = t^2(3\cos t - t\sin t)$

3. $\frac{d}{dx}\left[xe^{-x}\right] = x\left[e^{-x}(-1)\right] + e^{-x}(1) = -xe^{-x} + e^{-x} = e^{-x}(1 - x)$

5. $y' = e^{-5x} \cdot (-\sin 3x) \cdot 3 + \cos 3x \cdot e^{-5x} \cdot (-5) = -e^{-5x}[5\cos 3x + 3\sin 3x]$

7. $\frac{d}{dx}\left(x^3\sec x\right) = x^3\sec x\tan x + (\sec x)\left(3x^2\right) = x^2\sec x(x\tan x + 3)$

9. $D_x\left(x^2 \sin x + 2x\cos x\right)$

$$D_x\left(x^2 \cdot \sin x + 2x \cdot \cos x\right)$$
$$= x^2 \cdot \cos x + \sin x \cdot 2x + 2x \cdot -\sin x + \cos x \cdot 2$$
$$= x^2 \cos x + 2x\sin x - 2x\sin x + 2\cos x$$
$$= x^2 \cos x + \cancel{2x\sin x} \cancel{-2x\sin x} + 2\cos x$$
$$= x^2 \cos x + 2\cos x$$
$$= \cos x\left(x^2 + 2\right)$$

11. $f(x) = 2\sin^2 x\cos^2 x$

$$f'(x) = 2\left[\sin^2 x \cdot 2\cos x \cdot -\sin x + \cos^2 x \cdot 2\sin x \cdot \cos x\right]$$
$$= 2\left[-2\sin^3 x\cos x + 2\cos^3 x\sin x\right]$$
$$= 4\sin x\cos x\left[-\sin^2 x + \cos^2 x\right]$$
$$= 4\sin x\cos x\left[\cos^2 x - \sin^2 x\right]$$
$$= 2 \cdot 2\sin x\cos x\left[\cos 2x\right]$$
$$= 2\sin 2x \cdot \cos 2x$$
$$= \sin 4x$$

13. $f'(x) = \sec x\left(\sec^2 x\right) + \tan x\left(\sec x\tan x\right)$

$$f'\left(\frac{\pi}{4}\right) = \sec\left(\frac{\pi}{4}\right)\left(\sec^2\left(\frac{\pi}{4}\right)\right) + \tan\left(\frac{\pi}{4}\right)\left(\sec\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right)\right)$$
$$= \sqrt{2}\left(\left(\sqrt{2}\right)^2\right) + (1)\left(\left(\sqrt{2}\right)(1)\right) = 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$$

15. $\frac{d}{dx}\left[\left(x^2 - 2x - 8\right)e^x\right] = \left(x^2 - 2x - 8\right)e^x + e^x(2x - 2)$

$$= e^x\left(x^2 - 2x - 8 + 2x - 2\right) = e^x\left(x^2 - 10\right)$$

17. $\frac{d}{dx}\left[\left(x^2 - 1\right)e^{-1/2x}\right] = \left(x^2 - 1\right)\left[e^{-1/2x}\left(-1/2\right)\right] + e^{-1/2x}(2x)$

$$= -\frac{1}{2}e^{-1/2x}\left(x^2 - 1 + 4x\right) = -\frac{1}{2}e^{-\frac{1}{2}x}\left(x^2 - 4x - 1\right)$$

19. $\frac{d}{dx}\left[x^2e^{-4x}\right] = x^2e^{-4x}(-4) + e^{-4x}(2x) = -2xe^{-4x}\left[x^2(2) - 1\right] = -2xe^{-4x}(2x - 1)$

21. $\frac{d}{dx}\left[e^x\sqrt{7-x}\right]=\frac{d}{dx}\left[e^x(7-x)^{1/2}\right]=e^x\left[\frac{1}{2}(7-x)^{-1/2}(-1)\right]+(7-x)^{1/2}e^x$

$=e^x\left[\frac{-1}{2(7-x)^{1/2}}+(7-x)^{1/2}\right]=e^x\left[\frac{-1}{2(7-x)^{1/2}}+\frac{2(7-x)}{2(7-x)^{1/2}}\right]=e^x\left[\frac{-1+14-2x}{2(7-x)^{1/2}}\right]=e^x\left(\frac{13-2x}{2(7-x)^{1/2}}\right)$

