AB Calculus ’22-23
Volume Test Form I
Calculator Allowed

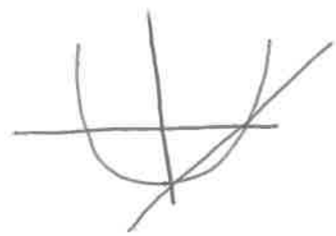
Name: Solution Key

Score: ______________

1. The area of the region enclosed by $y = x^2 - 4$ and $y = x - 4$ is given by

(a) $\int_0^1 (x - x^2)\,dx$ (b) $\int_0^1 (x^2 - x)\,dx$ (c) $\int_0^{2} (x - x^2)\,dx$

(d) $\int_0^{2} (x^2 - x)\,dx$ (e) $\int_0^{4} (x^2 - x)\,dx$



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2. Which of the following integrals gives the length of the graph $y = \tan x$ between $x=a$ to $x=b$ if $0 < a < b < \frac{\pi}{2}$?

$\frac{dy}{dx} = \sec^2 x$

(a) $\int_a^b \sqrt{x^2 + \tan^2 x}\,dx$ (b) $\int_a^b \sqrt{x + \tan x}\,dx$

(c) $\int_a^b \sqrt{1 + \sec^2 x}\,dx$ (d) $\int_a^b \sqrt{1 + \tan^2 x}\,dx$

(e) $\int_a^b \sqrt{1 + \sec^4 x}\,dx$

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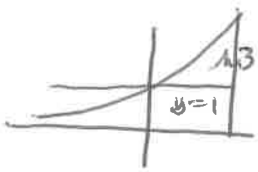
3. Let R be the region in the first quadrant bounded by $y = e^{x/2}$, $y = 1$ and $x = \ln 3$. What is the volume of the solid generated when R is rotated about the x-axis?

(a) 2.80 (b) 2.83 (c) 2.86 (d) 2.89 (e) 2.92



$V = \pi \int_0^{\ln 3} (e^{x/2})^2 - 1^2 \, dx$

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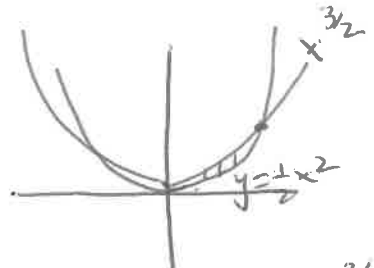
4. Consider the solid formed by revolving the region bounded by $y = \frac{1}{2}x^2$ and $y = \sqrt{x^3}$ about the x-axis. If the line $y = k$ divides the region such that the volumes on either side of $y = k$ are equal, which of the following set-ups would be used to solve for k?

AREA (a) $\int_0^k \left(x^{3/2} - \frac{1}{2}x^2\right) dx = \int_k^4 \left(x^{3/2} - \frac{1}{2}x^2\right) dx$

VOLUME (b) $\int_0^k \left(x^3 - \frac{1}{4}x^4\right) dx = \int_k^4 \left(x^3 - \frac{1}{4}x^4\right) dx$

~~(c)~~ $\int_0^k \left(y^{2/3} - \sqrt{2y}\right) dy = \int_k^8 \left(y^{2/3} - \sqrt{2y}\right) dy$

~~(d)~~ $\int_0^k \left(y^{4/3} - 2y\right) dy = \int_k^8 \left(y^{4/3} - 2y\right) dy$



$\frac{1}{2}x^2 = x^{3/2}$

$\frac{1}{2}x^2 - x^{3/2} = x^{3/2}\left(\frac{1}{2}x^{1/2} - 1\right)$

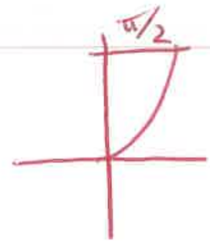
$\frac{1}{2}x^{1/2} \neq$

$x^{1/2} = 2$

$x = 4$

Siny = X

5. Let R be the region in the first quadrant **bounded by** $y = \sin^{-1}x$, the y-axis, and $y = \dfrac{\pi}{2}$. **Which of the** following integrals **gives the volume of the** solid **generated when** R **is rotated** about the y-axis?



(a) $\pi\int_0^{\pi/2} y^2\,dy$

~~(b)~~ $\pi\int_0^{1}(\sin^{-1}x)^2\,dx$

~~(c)~~ $\pi\int_0^{\pi/2}(\sin^{-1}x)^2\,dx$

(d) $\pi\int_0^{\pi/2}(\sin y)^2\,dy$

~~(e)~~ $\pi\int_0^{1}(\sin y)^2\,dy$

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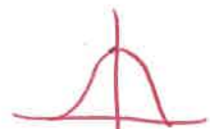
6. The base of a solid is the region enclosed by $y = \cos x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$. If each cross-section of the solid perpendicular to the x-axis is a square, the volume of the solid is

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi^2}{4}$ (c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi^2}{2}$ (e) 2



$V = \int_{-\pi/2}^{\pi/2} \cos^2 x = 2\left[\frac{1}{2}x + \frac{1}{4}\sin 2x\right]_0^{\pi/2}$

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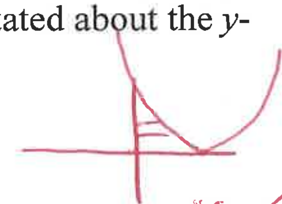
7. Let R be the region in the first quadrant bounded by $y = (x-3)^2$, $y = 0$, and $x = 0$. **What is the volume** of the solid generated when R is rotated about the y-axis?



