3.1 Free Response Solutions

1. $$\int_{-1}^{3} x^5 dx = \left[\frac{x^6}{6}\right]_{-1}^{6} = 6^5 - \frac{1}{6} = \frac{364}{3}$$

3. $$\int_{-5}^{5} \frac{2}{x^3} dx = \left[2\frac{x^{-2}}{-2}\right]_{-5}^{5} = \left(-\frac{1}{25}\right) - \left(-\frac{1}{25}\right) = 0$$

5. $$\int_{1}^{2} \frac{3}{t^4} dt = \left[3\frac{t^{-3}}{-3}\right]_{1}^{2} = \frac{7}{8}$$

7. $$\int_{0}^{\pi/4} \sec^2 y \; dy = [\tan y]_0^{\pi/4} = 1 - 0 = 1$$

9. $$\int_{0}^{e^2-1} \frac{1}{x+1} dx = \ln[x+1]_0^{e^2-1} = \ln\left(e^2 - 1 + 1\right) - \ln 1 = \ln\left(e^2\right) - \ln 1 = 2 - 0 = 2$$

11. $$\int_{3}^{5} \left(x^2 + 5x + 6\right) dx = \left[\frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x\right]_3^5 = \left(\frac{125}{3} + \frac{125}{2} + 30\right) - \left(9 + \frac{45}{2} + 18\right) = 84\frac{2}{3}$$

13. $$\int_{\pi}^{\frac{3\pi}{4}} \cos y dy = [\sin y]_{\pi}^{\frac{3\pi}{4}} = \sin\frac{3\pi}{4} - \sin\pi = \frac{1}{\sqrt{2}}.$$

15. $$\int_{1}^{2} \left(\frac{x^2 - 4x + 7}{x}\right) dx = \int_{1}^{2} \left(x - 4 + \frac{7}{x}\right) dx$$
$$= \left[x^2 - 4x + 7\ln x\right]_1^2 = (4 - 8 + 7\ln 2) - (1 - 4 + 0) = 7\ln 2 - 1$$

17. $$\int_{-2}^{1} f(x) dx = \int_{-2}^{5} f(x) dx - \int_{1}^{5} f(x) dx = -2 - 3 = -5$$

19. $$\int_{-2}^{1} h(x) dx = \int_{-2}^{5} h(x) dx - \int_{1}^{5} h(x) dx = \int_{5}^{-2} h(x) dx - \int_{1}^{5} h(x) dx = -(-6) - 7 = -1$$

21. $\int_{-2}^{5}[g(x)+h(x)]dx=\int_{-2}^{5}[g(x)]dx+\int_{-2}^{5}[h(x)]dx=\int_{-2}^{1}[g(x)]dx-\int_{5}^{1}[g(x)]dx-\int_{5}^{-2}[h(x)]dx=4-9-(-6)=1$

23. $\int_{-2}^{5}[h(x)+f(x)]dx=\int_{-2}^{5}[h(x)]dx+\int_{-2}^{5}[f(x)]dx=-\int_{5}^{-2}[h(x)]dx+\int_{-2}^{5}[f(x)]dx=-(-6)+(-2)=4$

25. $\int_{-2}^{1}[2f(x)-3g(x)]dx=2\int_{-2}^{1}[f(x)]dx-3\int_{-2}^{1}[g(x)]dx=2\left(\int_{-2}^{5}[f(x)]dx-\int_{1}^{5}[f(x)]dx\right)-3\int_{-2}^{1}[g(x)]dx=$
$=2(-2-3)-3(4)=-22$

27. $\int_{1}^{5}\left[\frac{1}{3}h(x)+2f(x)\right]dx=\frac{1}{3}\int_{1}^{5}[h(x)]dx+2\int_{1}^{5}[f(x)]dx=\frac{1}{3}(3)+2(7)=15$

29. $g(y)=\int_{2}^{y}t^2\sin t\,dt\Rightarrow g'(y)=y^2\sin y$

31. $F(x)=\int_{x}^{2}\cos(t^2)dt\Rightarrow F(x)=-\int_{2}^{x}\cos(t^2)dt$

33. $y=\int_{3}^{\sqrt{x}}\frac{\cos t}{t}dt\Rightarrow y'=\frac{\cos\sqrt{x}}{\sqrt{x}}\cdot\frac{1}{2}x^{-1/2}=\frac{\cos\sqrt{x}}{2x}$

35. $\frac{d}{dx}\left[\int_{e}^{x^2}\ln(t^2+1)dt\right]=\ln((x^2)^2+1)\frac{d}{dx}(x^2)=[\ln(x^4+1)](2x)=2x\ln(x^4+1)$

37. $\frac{d}{dx}\int_{10}^{x^2}t\ln(t)dt=x^2\ln x^2(2x)=4x^3\ln x$

39. $$h'(y) = \frac{d}{dx}\int_5^{\ln y} \frac{e^t}{t^4}\,dt = \frac{e^{\ln y}}{\ln^4 y} = \frac{y}{\ln^4 y}$$

3.1 Multiple Choice Solutions

1. $$\int_{-5}^{5} f(x)\ dx = \int_{-5}^{2} f(x)\ dx - \int_{5}^{2} f(x)\ dx = -17-(-4) = -13$$

The correct answer is D.

---

3. $$\int_7^3 P(t)dt = -\left[\int_3^7 P(t)dt\right] = -\left[\int_2^7 P(t)dt - \int_2^3 P(t)dt\right] = -[-2-(7)] = 9$$

The correct answer is D.

