AP Calculus BC
Practice Final 2008-9
Section I – Part A
No Calculator Allowed

name Solution Key

score __________

1.

$$\int_2^3 \frac{x}{x-1}\,dx =$$

$$\int_2^3 1 + \frac{1}{x-1}$$

$$x-1 \overline{)\,x\,}^{\;1} \quad -(x-1) \quad +1$$

$$= x + \ln(x-1)\Big]_2^3$$

$$= 1 + \ln 2$$

(A) $1 - \ln 2$

(B) $\ln 2$

[✓ (C) $1 + \ln 2$]

(D) $2 + \ln 2$

(E) $5 + \ln 2$

2.

The radius of convergence of the series $\frac{x}{4} + \frac{x^2}{4^2} + \frac{x^3}{4^3} + \ldots + \frac{x^n}{4^n} + \ldots$

(A) ∞

(B) 0

(C) 1

(D) 2

[✓ (E) 4]

$$\lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{4^{n+1}}}{x^n/4^n} \right| = \lim_{n\to\infty} \frac{1}{4}|x| < 1$$

3.

The position vector of a particle moving in the xy-plane at time t is given by $\mathbf{p} = (3t^2 - 4t)\,\mathbf{i} + (t^2 + 2t)\,\mathbf{j}$. The speed of the particle at $t = 2$ is

(A) 2 units/sec

(B) $2\sqrt{10}$ units/sec

[✓ (C) 10 units/sec]

(D) 14 units/sec

(E) 20 units/sec

$$v = (6t - 4)\hat{\imath} + (2t+2)\vec{\jmath}$$

$$s = \sqrt{8^2 + 6^2}$$

$$= 10$$

4.

If $f(x) = \ln(x^2 - e^{2x})$, then $f'(1) =$

(A) 0

(B) 1

[✓ (C)] 2

(D) e

(E) undefined

5.

The length of the curve $y = \int_0^x \sqrt{\frac{u}{3}}\, du$ from $x = 0$ to $x = 9$ is

$\frac{dy}{du} = \sqrt{u/3}$

(A) 16

[✓ (B)] 14

(C) $10\frac{1}{3}$

(D) $9\sqrt{3}$

(E) $4\frac{2}{3}$

$$L = \int_0^9 \sqrt{1 + \frac{u}{3}}\, du$$

$$= 3\int_1^4 \sqrt{y}\, dy$$

$$= 3 \left.\frac{y^{3/2}}{3/2}\right]_1^4 = 16 - 2 = 1$$

6.

If a population of wolves grows according to the logistic equation

$$\frac{dN}{dt} = 0.05N - 0.0005N^2 = .05n\left(1 - \frac{n}{100}\right)$$

where N is the number of wolves and t is measured in years, then $\lim_{t \to \infty} N(t) =$

(A) 50 (B) 75 [✓ (C)] 100 (D) 150 (E) 200

7.

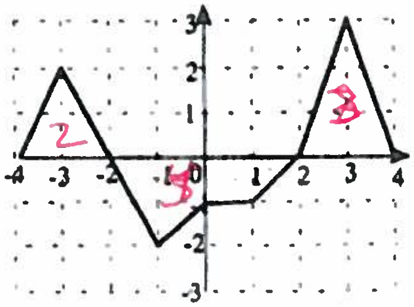
The slope of the line tangent to the graph of $y = 2[\text{Arc}\tan\sqrt{x}]^2$ at the point $\left(1, \frac{\pi^2}{8}\right)$ is

(A) 0 (B) $\frac{\pi}{8}$ [✓ (C)] $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$ (E) π

$$\frac{dy}{dx} = 4(\tan^{-1}\sqrt{x})\frac{1}{1+x}\,\frac{1}{2}x^{-1/2}\Big|_1 = \frac{\pi}{4}$$

8.

The graph of the function f on the interval $[-4,4]$ is shown at the right.



$$\int_{-4}^{4}|f(x)|\,dx =$$

~~(A) 1~~

(B) 2

(C) 5

(D) 8

[✓ (E) 9]

9.

Both x and y are functions of a third variable t and $y^2+x^2+y=10$. If $\frac{dx}{dt}=-5$ when $x=2$ and $y=2$, then $\frac{dy}{dt}=$

$$2y\frac{dy}{dt}+2x\frac{dx}{dt}+\frac{dy}{dt}=0$$

$$4\frac{dy}{dt}+(-20)+\frac{dy}{dt}=0$$

$$5\frac{dy}{dt}=20$$

(A) -1

(B) 1

(C) 2

(D) 3

[✓ (E) 4]

10.