23. $\frac{d}{dx}\left[x\sqrt{4-x^2}\right]=x\left(\frac{1}{2}(4-x^2)^{-1/2}(-2x)\right)+(4-x^2)^{1/2}(1)=\frac{-x^2}{(4-x^2)^{1/2}}+(4-x^2)^{1/2}=$

$$\frac{-x^2}{(4-x^2)^{1/2}}+\frac{(4-x^2)}{(4-x^2)^{1/2}}=\frac{4-2x^2}{(4-x^2)^{1/2}}$$

25. $\frac{d}{dx}\left[(x^2)\sqrt{9-x^2}\right]=\frac{d}{dx}\left[(x^2)(9-x^2)^{1/2}\right]=(x^2)\left[\frac{1}{2}(9-x^2)^{-1/2}(-2x)\right]+(9-x^2)^{1/2}(2x)$

$=(x^2)\left[\frac{-2x}{(9-x^2)^{1/2}}\right]+(9-x^2)^{1/2}(2x)=(x^2)\left[\frac{-2x}{(9-x^2)^{1/2}}+\frac{(9-x^2)(2x)}{(9-x^2)^{1/2}}\right]=$

$\frac{-x^3}{(9-x^2)^{1/2}}+\frac{(9-x^2)(2x)}{(9-x^2)^{1/2}}=\frac{-x^3+18x-2x^3}{(9-x^2)^{1/2}}=\frac{18x-3x^3}{(9-x^2)^{1/2}}$

27. $\frac{dy}{dx}=(4x^5-3)^7 5(7x^2+1)^4(14x)+(7x^2+1)^5 7(4x^5-3)^6(20x^4)=$

$=35x(4x^5-3)^6(7x^2+1)^4\left[2(4x^5-3)+4x^3(7x^2+1)\right]=$

$=35x(4x^5-3)^6(7x^2+1)^4\left[8x^5-6+28x^5+4x^3\right]=$

$=35x(4x^5-3)^6(7x^2+1)^4\left[36x^5+4x^3-6\right]$

29. $\frac{dy}{dx}=(3x^2-4)^3 2(6x^2+7)(12x)+(6x^2+7)^2 3(3x^2-4)^2(6x)=$

$6x(3x^2-4)^2(6x^2+7)\left[(3x^2-4)(4)+(6x^2+7)3\right]=$

$6x(3x^2-4)^2(6x^2+7)\left[12x^2-16+18x^2+21\right]=$

$6x(3x^2-4)^2(6x^2+7)\left[30x^2+5\right]=$

$30x(3x^2-4)^2(6x^2+7)(6x^2+1)$

31. $y'=e^{x\cos x}\left[x\cdot(-\sin x)+\cos x\right]=e^{x\cos x}\left[\cos x-x\sin x\right]$

33. Find the equation of the line tangent to $y=x^2e^{-x}$ at the point $\left(1,1/e\right)$.

$y' = x^2 \cdot e^{-x} \cdot (-1) + e^{-x} \cdot 2x \qquad m = y'\big|_{x=1} = -\frac{1}{e} + \frac{2}{e} = \frac{1}{e}$

$y - \frac{1}{e} = \frac{1}{e}(x-1)$

35. $f(3) = (3)\sqrt[4]{7+3^2} = 6$

$f'(x) = x\left[\frac{1}{4}(7+x^2)^{-3/4}(2x)\right] + \sqrt[4]{7+x^2}(1) \to m = f'(3) = \frac{9}{2} + 2 = \frac{13}{2}$

$y + 6 = \frac{13}{2}(x-3)$

37. $y\big|_{x=e} = e\sin\left(\frac{\pi}{2}\ln e\right) = e \Rightarrow (e,e) \qquad y' = x\cos\left(\frac{\pi}{2}\ln x\right) \cdot \frac{\pi}{2} \cdot \frac{1}{x} + \sin\left(\frac{\pi}{2}\ln x\right)$

$y'\big|_{x=e} = e\cos\left(\frac{\pi}{2}\ln e\right) \cdot \frac{\pi}{2} \cdot \frac{1}{e} + \sin\left(\frac{\pi}{2}\ln e\right) = 1$

Tangent line: $y - e = 1(x-e)$ Normal line: $y - e = -1(x-e)$

39. $y\big|_{x=4/\pi} = \frac{4}{\pi}\sin\left(\frac{\pi}{4}\right) = \frac{4}{\pi\sqrt{2}} = 2\sqrt{2}\,\pi \Rightarrow \left(\frac{4}{\pi}, 2\sqrt{2}\,\pi\right)$

$y' = x\cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) + \sin\left(\frac{1}{x}\right)$