---

5. $$= \frac{1}{2}\left[\int_0^6 [f(x)]\,dx - \left[-\int_3^6 \left[f(x)\right] dx\right]\right] - 3\left[\int_3^0 [g(x)]\,dx\right] = \frac{1}{2}[9+5] - 3[7] = 28$$

The correct answer is E.

---

7. $$\int_5^{-2} [g(x)-f(x)]dx = -\int_{-2}^{5} [g(x)]dx + \int_{-2}^{5} [f(x)]dx = -\left[\int_{-2}^{1} [g(x)]dx - \int_{5}^{1} [g(x)]dx\right] + \int_{-2}^{5} [f(x)]dx =$$

$$= -(4-9) + (-2) = 3$$

The correct answer is B.

3.2 Free Response Solutions

1. $$\int_0^1 x^2(1+2x^3)^5dx \quad \boxed{\begin{array}{l} u=1+2x^3 \\ du=6x^2dx \end{array}}$$

$$=\frac{1}{6}\int_0^1 6x^2(1+2x^3)^5dx=\frac{1}{6}\int_1^3 u^5du=\frac{1}{6}\cdot\ \frac{1}{6}u^6\Big|_1^3=\frac{1}{36}(728)=\frac{182}{9}$$

3. $$\int_{-1}^1 x\sqrt{4-x^2}\,dx \qquad \boxed{\begin{array}{l} u=4-x^2 \\ du=-2xdx \end{array}}$$

$$=-\frac{1}{2}\int_{-1}^1(-2x)(4-x^2)^{1/2}dx=-\frac{1}{2}\int_3^3 u^{1/2}du=0$$

5. $$\int_0^3 \frac{10t+15}{\sqrt[4]{t^2+3t+1}}dt \qquad \boxed{\begin{array}{l} u=t^2+3t+1 \\ du=(2t+3)dt \end{array}}$$

$$=\int_0^3 \frac{5(2t+3)}{\sqrt[4]{t^2+3t+1}}dt=5\int_1^{19}u^{-1/4}du=5\cdot\ \frac{4}{3}u^{3/4}\Big|_1^{19}=\frac{20}{3}\left(19^{3/4}-1\right)$$

7. $$\int_1^3 \frac{5t}{t^2+1}dt \qquad \boxed{\begin{array}{l} u=1+2x^3 \\ du=6x^2dx \end{array}}$$

$$=\frac{5}{2}\int_1^3\frac{2t}{t^2+1}dt=\frac{5}{2}\int_2^{10}\frac{du}{u}=\ \frac{1}{2}\ln u\Big|_2^{10}=\frac{5}{2}(\ln 10-\ln 2)=\frac{5}{2}\ln 5$$

9. $$\int_{\sqrt{3}}^{\sqrt{4}} ye^{y^2-3}dy=\frac{1}{2}\int_{\sqrt{3}}^{\sqrt{4}}2ye^{y^2-3}dy=\frac{1}{2}\int_0^1 e^u du=\frac{1}{2}\left[e^u\right]_0^1=\frac{1}{2}[e-1]$$

11. $\int_3^{e^2+2} \frac{1}{x-2}\,dx$ $\boxed{\begin{aligned} u &= x-2 \\ du &= dx \end{aligned}}$

$\int_1^{e^2} \frac{1}{u}\,du = [\ln u]_1^{e^2} = 2-0 = 2$

13. $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$ $\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}dx \end{aligned}}$

$= \int_1^4 \frac{du}{u^{1/2}} = \int_1^4 u^{-1/2}du = 2\,u^{1/2}\Big|_1^4 = 2\cdot 2 - 2\cdot 1 = 2$

15. $\int_0^{\pi} \frac{\sin x}{2-\cos x}\,dx$ $\boxed{\begin{aligned} u &= 2-\cos x \\ du &= \sin x dx \end{aligned}}$

$\int_0^2 f(x)dx = \int_0^1 x^4dx + \int_1^2 x^5dx = \frac{1}{5}x^5\Big|_0^1 + \frac{1}{6}x^6\Big|_1^2 = \left(\frac{1}{5}-0\right)+\left(\frac{32}{3}-\frac{1}{6}\right) = \frac{107}{10}$

17. $\int_0^{Ln2} \frac{e^x}{1+e^{2x}}\,dx$ $\boxed{\begin{aligned} u &= e^x \\ du &= e^x dx \end{aligned}}$

$= \int_1^2 \frac{du}{1+u^2} = \frac{1}{2}\int_1^2 \frac{du}{u} = \tan^{-1}u\Big|_1^2 = \tan^{-1}2 - \tan^{-1}1 = \tan^{-1}2 - \frac{\pi}{4}$

19. $\int_0^{\frac{\pi}{8}} \sec^2(2x)dx = \frac{1}{2}\int_0^{\pi/4}\sec^2(2x)(2dx) = \frac{1}{2}\int_0^{\pi/4}\sec^2 u du = \frac{1}{2}\tan u\Big]_0^{\pi/4} = \frac{1}{2}[1-0] = \frac{1}{2}$

21. $\int_0^{\pi} \frac{\cos x}{2+\sin x}\,dx$ $\boxed{\begin{aligned} u &= 2+\sin x \\ du &= \cos x dx \end{aligned}}$

$$\int_2^2 f(x)\,dx = 0$$

23. $$\int_0^{\sqrt{\frac{\pi}{4}}} m\sec(m^2)\tan(m^2)\,dm = \frac{1}{2}\int_0^{\sqrt{\frac{\pi}{4}}}\sec(m^2)\tan(m^2)2m\,dm = \frac{1}{2}\int_0^{\pi/4}\sec u\tan u\,du$$

$$= \frac{1}{2}[\sec u]_0^{\pi/4} = \frac{1}{2}\left[\frac{1}{\sqrt{2}} - 1\right] = \frac{1-\sqrt{2}}{2\sqrt{2}}$$