The substitution $u=\ln x$ transforms the definite integral $\int_1^e \frac{1-\ln x}{x^2}\,dx$ into

$e^u=x$

$$\int_0^1 \frac{1-u}{e^{2u}}$$

(A) $\int_0^1 (1-u)\,du$

~~(B)~~ $\int_0^e (1-u)\,du$

(C) $\int_0^1 \frac{1-u}{e^u}\,du$

[✓ (D) $\int_0^1 \frac{1-u}{e^{2u}}\,du$]

~~(E)~~ $\int_0^e \frac{1-u}{e^u}\,du$

11.

The number of cells of a certain type of bacteria increases continuously at a rate equal to two more than three times the number of bacteria present. If there are 10 present at the start and 42 present t hours later, the value of t is

(A) $3 \ln 4$

(B) $\ln 4$

(C) $\frac{1}{2} \ln 4$

[✓ (D)] $\frac{1}{3} \ln 4$

(E) $\frac{1}{4} \ln 4$

12.

If $\frac{dy}{dx} = x \cdot \sec y$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and $y = 0$ when $x = \sqrt{2}$, then when $x = 1$ the value of y is

[✓ (A)] $-\frac{\pi}{6}$

(B) 0

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

(E) $\frac{\pi}{2}$

13.

Which of the following are asymptotes of $y + xy - 2x = 0$?

I. $x = -1$ II. $x = 1$ III. $y = 2$

(A) I only

(B) II only

(C) III only

[✓ (D)] I and III only

(E) II and III only

14.

The curve passing through (1, 0) satisfies the differential equation $\frac{dy}{dx} = 4x + y$. An approximation to $y(2)$ using Euler's Method with two equal steps is

(A) 0 (B) 2 (C) 4 [✓ (D) 6] (E) 8

15.

The function $f(x) = \begin{cases} 4 - x^2 & \text{for } x \leq 1 \\ mx + b & \text{for } x > 1 \end{cases}$ is continuous and differentiable for all real numbers. The values of *m* and *b* are

(A) $m = 2, b = 1$
(B) $m = 2, b = 5$
(C) $m = -2, b = 1$
[✓ (D) $m = -2, b = 5$]
(E) none of these

16.

$$\int \frac{8}{(x-1)(x+3)}\,dx =$$

(A) $2 \ln\left|\frac{x+3}{x-1}\right| + C$

[✓ (B) $2 \ln\left|\frac{x-1}{x+3}\right| + C$]

(C) $2 \ln|(x+3)(x-1)| + C$

(D) $2 \ln\left|\frac{1}{(x+3)(x-1)}\right| + C$

(E) $8 \ln\left|\frac{1}{(x+3)(x-1)}\right| + C$

17.

If $f(x) = \dfrac{x-k}{x+k}$ and $k \neq 0$, then $f''(0) =$

[✓ (A) $-\frac{4}{k^2}$] (B) $-\frac{2}{k}$ (C) 0 (D) $\frac{2}{k}$ (E) $\frac{4}{k^2}$

18.

The base of a solid is a right triangle whose perpendicular sides have lengths 6 and 4. Each plane section of the solid perpendicular to the side of length 6 is a semicircle whose diameter lies in the plane of the triangle. The volume of the solid is

(A) 2π units^3

[✓ (B) 4π units^3]

(C) 8π units^3

(D) 16π units^3

(E) 24π units^3

19.

$$\lim_{h \to 0} \frac{(1+h)^3 - 1}{h} =$$

(A) undefined

[✓ (B) 3]

(C) 2

(D) 1

(E) 0

20.

Suppose a function f is defined so that it has derivatives $f'(x) = x^2(1-x)$ and $f''(x) = x(2-3x)$. Over which interval is the graph of f both increasing and concave up?

(A) $x < 0$ [✓ (B) $0 < x < \frac{2}{3}$] (C) $\frac{2}{3} < x < 1$ (D) $x > 1$ (E) none of these

21.

The average value of the function $f(x) = \sqrt[3]{x^2}$ on the interval $[0,8]$ is

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{4}$ [✓ (D) $\frac{12}{5}$] (E) $\frac{17}{6}$

22.

Let $f(x) = \begin{cases} 2 & \text{if } x < 0 \\ x+2 & \text{if } x \geq 0 \end{cases}$ and let $F(x) = \int_{-2}^{x} f(t)\,dt$. Which of the following statements are true?

I. $F(1) = 6.5$ II. $F'(1) = 3$ III. $F''(1) = 1$

(A) I only (B) II only (C) I and II only (D) II and III only [✓ (E) I, II, III]

23.

Which of the following three improper integrals converge?

I. $\int_{1}^{\infty} \frac{1}{x^3}\,dx$ II. $\int_{0}^{1} \frac{1}{\sqrt{x}}\,dx$ III. $\int_{0}^{1} \frac{1}{x^3}\,dx$

(A) II only [✓ (B) I and II only] (C) I and III only (D) II and III only (E) I, II, III

24.