$y'\big|_{x=4/\pi} = \frac{4}{\pi}\cos\left(\frac{\pi}{4}\right) \cdot \left(-\frac{\pi^2}{16}\right) + \sin\left(\frac{\pi}{4}\right) = -\frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{-\pi+4}{4\sqrt{2}} = \frac{\sqrt{2}(-\pi+4)}{8}$

Tangent line: $y - 2\sqrt{2}\,\pi = \frac{\sqrt{2}(-\pi+4)}{8}\left(x - 4/\pi\right)$

Normal line: $y - 2\sqrt{2}\,\pi = -\frac{4\sqrt{2}}{-\pi+4}\left(x - 4/\pi\right) \Rightarrow y - 2\sqrt{2}\,\pi = \frac{4\sqrt{2}}{\pi-4}\left(x - 4/\pi\right)$

41. $y' = x \cdot \frac{-1}{\sqrt{1-x^2}} + \cos^{-1}x - \frac{1}{2}(1-x)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}} + \cos^{-1}x = \cos^{-1}x$

43. $y' = \frac{-1}{\sqrt{1-x^2}} + x \cdot \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) + \sqrt{1-x^2} = \frac{-1}{\sqrt{1-x^2}} + \frac{-x^2}{\sqrt{1-x^2}} + \frac{1-x^2}{\sqrt{1-x^2}} = \frac{-2x^2}{\sqrt{1-x^2}}$

45. $\frac{dy}{dx}=\frac{3}{\sqrt{1-\left(\frac{x}{3}\right)^2}}\left(\frac{1}{3}\right)+\frac{1}{2}(9-x^2)^{-1/2}(-2x)=\frac{1}{\sqrt{1-\frac{x^2}{9}}}+\frac{-x}{(9-x^2)^{1/2}}=$

$$\frac{3}{(9-x^2)^{1/2}}+\frac{-x}{(9-x^2)^{1/2}}=\frac{3-x}{(9-x^2)^{1/2}}$$

1.5 Multiple Choice Solutions

1. $\frac{dy}{dx}=x^2(-\sin 2x)(2)+\cos 2x(2x)=-2x(\cos 2x-x\sin 2x)$

The correct answer is D

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3. $f'(x)=x\sec^2 x+(\tan x)(1)$

$f'(x)=x\sec^2 x+(\tan x)(1)$

The correct answer is B

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7. I. False: $\frac{d}{dx}(x\tan x)=x\sec^2 x+\tan x(1)$

II. True: $\frac{d}{dx}(x\ln x)=x\left(\frac{1}{x}\right)+(\ln x)(1)=1+\ln x$

III. False: $\frac{d}{dx}(1-x)^{1/2}=\frac{1}{2}(1-x)^{-1/2}(-1)=\frac{-1}{2\sqrt{1-x}}$

The correct answer is B

1.6 Free Response Solutions

1. $$\frac{dy}{dx}=\frac{\left(x^2-4\right)(2x)-\left(x^2-3\right)(2x)}{\left(x^2-4\right)^2}=\frac{2x^3-8x-2x^3+6x}{\left(x^2-4\right)^2}=\frac{-2x}{\left(x^2-4\right)^2}$$

3. $$f'(x)=\frac{\left(x^2-x-3\right)(2x+2)-\left(x^2+2x-8\right)(2x-1)}{\left(x^2-x-3\right)^2}$$
$$=\frac{2x^3-2x^2-6x+2x^2-2x-6-\left(2x^3+4x^2-16x-x^2-2x+8\right)}{\left(x^2-x-3\right)^2}=\frac{-3x^2+10x-14}{\left(x^2-x-3\right)^2}$$

5. $$\frac{d}{dx}\left(\frac{3x+3}{x^3+1}\right)=\frac{d}{dx}\left[\frac{3(x+1)}{(x+1)\left(x^2-x+1\right)}\right]=\frac{d}{dx}\left[\frac{3}{x^2-x+1}\right]$$
$$=\frac{\left(x^2-x+1\right)(0)-(3)(2x-1)}{\left(x^2-x+1\right)^2}=\frac{-6x+3}{\left(x^2-x+1\right)}$$

7. $$\frac{d}{dx}\left[\frac{x^5-12x^3-19x}{3x^3}\right]=\frac{d}{dx}\left[\frac{1}{3}x^2-4-19x^{-2}\right]=\frac{1}{9}x^3-4x+19x^{-1}=\frac{d}{dx}\left[\frac{x^4-12x^2-19}{3x^2}\right]$$