25. $$\int_{\frac{\pi}{2}}^{\pi}\cos^9(x)\sin(x)\,dx$$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x\,dx \end{aligned}}$$

$$\int_{\frac{\pi}{2}}^{\pi}\cos^9(x)\sin(x)\,dx = -\int_0^{-1}u^9\,du = \left[-\frac{1}{10}u^{10}\right]_0^{-1} = -\frac{1}{10}$$

27. $$\int_{\pi}^{2\pi}\cos\frac{1}{2}\theta\ d\theta$$

$$\boxed{\begin{aligned} u &= \frac{1}{2}\theta \\ du &= \frac{1}{2}d\theta \end{aligned}}$$

$$= 2\int_{\pi}^{2\pi}\frac{1}{2}\cos\frac{1}{2}\theta\,d\theta = 2\int_{\pi/2}^{\pi}\cos u\,du = 2\sin u|_{\pi/2}^{\pi} = 2\left(\sin\pi - \sin\frac{\pi}{2}\right) = -2$$

29. $$\int_0^{e^2-1}\frac{1}{x+1}\,dx$$

$$\boxed{\begin{aligned} u &= x+1 \\ du &= dx \end{aligned}}$$

$$\int_0^{e^2-1}\frac{1}{x+1}\,dx = \int_1^{e^2}\frac{1}{u}\,du = [\ln u]_1^{e^2} = 2-0 = 2$$

30. $\int_{5}^{e^3+4} \frac{1}{x-4}\,dx$ $\boxed{\begin{array}{l} u = x-4 \\ du = dx \end{array}}$

$$\int_{5}^{e^3+4} \frac{1}{x-4}\,dx = \int_{1}^{e^3} \frac{1}{u}\,du = [\ln u]_1^{e^3} = 3-0 = 3$$

31. $\text{Ave Value} = \frac{1}{7-3}\int_{3}^{7}(x-3)^2 dx = \frac{1}{4}\left[\frac{(x-3)^3}{3}\right]_3^7 = \frac{1}{4}\left[\frac{4^3}{3}-0\right] = \frac{16}{3}$

33. $\text{Ave Value} = \frac{1}{\pi/4 - 0}\int_{0}^{\pi/4}\sec^2 x\,dx = \frac{4}{\pi}[\tan x]_0^{\pi/4} = \frac{1}{4}[1-0] = \frac{4}{\pi}$

35. $\text{Ave Value} = \frac{1}{4-1}\int_{1}^{4}\left(t^2 - t^{1/2} + 5\right)dt = \frac{1}{3}\left[\frac{1}{3}t^3 - \frac{2}{3}t^{3/2} + 5t\right]_1^3 = 30\frac{4}{9}$

37. $f(x) = \cos x \sin^4 x$ on $x \in [0, \pi]$ $\boxed{\begin{array}{l} u = \sin x \\ du = \cos x dx \end{array}}$

$$f_{avg} = \frac{1}{\pi - 0}\int_{0}^{\pi}\cos x \sin^4 x dx = \frac{1}{\pi}\int_{0}^{0} u^4 du = 0$$

39. $G(x) = \frac{x}{\left(1+x^2\right)^3}$ on $x \in [0, 2]$ $\boxed{\begin{array}{l} u = 1+x^2 \\ du = 2x dx \end{array}}$

$$G_{avg} = \frac{1}{2-0}\int_{0}^{2}\frac{x}{\left(1+x^2\right)^3}dx$$

$$= \frac{1}{2}\cdot\frac{1}{2}\int_{0}^{2}\frac{2x}{\left(1+x^2\right)^3}dx$$

$$= \frac{1}{4}\int_{1}^{5}\frac{du}{u^3} = \frac{1}{4}\int_{1}^{5}u^{-3}du = \frac{1}{4}\left.\frac{u^{-2}}{-2}\right|_1^5 = -\frac{1}{8}\left(5^{-2}-1\right) = -\frac{1}{8}\left(\frac{-24}{25}\right) = \frac{3}{25}$$

41. Ave Temp $= \frac{1}{10}\int_0^{10}\left(60+390e^{-.205t}\right)\,dt = 225.753°F$

43. Ave Length $= \frac{1}{6}\int_9^{15}\left(T(t)=50+15\sin\frac{\pi}{12}t\right)\,dt = 63.045°F$

3.2 Multiple Choice Solutions

1. $$\int_1^4 \frac{dx}{(1+\sqrt{x})^2\sqrt{x}} \quad \boxed{\begin{array}{l} u = 1+\sqrt{x} \\ du = \dfrac{1}{2\sqrt{x}}dx \end{array}}$$

$$2\int_2^3 \frac{du}{u^2} = \left[\frac{-2}{u}\right]_2^3 = -\frac{2}{3} - (-1) = \frac{1}{3}$$

The correct answer is B

---

3. $\int_e^{e^2} \frac{1}{x\ln x}dx = \int_1^2 \frac{1}{u}\,du = [\ln u]_1^2 = \ln 2$

The correct answer is C

---

5. $$\int\left(2-\sin\frac{t}{5}\right)^2\cos\frac{t}{5}dt = -5\int\left(2-\sin\frac{t}{5}\right)^2\left(-\cos\frac{t}{5}\right)\left(\frac{1}{5}\right)dt = -5\int u^2du$$

$$= -\frac{5}{3}u^3 + c$$

The correct answer is A

7. $g_{avg} = \frac{1}{2-0}\int_0^2 e^{2x}dx = \frac{1}{2}\left(\frac{1}{7}\int_0^2 e^{7x}7dx\right) = \left[\frac{1}{14}e^{7x}\right]_0^2 = \frac{1}{14}\left(e^{14}-1\right)$

The correct answer is C

---

9. Ave $= \frac{1}{2-(-1)}\int_{-1}^{2}\left(x^2+5x+14\right)dx = \frac{1}{3}\left[\frac{1}{3}x^3+\frac{5}{2}x^2+14x\right]_{-1}^{2} = 6$

The correct answer is B.