The acceleration of a particle moving along the x-axis at time $t > 0$ is given by $a(t) = \frac{1}{t^2}$.

When $t = 1$ second, the particle is at $x = 2$ and moving with velocity -1 unit per second. The position when $t = e$ seconds is

(A) $x = -2$

(B) $x = -1$

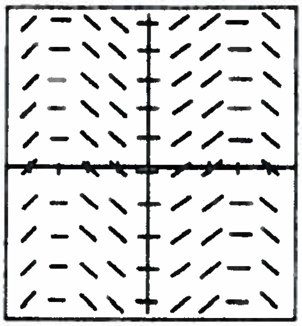
(C) $x = 0$

[✓ (D) $x = 1$]

(E) $x = 2$

25.

The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



(A) $\frac{dy}{dx} = \tan x \cdot \sec x$

[✓ (B) $\frac{dy}{dx} = \sin x$]

(C) $\frac{dy}{dx} = \sec^2 x$

(D) $\frac{dy}{dx} = \ln x$

(E) $\frac{dy}{dx} = e^{2x}$

26.

The area enclosed by the two curves $y = x^2 - 4$ and $y = x - 4$ is given by

[✓ (A) $\int_0^1 (x - x^2)\,dx$]

(B) $\int_0^1 (x^2 - x)\,dx$

(C) $\int_0^2 (x - x^2)\,dx$

(D) $\int_0^2 (x^2 - x)\,dx$

(E) $\int_0^4 (x^2 - x)\,dx$

27.

The coefficient of x^3 in the Taylor series for e^{2x} at $x = 0$ is

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

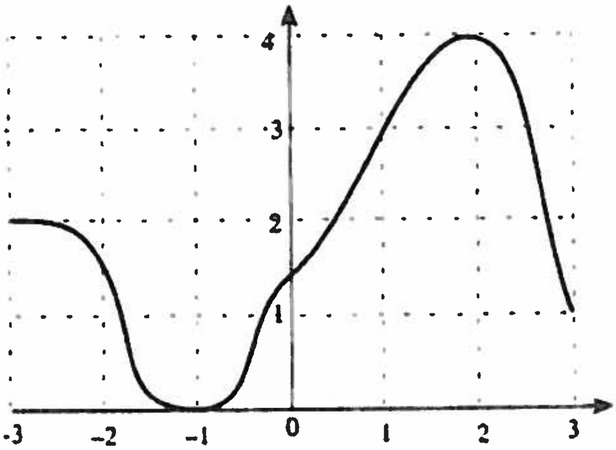
[✓ (D) $\frac{4}{3}$]

(E) $\frac{8}{3}$

28.

The graph of f is shown at the right.

Approximate $\int_{-3}^{3} f(x)\,dx$ using the Trapezoid Rule with 3 equal subdivisions.



(A) $2\frac{1}{4}$

(B) $4\frac{1}{2}$

[✓ (C) 9]

(D) 18

(E) 36 s

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76.

Which of the following is (are) true about a particle that starts at $t = 0$ and moves along a number line if its position at time t is given by $s(t) = (t-2)^3(t-6)$?

I. The particle is moving to the right for $t > 5$.
II. The particle is at rest at $t = 2$ and $t = 6$.
III. The particle changes direction at $t = 2$.

(A) I only (B) II only (C) III only [✓ (D) I and III only] (E) none

77.

The approximate *average* rate of change of the function $f(x) = \int_0^x \sin(t^2)\,dt$ over the interval $[1, 3]$ is

(A) 0.19 [✓ (B) 0.23] (C) 0.27 (D) 0.31 (E) 0.35

78.

$$\int \frac{1}{\sqrt{x}\,(1-\sqrt{x})}\,dx =$$

(A) $\frac{1}{2}\ln\left|1-\sqrt{x}\right| + C$

[✓ (B) $2\ln\left|1-\sqrt{x}\right| + C$]

(C) $4\sqrt{1-\sqrt{x}} + C$

(D) $-2\ln\left|1-\sqrt{x}\right| + C$

(E) none of these

79.

Let R be the region in the first quadrant that is enclosed by the graph of $f(x) = \ln(x+1)$, the x-axis and the line $x = e$. What is the volume of the solid generated when R is rotated about the line $y = -1$?