9. $$\frac{d}{dx}\left(\frac{\tan x+5}{\sin x}\right)=\frac{(\sin x)\left(\sec^2 x\right)-(\tan x+5)(\cos x)}{\sin x}=\left(\sec^2 x\right)-(\tan x+5)(\cot x)=$$
$$\sec^2 x-1-5\cot x=\tan^2 x-5\cot x$$

11. $$\frac{dy}{dx}=\frac{(\cos x-3)\sec^2 x-\tan x(-\sin x)}{(\cos x-3)^2}=\frac{\sec x-3\sec^2 x+\sin x\tan x}{(\cos x-3)^2}$$

13. $$y=\frac{\tan x-1}{\sec x}$$
$$y'=\frac{\sec x\cdot\sec^2 x-(\tan x-1)\cdot\sec x\tan x}{\sec^2 x}=\frac{\sec^2 x-(\tan x-1)\tan x}{\sec x}=\frac{\sec^2 x-\tan^2 x+\tan x}{\sec x}$$

Recall $1=\sec^2 x-\tan^2 x$, so substitute into numerator:

$$y' = \frac{1 + \tan x}{\sec x}$$

15. $f'(x) = \frac{(\tan x + 1)\sec^2 x + \tan x \sec^2 x}{(\tan x + 1)^2} \to f'\left(\frac{\pi}{4}\right) = \frac{(1+1)\left(\sqrt{2}\right)^2 + (1)\left(\sqrt{2}\right)^2}{(1+1)^2} = \frac{3}{2}$

17. $y' = \frac{\left(r^2+1\right)^{1/2} - r \cdot 1/2\left(r^2+1\right)^{-1/2} \cdot 2r}{r^2+1} = \frac{\left(r^2+1\right)^{-1/2}\left[r^2+1-r^2\right]}{r^2+1} = \frac{1}{\left(r^2+1\right)^{3/2}}$

19. At $x = -1$, the point on the function is $\left(-1,\ \frac{2}{17}\right)$

$\frac{dy}{dx} = \frac{\left(x^2+16\right)(-2) - (-2x)(2x)}{\left(x^2+16\right)^2} \to \left.\frac{dy}{dx}\right]_{x=-1} = \frac{17(-2) - (2)(-2)}{(17)^2} = -\frac{38}{289}$

The equation of the tangent line is the line $y - \frac{2}{17} = -\frac{38}{289}(x+1)$

The normal line is the line $y - \frac{2}{17} = \frac{289}{38}(x+1)$

21. At $x = -1$, the point on the function is $\left(-1,\ -\frac{3}{2}\right)$

$\frac{dy}{dx} = \frac{\left(x^2+1\right)(-3) - (-3x)(2x)}{\left(x^2+1\right)^2} \to \left.\frac{dy}{dx}\right]_{x=-1} = \frac{2(-6) - (3)(-2)}{(2)^2} = 0$

The equation of the tangent line is the line $y = -\frac{3}{2}$

The normal line is the line $x = -1$

1.6 Multiple Choice Solutions

1. $f'(x) = \frac{\sin\left(x^2\right)e^x - \left(1+e^x\right)\cos\left(x^2\right)(2x)}{\left(\sin\left(x^2\right)\right)^2}$; $f'(0) = \frac{\sin(0)e^0 - \left(1+e^0\right)\cos(0)(0)}{(\sin(0))^2} = \frac{0}{0}$

The correct answer is E

---

3. $\frac{dy}{dx}=\frac{(3x+2)(-2)-(3-2x)(3)}{(3x+2)^2}=\frac{(-6x-4)-(9-6x)}{(3x+2)^2}=\frac{-13}{(3x+2)^2}$

The correct answer is D

---

5. $\frac{dy}{dx}=\frac{(4x-3)(3)-(3x+4)(4)}{(4x-3)^2}\to m_{\tan}=\frac{(4-3)(3)-(3+4)(4)}{(4-3)^2}=-25\to m_{norm}=\frac{1}{25}$