---

11. Average rate of change $= \frac{f(b)-f(a)}{b-a}$. $f(3)=81-15=66$ and $f(0)=0$.

$\frac{f(b)-f(a)}{b-a} = \frac{66}{3} = 22$

The correct answer is C.

---

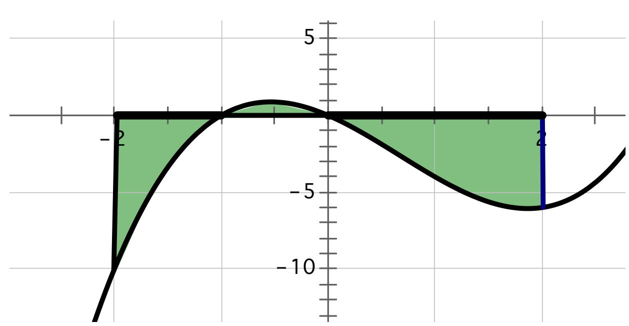
13. The average value will be zero if the areas below the x-axis and the areas above the x-axis match. For only E, this is true.

The correct answer is E

3.3 Free Response Solutions

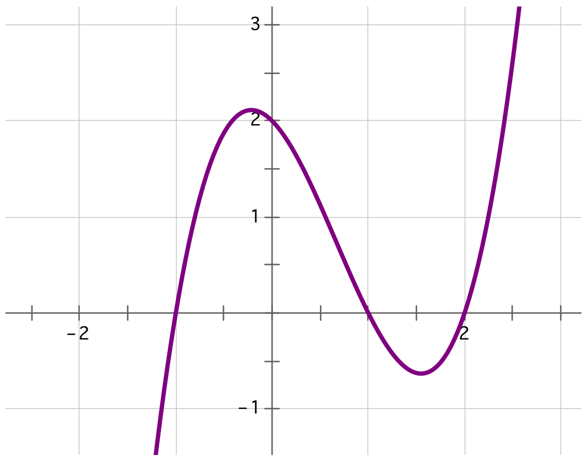
Find the area under the curve of the given equation on the given interval.

1. $y = x^3 - 2x^2 - 3x$ on $x \in [-2,2]$



$$
\begin{aligned}
A &= \int_{-2}^{-1}\left(x^3-2x^2-3x\right)dx-\int_{-1}^{0}\left(x^3-2x^2-3x\right)dx+\int_{0}^{2}\left(x^3-2x^2-3x\right)dx \\
&= \left[\frac{x^4}{4}-\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{-2}^{-1}-\left[\frac{x^4}{4}-\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{-1}^{0}+\left[\frac{x^4}{4}-\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{0}^{2} \\
&= \left[\left(4+\frac{16}{3}+6\right)-\left(\frac{1}{4}+\frac{2}{3}-\frac{3}{2}\right)\right]-\left[0-\left(\frac{1}{4}+\frac{2}{3}-\frac{3}{2}\right)\right]+\left[4-\frac{16}{3}-6\right] \\
&= 11.833
\end{aligned}
$$

3. $y = x^3 - 2x^2 - x + 2$ on $x \in [-3,3]$



$$A = -\int_{-3}^{-1}(x^3 - 2x^2 - x + 2)dx + \int_{-1}^{1}(x^3 - 2x^2 - x + 2)dx - \int_{1}^{2}(x^3 - 2x^2 - x + 2)dx$$
$$+ \int_{2}^{3}(x^3 - 2x^2 - x + 2)dx$$
$$= -\left[\frac{x^4}{4} - \frac{2}{3}x^3 - x^2 + 2x\right]_{-3}^{-1} + \left[\frac{x^4}{4} - \frac{2}{3}x^3 - x^2 + 2x\right]_{-1}^{1} - \left[\frac{x^4}{4} - \frac{2}{3}x^3 - x^2 + 2x\right]_{1}^{2}$$
$$+ \left[\frac{x^4}{4} - \frac{2}{3}x^3 - x^2 + 2x\right]_{2}^{3}$$
$$= 35.5$$

5. $y = \dfrac{-x}{x^2 + 4}$ on $x \in [-2,2]$

$$\boxed{\begin{array}{l} u = x^2 + 4 \\ du = 2xdx \end{array}}$$

$$A = \int_{-2}^{0}\frac{-x}{x^2+4}dx - \int_{0}^{2}\frac{-x}{x^2+4}dx$$
$$= 2\int_{-2}^{0}\frac{-x}{x^2+4}dx$$
$$= -\int_{-2}^{0}\frac{2x}{x^2+4}dx$$
$$= -\int_{8}^{4}\frac{1}{u}du = -\ln u\Big|_{8}^{4} = -\ln 4 - (-\ln 8) = -\ln\frac{1}{2} = \ln 2$$