~~(a)~~ $\pi\int_0^{3}(x-3)^2\,dx$

~~(b)~~ $\pi\int_0^{3}(x-3)^4\,dx$

(c) $\pi\int_0^{9}(3\mp\sqrt{y})\,dy$

(d) $\pi\int_0^{3}(3\mp\sqrt{y})^2\,dy$

(e) $\pi\int_0^{9}(3\mp\sqrt{y})^2\,dy$

$x = \pm\sqrt{3 \pm y}$

$x = 3 - \sqrt{y}$

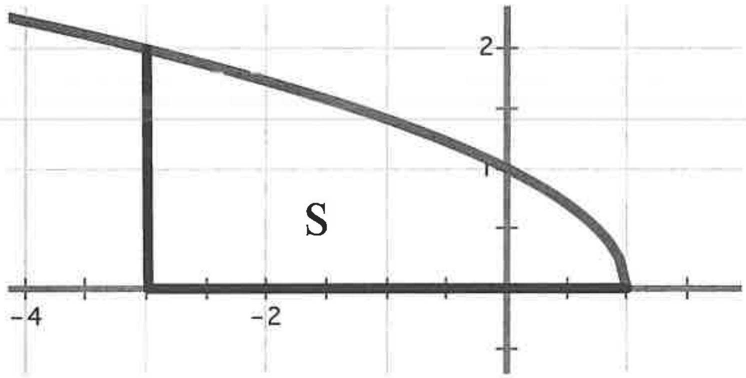
AP Calculus AB '22-23

Volume FRQ Test Form I

Calculator Allowed

Name: Solution Key

1. Let R be the region bounded by $y = \sqrt{1-x}$, $x = -3$, and $y = 0$.



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a) Find the area of region S. Show the anti-differentiation step.

$$A = -\int_{-3}^{1} \sqrt{1-x}\,(-dx)$$

$$= -\left.\frac{(1-x)^{3/2}}{3/2}\right]_{-3}^{1}$$

$$= -\frac{2}{3}\left[0-8\right] = \frac{16}{3}$$

b) Find the volume of the solid generated when S is rotated about the x – axis. Show the anti-differentiation steps.

4

$$V = \pi \int_{-3}^{1} \left(\sqrt{1-x}\right)^2 dx$$

$$= \pi \int_{-3}^{1} (1-x)\, dx$$

$$= \pi \left[x - \frac{x^2}{2}\right]_{-3}^{1}$$

$$= \pi \left[\left(1-\frac{1}{2}\right) - \left(-3-\frac{9}{2}\right)\right]$$

$$= 8\pi$$

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c) Find the volume of the solid with base S and cross-sections with are squares perpendicular to the y-axis. Show the set-up, but solve by calculator.

3

$$\boxed{s^2}$$

$$s = (y^2+1) - (-3)$$

$$= y^2 + 4$$

$y = \sqrt{1-x}$

$y^2 + 1 = x$

$$V = \int s^2\, dy$$

$$= \int_0^2 (y^2+4)^2\, dy$$

$$= 59.733$$

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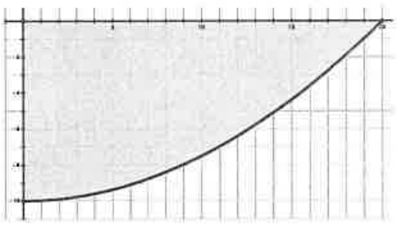
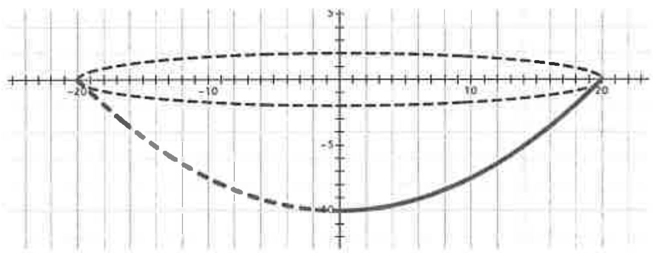
2. The Evaporation Pit Problem II

After the water from the Colorado River passes through the Yuma Desalting Plant to have its excess salts removed, the slurry (the densely salted byproduct water) is sent to evaporation pits in the desert where the salt sinks to the bottom and the water evaporates away. Assume the pit is in a shape formed by rotating the curve $y = \frac{x^2}{40} - 10$ on $x \in [0, \ 20]$ about the y-axis.





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a) How much can this pit hold?

$$V = \pi \int_{-10}^{0} \left(\sqrt{40(y+10)}\right)^2 dy = 40\pi \int_{-10}^{0} y + 10$$

$$= 40\pi \left[\frac{y^2}{2} + 10y\right]_{-10}^{0}$$

$$= 2000\pi$$

$$= 6283.183 \text{ FT}^3$$

1

b) Write an equation that would give the volume of the slurry in the pit at any depth *h,* where is the number of feet below the rim of the pit.

$$V = \pi \int_{-10}^{h} 40(y+10)\, dy$$

2

c) After a certain amount of time exposed to the desert heat, all the water evaporates, leaving just the salt. At that point, the level of salt is 3.9 feet below the rim of the pit. Use the equation in (b) to find the volume of the salt after all the water has evaporated.

$$V = \pi \int_{-10}^{-3.9} 40(y+10)\,dy$$

$$= 2337.973 \text{ FT}^3$$

3

d) The water in the slurry evaporates at a rate of $0.1 \, {}^{ft}\!/_{day}$. How fast is the volume of the slurry changing when $h = -1.9\,ft$?

$$\frac{d}{dt}\left\{V = \pi \int_{-10}^{h} 40(y+10)\,dy\right\}$$

$$\frac{dV}{dt} = 40\pi(h+10)\frac{dh}{dt} =$$

$$\frac{dV}{dt} = 40\pi(-1.9+10)(-.1) = -101.788 \frac{\text{FT}^3}{\text{DAY}}$$