$y-7=\frac{1}{25}(x-1)\to 25y-175=x-1$ is

The correct answer is D

1.7 Free Response Solutions

1. $f(x)=x^5+6x^2-7x$

$$f'(x)=5x^4+12x-7$$

$$f''(x)=20x^3+12$$

3. $y=\left(x^3+1\right)^{2/3}$

$$y'=\frac{2}{3}\left(x^3+1\right)^{-1/3}\cdot 3x^2=\frac{2x^2}{\left(x^3+1\right)^{1/3}}$$

$$y''=\frac{\left(x^3+1\right)^{1/3}\cdot 4x-2x^2\cdot 1/3\left(x^3+1\right)^{-2/3}\cdot 3x^2}{\left(x^3+1\right)^{2/3}}=\frac{\left(x^3+1\right)^{-2/3}\left[\left(x^3+1\right)\cdot 4x-2x^4\right]}{\left(x^3+1\right)^{2/3}}$$

$$=\frac{4x^4+4x-2x^4}{\left(x^3+1\right)^{4/3}}=\frac{2x\left(x^3+2\right)}{\left(x^3+1\right)^{4/3}}$$

5. $g(t)=t^3e^{5t}$

$$g'(t)=t^3\cdot e^{5t}\cdot 5+e^{5t}\cdot 3t^2=t^2e^{5t}[5t+3]$$

$$g''(t)=t^2e^{5t}\cdot 5+t^2(5t+3)\cdot e^{5t}\cdot 5+e^{5t}\cdot(5t+3)\cdot 2t$$

$$=te^{5t}[5t+5t(5t+3)+2(5t+3)]$$

$$=te^{5t}\left[25t^2+30t+6\right]$$

7. $y=\sin^3 x$

$$y'=3\sin^2 x\cos x$$

$$y''=3\left[\sin^2 x\cdot -\sin x+\cos x\cdot 2\sin x\cos x\right]=3\sin x\left[2\cos^2 x-\sin^2 x\right]$$

9. $$\frac{d^2}{dx^2}\left[5x^4+9x^3-4x^2+x-8\right]$$

$$=\frac{d}{dx}\left[-20x^3+27x^2-8x+1\right]$$

$$=-60x^2+54x-8$$

11. $y=\cos x^2$, find y''

$$\frac{dy}{dx}=-\sin x^2(2x)=-2x\sin x^2$$

$$\frac{d^2y}{dx^2}=-2x\left[\cos x^2(2x)\right]+\sin x^2(-2)=-2\left[2x^2\cos x^2+\sin x^2\right]$$

13. $y = \sec 3x$, find $\dfrac{d^2y}{dx^2}$

$$\frac{dy}{dx} = \sec 3x \tan 3x(3) = 3\sec 3x \tan 3x$$

$$\frac{d^2y}{dx^2} = 3\sec 3x\left(\sec^2 3x(3)\right) + \tan 3x(3\sec 3x \tan 3x(3)) = 9\sec 3x\left(\sec^2 3x + \tan^2 3x\right)$$

15. $f(x) = \ln\left(x^2+3\right)$, find $f''(x)$

$$f'(x) = \frac{2x}{x^2+3}$$

$$f''(x) = \frac{\left(x^2+3\right)(2)-(2x)(2x)}{\left(x^2+3\right)^2} = \frac{\left(2x^2+6\right)-\left(4x^2\right)}{\left(x^2+3\right)^2} = \frac{-2\left(x^2-3\right)}{\left(x^2+3\right)^2}$$

17. $h(x) = \sqrt{x^2+5}$, find $h''(x)$

$$h(x) = \sqrt{x^2+5} = \left(x^2+5\right)^{1/2}$$

$$h'(x) = \sqrt{x^2+5} = \frac{1}{2}\left(x^2+5\right)^{-1/2}(2x) = \frac{x}{\left(x^2+5\right)^{1/2}}$$

$$h''(x) = \frac{\left(x^2+5\right)^{1/2}(1) - x \cdot \dfrac{x}{\left(x^2+5\right)^{1/2}}}{\left(x^2+5\right)^1}$$

$$= \frac{\left(x^2+5\right)^1 - x^2}{\left(x^2+5\right)^{3/2}}$$

$$= \frac{5}{\left(x^2+5\right)^{3/2}}$$

19. $y = \dfrac{x^2-3}{x^2-10}$, find $\dfrac{d^2y}{dx^2}$

$$\frac{dy}{dx} = \frac{\left(x^2-10\right)(2x)-\left(x^2-3\right)(2x)}{\left(x^2-10\right)^2} = \frac{(2x)\left[\left(x^2-10\right)-\left(x^2-3\right)\right]}{\left(x^2-10\right)^2} = \frac{-14x}{\left(x^2-10\right)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x^2-10)^2(-14)-(-14x)2(x^2-10)^1(2x)}{(x^2-10)^4}$$