7. $y=\dfrac{\sin\sqrt{x}}{\sqrt{x}}$ on $x\in\left[.01,\pi^2\right]$

$$\boxed{\begin{aligned} u &= x^{1/2} \\ du &= \frac{1}{2}x^{-1/2}dx \end{aligned}}$$

$$\begin{aligned}
A &= \int_{.01}^{\pi}\frac{\sin\sqrt{x}}{\sqrt{x}}dx \\
&= \int_{.01}^{\pi}\frac{\sin x^{1/2}}{x^{1/2}}dx \\
&= 2\int_{.01}^{\pi}\frac{\sin x^{1/2}}{2x^{1/2}}dx = 2\int_{.01}^{\sqrt{\pi}}\sin u\,du \\
&= -2\cos u\Big|_{.1}^{\sqrt{\pi}} = -2\cos\sqrt{\pi}+2\cos(.1) = 1.627
\end{aligned}$$

9. $y=3\sin x\sqrt{1-\cos x}$ on $x\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{3}\right]$

$$\boxed{\begin{aligned} u &= 1-\cos x \\ du &= \sin x\,dx \end{aligned}}$$

$$\begin{aligned}
A &= -\int_{-\pi/3}^{0}3\sin x\sqrt{1-\cos x}\,dx+\int_{0}^{\pi/2}3\sin x\sqrt{1-\cos x}\,dx \\
&= -3\int_{1/2}^{0}u^{1/2}du+3\int_{0}^{1}u^{1/2}du = -3\cdot\frac{2}{3}u^{3/2}\Big|_{1/2}^{0}+3\cdot\frac{2}{3}u^{3/2}\Big|_{0}^{1} = 2.707
\end{aligned}$$

11. The velocity function (in meters per second) for a particle moving along a line is $v(t)=3t-5$ for $0\le t\le 3$. Find (a) the displacement and (b) the distance traveled by the particle during the given time interval.

(a) $$\int_0^3(3t-5)dt = \frac{3}{2}t^2-5t\Big|_0^3 = \left(\frac{27}{2}-15\right) = -\frac{3}{2}m$$

(b) $$\int_0^3|(3t-5)|dt = \int_0^{5/3}-(3t-5)dt+\int_{5/3}^{3}(3t-5)dt$$

$$= -\frac{3}{2}t^2+5t\Big|_0^{5/3}+\frac{3}{2}t^2-5t\Big|_{5/3}^{3} = \frac{41}{6}m$$

13. Find the area under the curve $f(x) = e^{-x^2} - x$ on $x \in [-1, 2]$ (do not use absolute values in your setup, break it into multiple integrals).

$\int_0^a \left(e^{-x^2} - x\right)dx - \int_a^2 \left(e^{-x^2} - x\right)dx = 3.080$, where $a = 0.65291864$

15. Find the area under the curve $f(x) = \dfrac{x}{x^2+1} + \cos(x)$ on $x \in [0, \pi]$

$\int_0^b \left(\frac{x}{x^2+1} + \cos(x)\right)dx - \int_b^\pi \left(\frac{x}{x^2+1} + \cos(x)\right)dx = 2.235$, where $b = 0.1.9843654$

3.3 Multiple Choice Homework

1. $\int_{-5}^{3} f(x)\, dx + \int_{-2}^{3} f(x)\, dx = A + (-B)$

The correct answer is B

---

3. $\int_0^4 f(x)dx = \int_0^2 f(x)dx + \int_2^4 f(x) = 6 + (-6) = 0?$

The correct answer is C

---

5. $\int_0^2 \left|3e^{-t^2}\sin(2t)\right|dt = 1.661$ by calculator

The correct answer is C

---

7. $\int_0^2 \left(3e^{-t^2}\sin(2t)\right)dt = 1.625$ by calculator

The correct answer is B

3.4 Free Response Solutions

1. $\int_5^{10} w'(t)\ dt$ represents the change of the child's weight in pounds from ages 5 to 10 years

3. Newtons-meters, or joules

5a) $\int_0^{6.2}\left[8843\left(\frac{t}{5}\right)^4\left(1-\frac{t}{10}\right)^5\right]\ dt = 971.595$

971 or 972 passengers have gotten into line.

5b) $E'(6.2) = 8843\left(\frac{6.2}{5}\right)^4\left(1-\frac{6.2}{10}\right)^5 = -111.092$, so the rate of change of people entering the processing line is decreasing because $E'(6.2) < 0$.

5c) $Total\ people\ in\ line = 2500 + \int_0^{6.2}\left[8843\left(\frac{t}{5}\right)^4\left(1-\frac{t}{10}\right)^5 - 250\right]\ dt = 1921.595$

There are 1921 or 1922 people in line

7a) $M(t) = \int_0^{12}\left(8 - \frac{e^{0.47t}}{t+6}\right)dt = 76.844\ cg$

7b) $Total = \int_0^{9}\left(8 - \frac{e^{0.47t}}{t+6}\right) - (7 - .46t\cos(t))dt = 4.143\ cg$

7c) $L'(6) = -1.213$. The rate at which the liver is cleansing the blood is decreasing at a rate of $-1.213\ {}^{cg}\!/_{hr^2}$ when $t = 6$.

d) $\int_0^{t}\left(8 - \frac{e^{0.47x}}{x+6}\right) - (7 - .46x\cos(x))dx = 0$

9a) $P'(2) = 0.398$ hundreds of letters per hour per hour. The rate at which the 100s of letters per hour are coming intot he post office is increasing by approximately 40 letters per hr per hr.

9b) $\int_0^{3} P(t)\ dt = 24.604\ hundreds\ of\ letters$ or 2460 letters.

9c) Write an expression for $L(t) = 3 + \int_0^t [P(x) - 5]\, dx$, the total number of letters in the post office at time *t*.

