AP Calculus BC ’15-16

Fall Final
Part IIa

Calculator Required

Name:

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1. At an electric substation, readings of the rate at which power is being used were recorded every three hours over a 24-hour period. The readings are listed in the table below. The rate is in kilowatts per hour.

t	0	3	6	9	12	15	18	21	24
P(t)	1245	1268	1321	1316	1393	1369	1369	1451	1428

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a. Using midpoint Rheiman rectangles with four equal subintervals, approximate $\int_0^{24} P(t)\, dt$.

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b. Using the correct units, explain the meaning of $\int_0^{24} P(t)\, dt$ in terms of power usage.

c. Estimate how fast the rate of change of the power usage is increasing at t = 12. Show the computations and indicate units of measure.

d) Assume the function $F(t) = 1245 + 10te^{.25\cos t}$ is an accurate model of the rate of power usage at time t, where $F(t)$ is measured in kilowatts per hour and t is measured in hours. Use $F(t)$ to approximate the average rate of power usage during the 24-hour period. Indicate units of measure.

2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-.2t} & \text{for } 5 < t \end{cases}$$

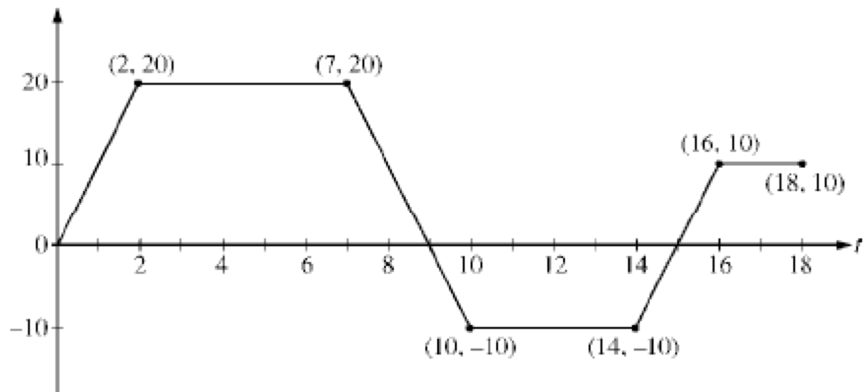
a. Is $r(t)$ continuous at $t = 5$? Show the work that leads to your answer.

b. Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

c. Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem

d. Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

3. A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B. For $0 \leq t \leq 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph below.



Graph of v

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a. At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

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b. At what time in the interval $0 \leq t \leq 18$ is the squirrel farthest from building A? How far from building A is the squirrel at this time?

c. Find the total distance the squirrel travels during the time interval $0 \leq t \leq 18$.

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d. Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

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No Calculator Allowed

Name:

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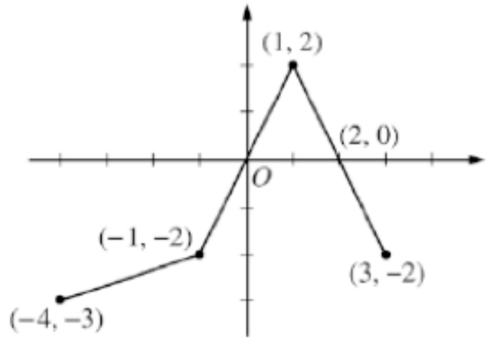
4. The graph of f below consists of three line segments:



Graph of f

Let $g(x)=\int_{-4}^{x} f(t)\,dt$.

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a. Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

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b. Find the x-coordinate for each point of inflection of $g(x)$ on $-4 < x < 3$. Explain your reasoning.

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c. Let $h(x)=\int_{x}^{3} f(t)\,dt$. Find all values of $x \in [-4,\ 3]$ for which $h(x)=0$.

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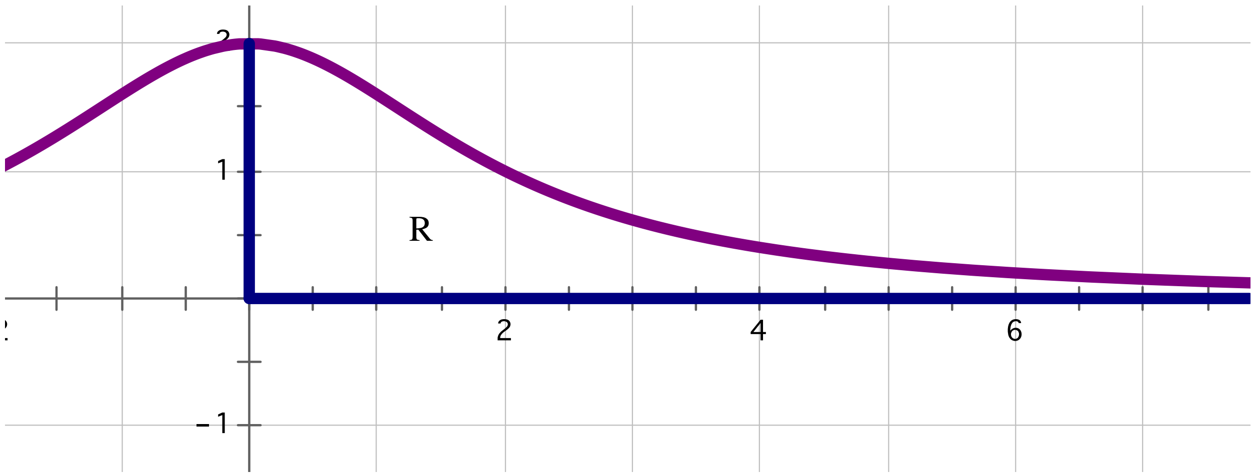
d. Find all intervals for which $h(x)$ is decreasing. Explain your reasoning.



5. The picture above is the graph of $f(x)=\dfrac{8}{x^2+4}$.

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a. Find the area of the unbounded region in Quadrant I between $f(x)=\dfrac{8}{x^2+4}$ and the x-axis.

b. The formula to find the volume of a solid formed by rotating a region about the x-axis is $V = \pi \int_a^b \left[f(x) \right]^2 dx$. Set up, but do not solve, the formula for the volume of the solid formed by revolving region R above about the x-axis.

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c. Show that $\sum_{n=0}^{\infty} a_n$ where $a_n = f(n)$ converges. Will the volume in part b converge? Why or why not?

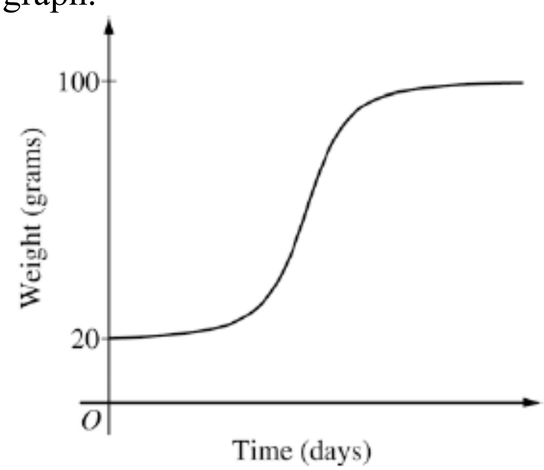
6. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B and use it to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

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