(A) 5.037 (B) 6.545 (C) 10.073 (D) 20.146 [✓ (E) 28.686]

80.

$$\lim_{h\to 0}\frac{\int_1^{1+h}\sqrt{x^3+8}\ dx}{h} \quad \text{is}$$

(A) 0

[✓ (B) 1]

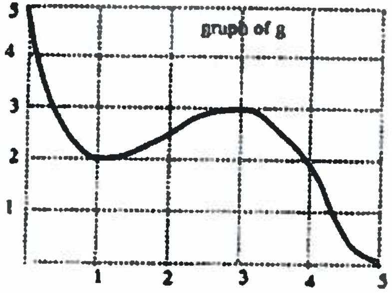
(C) 3

(D) $2\sqrt{2}$

(E) nonexistent

81.

A graph of the function g is shown in the figure. If the function h is defined by $h(x) = g(x^2)$, use the graph to estimate $h'(2)$.



(A) –8 [✓ (B) –4] (C) –2 (D) 2 (E) 4

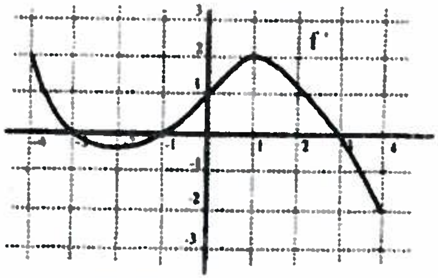
82.

$$\int_0^\infty xe^{-x^2}\,dx \quad \text{is}$$

(A) –1 (B) 0 [✓ (C) 1] (D) $\frac{1}{4}$ (E) $\frac{1}{2}$

83.

The graph of the **derivative** of a function f is shown to the right. Which of the following are true about the original function f?



The derivative of f

I. f is increasing on the interval $(-2, 1)$.
II. f is continuous at $x = 0$.
III. f has an inflection point at $x = -2$.

(A) I only (B) II only (C) III only (D) II and III only [✓ (E) I, II and III]

84.
A curve is defined parametrically by $x = e^t$ and $y = 2e^{-t}$. An equation of the tangent line to the curve at $t = \ln 2$ is

[✓ (A) $x - 2y + 3 = 0$]
(B) $x + 2y - 4 = 0$
(C) $x + 2y - 5 = 0$
(D) $x - 2y - 4 = 0$
(E) $2x + y - 5 = 0$

85.

If $x^2 - y^2 = 25$ then $\frac{d^2y}{dx^2} =$

[✓ (A) $-\frac{x}{y}$] (B) $\frac{5}{y^2}$ (C) $-\frac{x^2}{y^3}$ (D) $-\frac{25}{y^3}$ (E) $\frac{4}{y^3}$

86.
Which of the following series are convergent?

I. $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} + \dots$

II. $1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{(-1)^n}{n} + \dots$

III. $2 + 1 + \frac{8}{9} + \dots + \frac{2^n}{n^2} + \dots$

(A) I only
[✓ (B) III only]
(C) I and II only
(D) II and III only
(E) I, II and III

87.

If $\lim\limits_{h\to 0}\dfrac{g(x+h)-g(x)}{h}=\dfrac{x^2+1}{x^2}$, then $g(x)$ could be equal to

(A) x^{-3} (B) $-2x^{-3}$ (C) $\dfrac{x^2-1}{x}$ (D) $x-x^2$ [✓ (E) $1+x^{-2}$]

88.

Two particles move along the x-axis and their positions at time $0\le t\le 2\pi$ are given by $x_1=\cos t$ and $x_2=e^{(t-3)/2}-0.75$. For how many values of t do the two particles have the same velocity?

(A) 0

(B) 1

(C) 2

[✓ (D) 3]

(E) 4

89.

A rectangle with one side on the x-axis has its upper vertices on the graph of the parabola $y=4-x^2$. The maximum area of such a rectangle is

(A) 1.155 (B) 1.855 (C) 3.709 (D) 6.158 [✓ (E) 12.316]

90.

The radius of convergence of the series $x+\dfrac{2x^2}{2^2}+\dfrac{6x^3}{3^3}+\cdots+\dfrac{n!x^n}{n^n}+\cdots$ is

(A) ∞ (B) e^2 [✓ (C) e] (D) $\frac{e}{2}$ (E) 0

91.

When using the method of partial fractions to decompose $\dfrac{8x-4}{x^2+2x-3}$, one of the fractions obtained is

(A) $\dfrac{1}{x+3}$ [✓ (B) $\dfrac{7}{x-1}$] (C) $\dfrac{7}{x+3}$ (D) $\dfrac{1}{x-3}$ (E) $\dfrac{7}{x+1}$

92.

A particle moves on the xy-plane so that at time t, $0 \le t \le 5$, its acceleration vector is $\langle \sin t,\ e^{-t} \rangle$. If the particle is at rest when $t = 0$, what is the maximum speed it obtains?

(A) 2.10

(B) 2.22

(C) 2.34

[✓ (D) 2.46]

(E) 2.58