11a) $A = \int_0^{12} \left[2 + \dfrac{10}{1 + \ln(t+1)}\right] dt = 70.571$

11b) $E(6) - L(6) = -2.924 > 0 \therefore \text{falling}$

11c) $A = 125 + \int_0^{12} \left[\left(2 + \dfrac{10}{1+\ln(t+1)}\right) - \left(12\sin\left(\dfrac{t^2}{47}\right)\right)\right] dt = 122.026 \ \textit{gallons}$

11d) $\left(2 + \dfrac{10}{1+\ln(t+1)}\right) - \left(12\sin\left(\dfrac{t^2}{47}\right)\right) = 0 \rightarrow t = 4.790 \ \text{and} \ \ 11.318.$

Time for the maximum occur at $t = 4.790 \ \text{and} \ 12$

$A(4.790) = 149.408$

$A(12) = 122.026$

$Abs \ \ Max = 149.408 \therefore$ no, the tank never overflows.

3.4 Multiple Choice Homework

1. $H'(24)$ would be an instantaneous rate of change.

The correct answer is E

---

3. Since $E(t)$ and $L(t)$ are rates, so is $F(t)$. $F(16)$ would be the rate of change of the number of people in the park at 4 pm.

The correct answer is D

---

5. Acre-feet of this field = $\int_0^3 (4 - \sin^3 t)\, dt = 10.667$

The correct answer is B

---

7. Average rate of change = $\dfrac{P(365)-P(0)}{365-0} = 154{,}120$

The correct answer is C

---

9. $C(9) = 10 + \int_0^9 \left(1 - 3e^{-0.2\sqrt{t}}\right) dt = 0.715$

The correct answer is E

---

11. $T(5) = \int_0^5 20e^{0.02t}\, dt = 105$

The correct answer is D

3.5 Free Response Solutions

1a. $F_{avg}(x) = \dfrac{1}{8-0}\displaystyle\int_0^8 F(x)dx$. Right Hand Rectangles, n = 8

$$\approx \frac{1}{8}\left[1\cdot F(1)+1\cdot F(2)+1\cdot F(3)+1\cdot F(4)+1\cdot F(5)+1\cdot F(6)+1\cdot F(7)+1\cdot F(8)\right]$$
$$= \frac{1}{8}[15+17+12+3-5+8-2+10]$$
$$= 7.25$$

1b. Left Hand Rectangles, n = 8

$$\approx \frac{1}{8}\left[1\cdot F(0)+1\cdot F(1)+1\cdot F(2)+1\cdot F(3)+1\cdot F(4)+1\cdot F(5)+1\cdot F(6)+1\cdot F(7)\right]$$
$$= \frac{1}{8}[10+15+17+12+3-5+8-2]$$
$$= 7.25$$

1c. Midpoint Rectangles, n = 4

$$\frac{1}{8-0}\int_0^8 F(x)dx \approx \frac{1}{8}\left[2\cdot F(1)+2\cdot F(3)+2\cdot F(5)+2\cdot F(7)\right]$$
$$= 30$$

3a. $$\int_0^{60} v(t)dt \approx 20\cdot v(10)+20\cdot v(30)+20\cdot v(50)$$

$$= 20\cdot(28)+20\cdot(18)+20\cdot(48)$$
$$= 1{,}880\ km$$

b) Find an approximation for $\displaystyle\int_0^{30} v(t)dt$ using trapezoids.

3b. $$\int_0^{30} v(t)dt \approx \frac{1}{2}[v(0)+v(10)]\cdot(10)+\frac{1}{2}[v(10)+v(20)]\cdot(10)+\frac{1}{2}[v(20)+v(30)]\cdot(10)$$

$$= \frac{1}{2}[30+28]\cdot(10)+\frac{1}{2}[28+32]\cdot(10)+\frac{1}{2}[32+18]\cdot(10)$$
$$= 840\ km$$

c) Find an approximation for $\int_{30}^{60} v(t)dt$ using left rectangles.

3c. $$\int_{30}^{60} v(t)dt \approx 20 \cdot v(30) + 20 \cdot v(40) + 20 \cdot v(50)$$

$$= 10 \cdot (18) + 10 \cdot (52) + 10 \cdot (48)$$
$$= 1{,}880 \;\; km$$

d) Find an approximation for $\int_{0}^{40} v(t)dt$ using right rectangles.

$$\int_{0}^{40} v(t)dt \approx 10 \cdot v(10) + 10 \cdot v(20) + 10 \cdot v(30)$$

$$= 10 \cdot (28) + 10 \cdot (32) + 10 \cdot (18)$$
$$= 1{,}300 \;\; km$$

5a) Find an approximation for $\int_{0}^{360} v(t)dt$ using left Riemann rectangles.

$$\int_{0}^{360} v(t)dt \approx 30 \cdot v(0) + 60 \cdot v(30) + 30 \cdot v(90) + 100 \cdot v(120) + 80 \cdot v(220) + 60 \cdot v(300)$$

$$= 30 \cdot (0) + 60 \cdot (21) + 30 \cdot (43) + 100 \cdot (38) + 80 \cdot (30) + 60 \cdot (24)$$
$$= 10{,}190m$$

b) Find an approximation for $\int_{0}^{220} v(t)dt$ using trapezoids.

$$\int_{0}^{220} v(t)dt \approx \frac{1}{2}[v(0) + v(30)] \cdot (30) + \frac{1}{2}[v(30) + v(90)] \cdot (60) + \frac{1}{2}[v(90) + v(120)] \cdot (30) + \frac{1}{2}[v(120) + v(220)] \cdot (100)$$

$$= \frac{1}{2}[0 + 21] \cdot (30) + \frac{1}{2}[21 + 43] \cdot (60) + \frac{1}{2}[43 + 38] \cdot (30) + \frac{1}{2}[38 + 30] \cdot (100)$$
$$= 6850 \;\; m$$

