AB Calculus '23-24 Name ___________________________
Dr. Quattrin
Volume Test Form H
Calculator Allowed Score: _______________
20 minutes

1. The region R is enclosed by the graphs of $y = -x^5$, $y = 1$ and $x = 1$. The area of region R is

a) $\int_0^1 \left(-x^5 - 1\right) dx$

b) $\int_0^1 \left(1 - x^5\right) dx$

c) $\int_{-1}^1 \left(-x^5 - 1\right) dx$

d) $\int_{-1}^1 \left(1 + x^5\right) dx$

e) $\int_0^1 \left(1 + x^5\right) dx$

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2. Let ***M*** represent the region in the first quadrant bounded by $y = 2 - e^{x^2}$. Find the volume of the solid formed by revolving ***M*** around the y-axis**.**

a) 0.833

b) 0.593

c) 1.214

d) 0.386

e) 1.535

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3. A region is bounded by $y = \dfrac{1}{\sqrt{x^3}}$, the x-axis, the line $x = m^2$, and the line $x = m^4$, where $m > 1$.

The volume of the solid

(a) is independent of m.

(b) increases as m increases.

(c) decreases as m increases.

(d) increases until $m = 1/2$, then decreases.

(e) decreases until$m = 1/2$, then increases.

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4. The region in Quadrant I is bounded by the graphs of $y = 2e^x$ and $y = 4$ is revolved about the x-axis. The volume of this solid is

(a) $\pi \int_0^{\ln 2} (4 - 2e^x)^2 dx$

(b) $\pi \int_2^4 \left(4 - \ln\left(\frac{y}{2}\right)\right)^2 dy$

(c) $\pi \int_2^{\ln 2} \left[1 - \ln^2\left(\frac{y}{2}\right)\right] dy$

(d) $\pi \int_2^4 \left[\ln^2\left(\frac{y}{2}\right)\right] dy$

(e) $\pi \int_0^{\ln 2} \left(16 - 4e^{2x}\right) dx$

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5. Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

(a) $\pi \int_0^1 \left((3-2x^2)^2 - (3-2x)^2\right)\, dx$

(b) $\pi \int_0^1 \left((3-2x)^2 - (3-2x^2)^2\right)\, dx$

(c) $\pi \int_0^1 \left(4x^4 - 4x^2\right)\, dx$

(d) $\pi \int_0^2 \left(\left(3 - \frac{y}{2}\right)^2 - \left(3 - \sqrt{\frac{y}{2}}\right)^2\right) dy$

(e) $\pi \int_0^2 \left(\left(3 - \sqrt{\frac{y}{2}}\right)^2 - \left(3 - \frac{y}{2}\right)^2\right) dx$

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6. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are rectangles with a height twice as big as the base, the volume of S is

(a) 0.5 (b) 0.667 (c) 1.140 (d) 2 (e) 6.362

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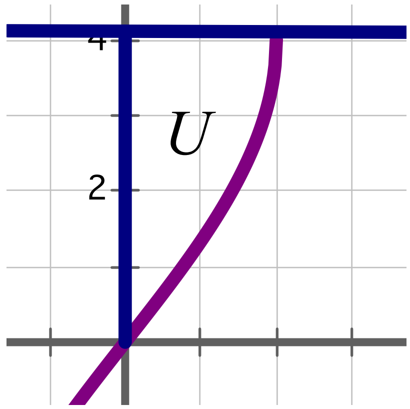
7. Let U be the region bounded by the graph of $y=\frac{8}{\pi}\sin^{-1}\left(\frac{1}{2}x\right)$, the graph of $y=4$, and $x=0$.



The volume of this solid with base U and cross sections which are squares perpendicular to the y-axis would be found by

(a) $\int_0^2 \left(\left(\frac{8}{\pi}\sin^{-1}\left(\frac{1}{2}x\right)\right)^2\right)dx$

(b) $\int_0^2 \left((4)^2-\left(\frac{8}{\pi}\sin^{-1}\left(\frac{1}{2}x\right)\right)^2\right)dx$

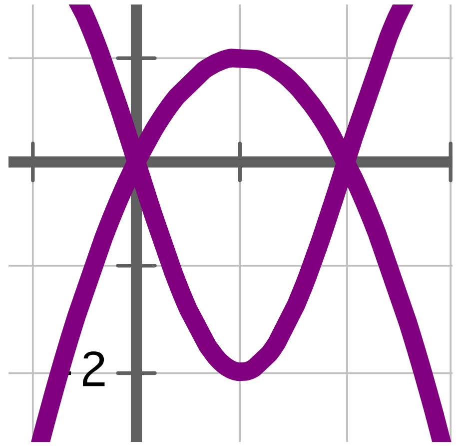
(c) $\int_0^4 \left(\left(\frac{\pi}{4}\sin(y)\right)^2\right)dy$

(d) $\int_0^4 \left(\left(2\sin\left(\frac{\pi}{8}y\right)\right)^2\right)dy$

AP Calculus AB ’23-24
Dr Quattrin
Volume FRQ Test Form H
Calculator Allowed
30 minutes

Name:______________________________

1. Let S be the region shown above bounded above by the graph of $y = -2\sin\left(\frac{\pi}{2}x\right)$ and below the graph of $y = 2x - x^2$.



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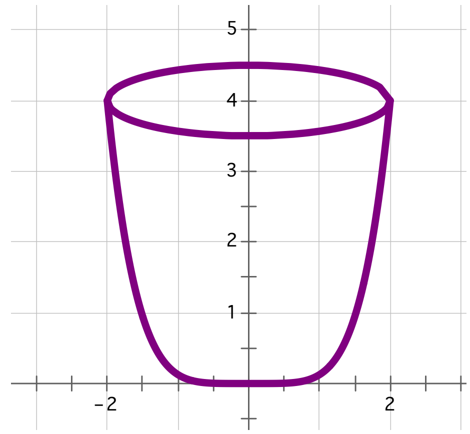
a) Find the area of region *S*. Show the set-up and the antiderivative.

(b) Find the volume of the solid generated when S is revolved about the line $y = 2$. Show the set-up, but use the calculator to solve.

(c) Let the base of the solid be the region S. Find the volume of the solid where the cross-sections perpendicular to the x-axis are rectangles that are three times as tall as they are wide. Show the set-up, but use the calculator to solve.

2. The inner surface of an ancient crucible, used to smelt iron ore, is found to have an inner surface that conforms to the solid formed by the region bounded by $y = \frac{1}{8}x^5$, $x = 0$, and $y = 4$ being revolved about the y-axis. The width and height are measured in feet.



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a) Find the volume of the solid.

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b) Find an equation for the volume $V(h)$ of molten metal in the crucible, where h is the depth of the liquid.

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c) Assume the same size and shape vessel is used to store water, and that the water evaporates at a constant rate such that $\frac{dh}{dt} = -\frac{1}{5} \text{ft}/\text{hr}$. How fast is the volume changing when $h = 3$.

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d) Show that the rate of change of the volume of the container is directly proportional to the exposed surface area of the liquid.