$$= \frac{-14(x^2-10)\left[(x^2-10)-(4x^2)\right]}{(x^2-10)^4}$$

$$= \frac{-14(-3x^2-10)}{(x^2-10)^3} = \frac{14(3x^2+10)}{(x^2-10)^3}$$

21. $y = x^3 + x^2 - 7x - 15$

$y' = 3x^2 + 2x - 7$

$y'' = 6x + 2$

23. $$\frac{dy}{dx} = \frac{(x^2+4)(-4)-(-4x)(2x)}{(x^2+4)^2} = \frac{(-4x^2-16)-(-8x^2)}{(x^2+4)^2} = \frac{4x^2-16}{(x^2+4)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x^2+4)^2(8x)-(4x^2-16)2(x^2+4)^1(2x)}{(x^2+4)^4}$$

$$= \frac{(x^2+4)(8x)-(4x^2-16)(4x)}{(x^2+4)^3}$$

$$= \frac{(8x^3+32x)-(16x^3-64)}{(x^2+4)^3}$$

$$= \frac{(-8x^3+96x)}{(x^2+4)^3} = \frac{-8x(x^2-12)}{(x^2+4)^3}$$

25. $y = x\sqrt{8-x^2} = x(8-x^2)^{1/2}$

$$\frac{dy}{dx} = x \cdot \frac{1}{2} \cdot (8-x^2)^{-1/2}(-2x) + (8-x^2)^{1/2}(1)$$

$$= \frac{-x^2}{(8-x^2)^{1/2}} + (8-x^2)^{1/2}$$

$$= \frac{-x^2+(8-x^2)^1}{(8-x^2)^{1/2}} = \frac{8-2x^2}{(8-x^2)^{1/2}}$$

$$\frac{d^2y}{dx^2} = \frac{\left(8-x^2\right)^{1/2}(-4x) - \left(8-2x^2\right)\frac{-x}{\left(8-x^2\right)^{1/2}}}{\left(8-x^2\right)^1}$$

$$= \frac{\left(8-x^2\right)^1(-4x) - (-x)\left(8-2x^2\right)}{\left(8-x^2\right)^{3/2}}$$

$$= \frac{2x^3-24x}{\left(8-x^2\right)^{3/2}} = \frac{(2x)\left(x^2-24\right)}{\left(8-x^2\right)^{3/2}}$$

27. $y = xe^{-x}$

$$\frac{dy}{dx} = xe^{-x}(-1) + e^{-x}(1) = e^{-x}(x+1)$$

$$\frac{d^2y}{dx^2} = e^{-x}(1) + (x+1)e^{-x}(-1) = e^{-x}(x-2)$$

29. $y = \dfrac{x}{x^2-9}$

$$\frac{dy}{dx} = \frac{\left(x^2-9\right)(1) - (x)(2x)}{\left(x^2-9\right)^2} = \frac{-x^2-9}{\left(x^2-9\right)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left(x^2-9\right)^2(-2x) - \left(-x^2-9\right)2\left(x^2-9\right)^1(2x)}{\left(x^2-9\right)^4}$$

$$= \frac{\left(x^2-9\right)(-2x) - \left(-x^2-9\right)2(2x)}{\left(x^2-9\right)^3}$$

$$= \frac{\left(-2x^3+18x\right) - \left(-4x^3-36x\right)}{\left(x^2-9\right)^3}$$

$$= \frac{\left(2x^3+54x\right)}{\left(x^2-9\right)^3} = \frac{2x\left(x^2+27\right)}{\left(x^2-9\right)^3}$$

1.7 Multiple Choice Solutions

1. $h'(x) = g'(f(x)) \cdot f'(x)$

$$h''(x) = g'(f(x)) \cdot f''(x) + f'(x) \cdot g''(f(x)) \cdot f'(x) = g'(f(x)) \cdot f''(x) + \left[f'(x)\right]^2 \cdot g''(f(x))$$

The correct answer is E

---

3. $$\frac{dy}{dx} = \frac{1}{\cos x}(-\sin x) = -\tan x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}[-\tan x] = -\sec^2 x$$

The correct answer is B

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5. $$\frac{dy}{dx} = e^{x^2}(2x) \rightarrow \frac{d^2y}{dx^2} = e^{x^2}(2) + 2x\left(e^{x^2}(2x)\right) = 2e^{x^2}\left(1 + 2x^2\right)$$

The correct answer is B

1.8 Free Response Homework

1a. $g'(x) = 3[f(x)]^2 \cdot f'(x) \Rightarrow g'(2) = 3[f(2)]^2 \cdot f'(2) = 3 \cdot 1^2 \cdot 7 = 21$