7a. Find an approximation for $\int_0^{20} V(t)dt$ using trapezoids.

$$\int_0^{20} V(t)dt \approx \frac{1}{2}[v(0)+v(4)]\cdot(4)+\frac{1}{2}[v(4)+v(6)]\cdot(2)+\frac{1}{2}[v(6)+v(10)]\cdot(4)$$

$$+\frac{1}{2}[v(10)+v(13)]\cdot(3)+\frac{1}{2}[v(13)+v(15)]\cdot(2)+\frac{1}{2}[v(15)+v(20)]\cdot(5)$$

$$=\frac{1}{2}[83+68]\cdot(4)+\frac{1}{2}[68+82]\cdot(2)+\frac{1}{2}[82+40]\cdot(4)+\frac{1}{2}[40+38]\cdot(3)$$

$$+\frac{1}{2}[38+30]\cdot(2)+\frac{1}{2}[30+68]\cdot(5)$$

$$=1{,}130 \ \ gallons$$

7b) Find an approximation for $\int_0^{20} V(t)dt$ using left Riemann rectangles.

$$\int_0^{20} V(t)dt \approx V(0)\cdot 4+V(4)\cdot 2+V(6)\cdot 4+V(10)\cdot 3+V(13)\cdot 2+V(15)\cdot 5$$

$$=(83)\cdot 4+(68)\cdot 2+(82)\cdot 4+(40)\cdot 3+(38)\cdot 2+(30)\cdot 5$$

$$=1{,}134 gallons$$

9a. $$\int_0^2 \sqrt[4]{1+x^2}\,dx \approx \frac{2-0}{2\cdot 8}\left[\begin{array}{l} f(0)+2f\left(\frac{1}{4}\right)+2f\left(\frac{1}{2}\right)+2f\left(\frac{3}{4}\right) \\ +2f(1)+2f\left(\frac{5}{4}\right)+2f\left(\frac{3}{2}\right)+2f\left(\frac{7}{4}\right)+f(2)\end{array}\right]$$

$$=\frac{1}{8}\left[1+2(1.015)+2(1.057)+2(1.118)+2(1.189)+2(1.265)+2(1.343)+2(1.420)+1.495\right]$$

$$=2.414$$

b. $$\int_0^2 \sqrt[4]{1+x^2}\,dx \approx \frac{1}{4}\left[\begin{array}{l} f\left(\frac{1}{8}\right)+f\left(\frac{3}{8}\right)+f\left(\frac{5}{8}\right)+f\left(\frac{7}{8}\right)+ \\ \quad f\left(\frac{9}{8}\right)+f\left(\frac{11}{8}\right)+f\left(\frac{13}{8}\right)+f\left(\frac{15}{8}\right)\end{array}\right]$$

$$=2.411$$

11. See AP Central

3.5 Multiple Choice Solutions

1. Because the curve is decreasing, the right-hand sum will be the lowest value.

The correct answer is C

---

3. $10(90)+20(88)+30(100)+10(90)=6560$

The correct answer is E

---

5. $\int_0^8 f(x) \approx 2(8)+2(4)+2(4)+2(4)=40$

The correct answer is C

---

7. $\int_0^8 f(x) \approx 1(7)+1(8)+1(7)+1(4)+1(2)+1(4)+1(8)+1(4)=44$

The correct answer is E

---

9. $\approx 1\left[\frac{1}{2}(7+2(8)+2(7)+2(4)+2(2)+2(4)+2(8)+2(4)+0)\right]=40.5$

The correct answer is C

---

11. The Trapezoidal Rule $\approx \frac{b-a}{2n}\left[\frac{1}{2}\left(f(a)+2\left(f\left(x_1\right)\right)+2\left(f\left(x_2\right)\right)+ + (f(b))\right)\right]$

The correct answer is C

3.6 Free Response Solutions

1a) $V'(7)=\dfrac{V(8)-V(0)}{8-0}=\dfrac{32-26}{8-0}=\dfrac{6}{8}=\dfrac{3}{4}\ m^3/\text{min}^2$

1b) $\int_8^{40} V'(t)dt = V(40)-V(8)=24-32=-8$

1c) $1344\ m^3$ of water flows through the pipeline between $t=0$ and $t=48$ minutes.

$$\int_0^{48} V(t)dt \approx 8\left(\frac{26+32}{2}\right)+8\left(\frac{32+43}{2}\right)+8\left(\frac{43+24}{2}\right)$$
$$+8\left(\frac{24+19}{2}\right)+8\left(\frac{19+24}{2}\right)+8\left(\frac{24+26}{2}\right)$$
$$=1344$$

1d) $\dfrac{1}{48}\int_0^{48} V(t)dt$ represents the average rate, in m^3/min, of the flow of water through the pipeline between $t=0$ and $t=48$ minutes.

3a) Estimate $R'(30)$. Show the work that leads to your answer. Indicate the units.

$$R'(30)\approx\frac{R(40)-R(20)}{40-20}=\frac{40-30}{40-20}-\frac{10}{20}=\frac{1}{2}\,gallons/\text{min}^2$$

3b) Use right hand Riemann rectangles to approximate $\int_0^{90} R(t)\ dt$ and indicate units of measure. Explain the meaning of $\int_0^{90} R(t)\ dt$ in terms of the fuel consumption.

$$\int_0^{90} R(t)\ dt \approx 20(30)+20(40)+10(55)+10(65)+30(70)=4700\,gallons$$

$\int_0^{90} R(t)\ dt$ it the total consumption of gallons of fuel between $t=0$ and $t=90$ minutes.

3c) Use left hand rectangles to find $\dfrac{1}{70}\int_{20}^{90} R(t)\ dt$. Using the correct units, explain the meaning of $\dfrac{1}{70}\int_{20}^{90} R(t)\ dt$ in terms of the fuel consumption

$$\frac{1}{70}\int_{20}^{90} R(t)\,dt = \frac{1}{70}[20(30)+10(40)+10(55)+30(65)] = 50\,{}^{gal}\!/_{\min}$$

The average rate of consumption of fuel, in gallons per minute, between $t = 0$ and $t = 90$ minutes is 50.

5a) $\int_0^{21} A'(t)\,dt = A(21) - A(0) = 8.6 - 10.3 = -1.7$. The patient's A1c score hase dropped 1.7 over the span from $\mathrm{t} = 0$ to $\mathrm{t} = 21$ months.

5b) $\int_0^{21} A(t)\,dt \approx 3(10.0)+3(10.5)+3(9.1)+3(8.0)+3(8.9)+3(8.3)+3(8.6) = 189.3$.

$\frac{1}{21}\int_0^{21} A(t)\,dt \approx 9.01$ is the patient's average A1c score per month from $\mathrm{t} = 0$ to $\mathrm{t} = 21$ months.

5c) $\frac{1}{21}\int_0^{21} B(t)\,dt = 7.372$.

7a) $C_e'(3.4) = \frac{420.5 - 538.4}{4-3} = -117.9\,{}^{kW}\!/_{month}$ and $C_g'(3.4) = \frac{79.8 - 116}{4-3} = -36.2\,{}^{therms}\!/_{month}$. Since both values are negative, consumption of both commodities were decreasing at $t = 3.4$ months.

7b) $\int_0^{12} C_e(t)\,dt \approx 660 + 667.1 + 538.4 + 420.5 + 412.1 + 347.8 + 287.5$

$+303.1 + 322.4 + 342.5 + 390.3 + 384.2 = 5075.3\ kW$

7c) $\int_0^{12} C_g(t)\,dt \approx 2(84.6) + 2(116) + 2(53.9) + 2(24.9) + 2(18) + 2(48.9) = 692.6$ therms.

7d) $\frac{1}{12}\int_0^{12} C_g(t)\,dt$ is the average gas consumption in therms per month during these 12 months.

---

9 a) $W(2) \approx \frac{-1.2 - (-3.1)}{3-1} = 0.45\ {}^{\circ C}\!/_{hr^2}$

9b) $\int_0^8 W(t)dt \approx 1(-3.1) + 2(-1.2) + 3(1.9) + 2(2.5) = 2.1\,°C$ The temperature in Sauris on this night has risen approximately $2.1\,°C$ between midnight and 8:00am.

9c) $T(1pm) = T(13) = -8 + \int_0^{13} W(t)\,dt.$

Definite Integral Practice Chapter Test

1. Find $\int_1^4 \frac{6}{\sqrt{x}}\,dx = \left[6\frac{x^{1/2}}{1/2}\right]_1^4 = 12[2-1] = 12$

The correct answer is A

---

3. $\int_1^2 \frac{1}{\sqrt{1-\frac{1}{4}t^2}}\,dt = 2\int_1^2 \frac{1}{\sqrt{1-\left(\frac{1}{2}t\right)^2}}\,\frac{1}{2}dt = 2\int_{1/2}^1 \frac{1}{\sqrt{1-u^2}}\,du$

$= 2\left[\sin^{-1}u\right]_{1/2}^1 = 2\left[\frac{\pi}{2}-\frac{\pi}{6}\right] = \frac{2\pi}{3}$

The correct answer is E

---

5. $\int_0^{\pi} \cos x\,dx = [\sin x]_0^{\pi} = 0-0=0?$

The correct answer is A

---

7. Average rate of change $= \frac{f(b)-f(a)}{b-a} = \frac{28-10}{2-(-1)} = 6$ on $x \in [-1,\ 2]$

The correct answer is B

Free Response

1. $\int_0^{\pi/9}\left(\tan 3x + \frac{x}{4+x^2}\right)dx = \frac{1}{3}\int_0^{\pi/9}(\tan 3x)\,3dx + \frac{1}{2}\int_0^{\pi/9}\left(\frac{2xdx}{4+x^2}\right)$

$= \frac{1}{3}\ln|\sec 3x|\Big|_0^{\pi/9} + \frac{1}{2}\ln\left|4+x^2\right|\Big|_0^{\pi/9} = 0.246$

3. $A = -\int_{-2}^{0} xe^{3x^2}dx + \int_{0}^{1} xe^{3x^2}dx = -\frac{1}{6}\int_{12}^{0} e^u du + \frac{1}{6}\int_{0}^{3} e^u du$

$A = -\frac{1}{6}\left[e^u\right]_{12}^{0} + \frac{1}{6}\left[e^u\right]_{0}^{3} = 27128.812$

5a. $T = \int_{0}^{2} f(x)dx = 20.051 \ \ lbs$

5b. $f'(7) = -8.120 \ \ lbs/hr/hr$. $f'(7)$ means that, after the store has been open 7 hours, the rate at which the bananas are being removed is decreasing by 8.12 lbs per hour per hour.

5c. $g(5) - f(5) = -2.264 \ \ \frac{lbs}{hr}$

5d. $Total = 50 + \int_{3}^{8} g(t)dt - \int_{0}^{8} f(t)dt = 23.347 \ \ lbs$