1b. $h'(x) = f'(x^3) \cdot 3x^2 \Rightarrow h'(2) = f'(2^3) \cdot 3 \cdot 2^2 = 12f'(8) = 12 \cdot -3 = -36$

3. $h'(x) = f'(g(x)) \cdot g'(x)$
$h'(2) = f'(g(2)) \cdot g'(2) = f'(8) \cdot (1) = (-12) \cdot (1) = -12$

5. $p'(x) = f'(f(x)) \cdot f'(x)$
$p'(4) = f'(f(4)) \cdot f'(4) = f'(2) \cdot (8) = (-2) \cdot (8) = -16$

7. $P_1'(8) = f(8)g'(8) + g(8)f'(8) = 8(-4) + (2)(-12) = -56$

9. $P_3'(4) = f(4)g'(2)\left(\frac{1}{2}\right) + g(2)f'(8)(2) = 2(8)\left(\frac{1}{2}\right) + (-2)(8) = -8$

11. $$Q_2'(8) = \frac{f(8)g'(8) - g(8)f'(8)}{[f(8)]^2} = \frac{8(4) - (4(-12))}{8^2} = \frac{80}{64} = \frac{5}{4}$$

13. $$Q_4'(4) = \frac{f(8)g'(2)\left(\frac{1}{2}\right) - g(2)f'(8)(2)}{[f(8)]^2} = \frac{8(1)\left(\frac{1}{2}\right) - (8)(-12)(2)}{8^2} = \frac{196}{64} = \frac{49}{16}$$

15. $v'(x) = g'(f(x)) \cdot f'(x)$
$$v'(4) = g'\left(\frac{2}{3}\right) \cdot f'(4) = \left(-\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) = -\frac{4}{9}$$

17. $t'(x) = f'(f(x)) \cdot f'(x)$
$t'(8) = f'(f(8)) \cdot f'(8) = f'(0) \cdot (-3) = 1/2 \cdot (dne) = dne$

19. $$P_1'(8) = f(8)g'(8) + g(8)f'(8) = 0\left(-\frac{2}{3}\right) + \left(-\frac{2}{3}\right)(dne) = dne$$

21. $P_3{}'(2) = f(2)g'(1)\left(\frac{1}{2}\right) + g(1)f'(2)(2) = -\frac{2}{3}(dne)\left(\frac{1}{2}\right) + (2)\left(\frac{2}{3}\right) = dne$

23. $Q_2{}'(8) = \frac{f(8)g'(8) - g(8)f'(8)}{[f(8)]^2} = \frac{0\left(-\frac{2}{3}\right) - \left(-\frac{2}{3}\right)(dne)}{0^2} = dne$

25. $Q_4{}'(4) = \frac{f(8)g'(2)\left(\frac{1}{2}\right) - g(2)f'(8)(2)}{[f(8)]^2} = \frac{0(0)\left(\frac{1}{2}\right) - (2)(dne)(2)}{0^2} = dne$

27a. $g(4) = 3;\ g'(4) = 6 \rightarrow y - 3 = 6(x - 4)$

27b. $K'(8) = g'(g(8))\ g'(8) = g'(4)\ g'(8) = 6\ (8) = 48$

27c. $M'(4) = g(4)\ f'(4) + f(4)\ g'(4) = 3(1) + (1)(6) = 9$

27d. $J'(4) = \frac{f(1)g'(2)(2) - g(2)f'(1)}{[1(1)]^2} = \frac{(-1)(3)(2) - 1(0)}{(-1)^2} = -6$

29a. $h\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) + e^{\cos\left(\frac{3\pi}{2}\right)} = 1 + e^0 = 2$

$h'(x) = \cos(x) + e^{\cos 3x}(\sin 3x)(-3)$

$m = h'\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) + e^{\cos\frac{3\pi}{2}}\left(\sin\frac{3\pi}{2}\right)(3) = 0 + (1)(-1)(3) = 3$

$y - 2 = 3\left(x - \frac{\pi}{2}\right)$

29b. $K'\left(\frac{\pi}{2}\right) = f'\left(h\left(\frac{\pi}{2}\right)\right) \cdot h'\left(\frac{\pi}{2}\right) = f'(2) \cdot h'\left(\frac{\pi}{2}\right) = \frac{2}{3} \cdot 3 = 2$

29c. $M'(2)=g(4)\cdot f'(2)+f(2)\cdot g'(4)\cdot(2)=3\left(\frac{2}{3}\right)+\left(-\frac{2}{3}\right)(-2)(2)=\frac{14}{3}$

29d. $J'(4)=\dfrac{f(4)g'(4)-g(4)f'(4)}{[f(4)]^2}=\dfrac{\left(\frac{2}{3}\right)(2)-3\left(\frac{2}{3}\right)}{(2/3)^2}=\dfrac{1/3}{4/9}=\dfrac{3}{4}$

31. See AP Central

1.8 Multiple Choice Homework

1. $g'(1.5)=f'(f(1.5))\,f'(1.5)=f'(2.5)\,f'(1.5)=0(.6)=0$

The correct answer is A

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3. $h'(x)=\dfrac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}\rightarrow$

$h'(2)=\dfrac{g(2)f'(2)-f(2)g'(2)}{(g(2))^2}=\dfrac{(8)(-5)-(1)(7)}{(8)^2}=\dfrac{-47}{64}$

The correct answer is D

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5. $B'(1)=f(1)g'(1)+g(1)f'(1)=\left(\frac{3}{2}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)(1)=\frac{5}{6}$

The correct answer is A

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7. $B'(1)=f'(g(1))\,g'(1)=f'(-2)\,f'(1)=\frac{1}{3}\left(-\frac{1}{2}\right)=-\frac{1}{6}$

The correct answer is B

Derivative Review Practice Test Solutions

1. $y=\ln(\sin x)\rightarrow \dfrac{dy}{dx}=\dfrac{1}{\sin x}(\cos x)=\dfrac{\cos x}{\sin x}=\cot x.$

So, the correct answer is D

3. $h(x)=g(g(x))=g'(g(x))\cdot g'(x)$

$h'(8)=g'(g(8))\cdot g'(8)=g'(2)\cdot g'(8)=(1)\cdot(4)=4$

So, the correct answer is D.

5. $f(2)=4$ and $m=f'(2)=\sqrt{2^3+1}=\sqrt{9}=3.$

Tangent line: $y-4=3(x-2)$

$f(2.2)\approx y(2.2)=3(2.2-2)+4=4.6.$

So, the correct answer is D

7. $y=\dfrac{(x^2-3)^3}{(5x-9)^2}\rightarrow \dfrac{dy}{dx}=\dfrac{(5x-9)^2 3(x^2-3)^2(2x)-(x^2-3)^3 2(5x-9)(5)}{(5x-9)^4}$

$\left.\dfrac{dy}{dx}\right|_{x=2}=\dfrac{(5(2)-9)^2 3((2)^2-3)^2(2(2))-((2)^2-3)^3 2(5(2)-9)(5)}{(5(2)-9)^4}=\dfrac{(1)(3)(1)(4)-(1)(2)(1)(5)}{1^4}=12-10=2$

So, the correct answer is D

9. $D_x\left[e^{3x^2}\cos 4x\right]=e^{3x^2}(-\sin 4x)(4)+\cos 4x e^{3x^2}(6x)=2e^{3x^2}(3\cos 4x-2\sin 4x)$

11a. $f(x)=e^{k(x-1)}+g(2x)\rightarrow f(1)=e^{k(1-1)}+g(2)=e^0+5=6$

$f(x)=e^{k(x-1)}+g(2x)\rightarrow f'(x)=e^{k(x-1)}k+g'(2x)\cdot 2=ke^{k(x-1)}+2g'(2x)$

$f'(1)=ke^{k(1-1)}+2g'(2)=ke^0-2(2)=k-4$

$f'(x)=ke^{k(x-1)}+2g'(2x)\rightarrow f''(x)=ke^{k(x-1)}(k)+2g''(2x)(2)=k^2e^{k(x-1)}+4g''(2x)$

$f''(x)=ke^{k(x-1)}(k)+2g''(2x)(2)\rightarrow f''(1)=k^2e^{k(1-1)}+4g''(2)=k^2+12$

11b. Show that the fourth derivative of f is $k^4e^{k(x-1)}+16g^{IV}(2x)$

$f''(x)=k^2e^{k(x-1)}+4g''(2x)\rightarrow f'''(x)=k^2e^{k(x-1)}(k)+4g'''(2x)(2)=k^3e^{k(x-1)}+8g'''(2x)$

$f'''(x)=k^3e^{k(x-1)}+8g'''(2x)\rightarrow f^{IV}(x)=k^3x^{k(x-1)}(k)+8g^{IV}(2x)\cdot(2)=k^4x^{k(x-1)}+16g^{IV}(2x)$